ITM 1010
Computer and Communication Technologies

Lecture #12
Part II Introduction to Communication Technologies:
Modulation
Modulation

- Modulation is the term used to describe how an information carrying signal \( m(t) \) is placed on a continuous “carrier” wave (c.w.) such as a radio wave or a light wave.

- Many different types of modulation have been invented: we shall introduce some of the basic concepts for
  - Amplitude Modulation (AM)
  - Frequency Modulation (FM)
Amplitude Modulation

- Amplitude modulation of a sinusoidal carrier wave, $A_c \cos (2\pi f_c t)$, by a base band signal $m(t)$ will produce a signal $s(t)$ which may be depicted as follows:

$$s(t) = A_c [1 + k_a m(t)] \cos (2\pi f_c t)$$

- $k_a$ is called the modulation sensitivity.

- Original message is simply the envelope of $s(t)$ if:
  - $|k_a m(t)| < 1$ so that $(1 + k_a m(t))$ is positive;
  - frequency of carrier is greater than base band bandwidth.
Over-modulated AM signal

- If the modulation sensitivity $k_a$ is too large $(1+k_a m(t))$ may become negative leading to an over-modulated AM signal.
- Envelope of over-modulated signal is distorted by amplitude sign change which occurs whenever $(1+k_a m(t)) < 0$.

$(1+k_a m(t)) > 0$ for all $t$

normal AM signal

$(1+k_a m(t)) < 0$ between 2 arrows

Over-modulated AM signal
AM is wasteful of bandwidth because:
- Carrier frequency contains power which must be sent by the transmitter but there is no information in this signal
- Identical information is contained in the upper and lower sideband
- If base band bandwidth is w, the AM signal occupies a bandwidth of 2w (from $f_c - w$ to $f_c + w$)

AM is widely used for broadcast radio because of simplicity in receiver design (only an envelope detector is needed)
Double sideband and single sideband

- More efficient usage of bandwidth and transmitter power can be achieved by filtering out the un-needed frequency components from the transmitted spectrum but these need more complicated receivers.

- Examples of such techniques include:
  - Double sideband suppressed carrier (DSB-SC) which removes the carrier frequency from the transmitted signal
  - Single sideband (SSB) transmission which filters out both the carrier and either the lower sideband or upper sideband

- SSB and DSB receivers must multiply the received signal with a local oscillator to recover the base band signal.
Double sideband modulation

- Double sideband suppressed carrier modulation may be represented by
  \[ s(t) = m(t)A_c \cos(2\pi f_c t) \]

- \( s(t) \) undergoes a phase jump whenever the sign of \( m(t) \) changes: envelope of DSB-SC is NOT the same as \( m(t) \)

- In DSB-SC modulation the base band signal \( m(t) \) is simply translated in frequency and no carrier frequency is present.
Frequency Modulation (FM)

- FM involves varying the instantaneous frequency of the carrier wave, $f_i$, in proportion to amplitude of base band signal $m(t)$:
  \[ f_i = f_c + k_f m(t) \]

- $k_f$ is the modulation sensitivity (if $m(t)$ is in volts, $k_f$ has units of Hz/V).

- FM signal has constant amplitude and thus is less prone to noise than AM (noise usually only appears in the amplitude of a received radio signal).
Mathematical description of FM

- An easy mistake is to describe an FM signal $s(t)$ by

$$s(t) = A_c \cos(2\pi f_i t) \quad \text{WRONG}$$

- Need to consider the effect of previous $f_i$ which will contribute to the phase $\theta_i$ of the carrier:

$$s(t) = A_c \cos(\theta_i)$$

The phase change in a short time interval $dt$ is:

$$d\theta_i = 2\pi f_i dt = \omega_i dt$$

Hence

$$\theta_i = \int_{0}^{t} \omega_i dt = \omega_c t + 2\pi f \int_{0}^{t} m(t) dt$$

- The general expression for FM is thus:

$$s(t) = A_c \cos \left( \omega_c t + 2\pi f \int_{0}^{t} m(t) dt \right)$$
Analysis of FM

- Suppose the base band signal \( m(t) \) is represented by a sinusoid with a frequency equal to its highest frequency Fourier component \( f_m \) and an amplitude \( V_m \) which equals the maximum voltage in \( m(t) \):
  \[
m(t) = V_m \cos \omega_m t
  \]

- The FM signal may then be written as:
  \[
s(t) = A_c \cos \left( \omega_c t + 2\pi k f \int_0^t V_m \cos(\omega_m t) \, dt \right)
  \]
  \[
s(t) = A_c \cos \left( \omega_c t + \frac{2\pi k f V_m}{\omega_m} \sin(\omega_m t) \right) = A_c \cos \left( \omega_c t + \frac{k f V_m}{f_m} \sin(\omega_m t) \right)
  \]

- We define the modulation index, \( \beta \), as the maximum FM frequency deviation divided by the base band bandwidth:
  \[
  \beta = \frac{k f V_m}{f_m}
  \]

  Hence
  \[
s(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))
  \]
Bessel function expansion of FM

It can be shown that the FM signal may be expressed as a sum of Bessel functions of the first kind, $J_n(\beta)$:

$$s(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c t + n\omega_m t)$$

Since for negative $n$, $(-1)^n J_{-n}(\beta) = J_n(\beta)$, the spectrum of the FM signal $s(t)$ contains an infinite set of side frequencies located symmetrically about the carrier frequency.
Carson’s rule for bandwidth of FM signal

- Bandwidth depends on the modulation index

- Carson defined the bandwidth as that containing 98% of the signal power and arrived at the following approximation for the bandwidth $B$ of an FM signal as a function of modulation index $\beta$ and bandwidth $f_m$ of base band message:

  $$ B = 2(1 + \beta)f_m $$

- Carson’s rule gives a slight underestimate of the actual bandwidth needed for an FM signal

- If modulation index $\beta$ is small (much less than 1) the bandwidth needed for narrowband FM is $2f_m$, the same as for AM

- In practice broadcast FM radio employs a maximum frequency deviation of 75KHz, and requires a bandwidth of 200KHz (225KHz in UK). Broadcast FM use a carrier frequency between 88MHz to 108MHz, and provides a base band bandwidth of 15KHz.
Summary

- A signal’s bandwidth is the range of positive frequencies which contain most of the signal’s power.
- Signals may be transmitted by modulating a carrier wave, such as a continuous EM wave (radio wave or light signal).
- AM uses amplitude of carrier wave to carry the base band signal.
  - Required bandwidth is 2w where w is base band bandwidth
  - Variants on AM include DSB-SC (to avoid wasting transmitter power) and SSB which reduces bandwidth needed from 2w to w.
- FM uses the frequency of the carrier wave to carry the base-band signal
  - Required bandwidth of FM is NOT just the maximum frequency deviation of the modulation
  - FM theoretically generates an infinitely wide spectrum (but the power in higher order Bessel components falls rapidly).
  - Bandwidth needed by FM is at least that of AM and depends on the modulation index (defined as maximum frequency deviation divided by the bandwidth of the base band signal).