3. Example 2.7 An instrumentation amplifier

The simple version of differential Amp has two major drawbacks:

(1) low input resistance; and (2) its gain is not easily varied.

A much superior circuit is shown in Fig.(a). To have a good understand, we need

(1) determine $v_O$ as a function of $v_1$ and $v_2$;
(2) determine the differential gain;
(3) suggest a way to make a variable gain;
(4) find $R_{in}$; and
(5) design a circuit to provide a gain that can be varied over the range 2 to 1000 using a 100-kΩ variable resistance.
Solutions:

step 1: \( \because \) the virtual short circuits at input of \( A_1 \) and \( A_2 \), \( \because \) \( i = (v_1 - v_2)/R_i \) flowing through \( R_1 \) and \( R_2 \)

therefore, \( (v_1 - v_2) = \frac{R_1}{R_1 + 2R_2}(v_{o1} - v_{o2}) \)

step 2: \( \because \) Op Amp \( A_3 \) is a differential amp configuration

\( \therefore \) \( v_O = -\frac{R_4}{R_3}(v_{o1} - v_{o2}) \)

combining step 1 and step 2 results in the differential voltage gain

\[
A_d = \frac{v_O}{(v_2 - v_1)} = \left(1 + \frac{2R_2}{R_1} \right) \frac{R_4}{R_3}
\]

step 3: It can be found that the gain can be varied by adjusting the single resistor \( R_1 \), any other arrangement involves varying two resistors simultaneously.

step 4: Since the input-stage op amps are connected in the noninverting configuration, \( R_{in} = \infty \)
Now we turn to the design problem. Engineering design is an art of trade-off. In this case, it is preferable to obtain all the required gain in the first stage, leaving the second stage to perform the rejection of the common-mode signal.

Adopting this approach, we select all the second-stage resistors to be equal to a practically convenient value, say 10 kΩ. The problem then reduces to designing the first stage to realize a gain adjustable over the range 2 to 1000.

Implementing $R_1$ as the series combination of a fixed resistor $R_{1f}$ and the variable resistor $R_{1v}$ obtained using the 100 kΩ pot, we have

$$1 + \frac{2R_2}{R_{1f} + R_{1v}} = 2 \text{ to 1000}$$

Thus, $1 + \frac{2R_2}{R_{1f}} = 1000$ and $1 + \frac{2R_2}{R_{1f} + 100k\Omega} = 2$

These two equations yield $R_{1f} = 100.2$ kΩ and $R_2 = 50.05$ kΩ. Other practical values may be selected: for instance, $R_{1f} = 100$ kΩ and $R_2 = 49.9$ kΩ.
4. Voltage to Current Converter

\[ V_0 = \left(1 + \frac{R_2}{R_1}\right) V_x = 2 V_x \text{ if } R_1 = R_2 \]

\[ i_L = \frac{V_5(t)}{R} \text{ is independent on } Z_L \text{ if } R_1 = R_2 \]

**Proof:** For ideal Op-Amp with \( A \gg 0 \), there is a virtual short-circuit connecting \( V_- \) and \( V_+ \) terminals, i.e., \( V_+ \approx V_- = V_x \).

Since Op-Amp draws no input current, KCL at \( V_- \) and \( V_+ \) nodes yields:

\[ \frac{V_x}{R_1} + \frac{V_x - V_0}{R_2} = 0 \quad (1) \quad : \quad V_x \leftarrow \frac{R_1}{V_x} \frac{R_2}{R_2} \rightarrow V_0 \]

\[ \frac{V_x - V_5}{R} + \frac{V_x - V_0}{Z_L} = 0 \quad (2) \quad : \quad V_5 \rightarrow \frac{R}{Z_L} \leftarrow \frac{V_x}{R} \rightarrow V_0 \]

From (1), we get...
Applications of non-inverting configuration

1.3 Operational Amplifiers

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_o}{R_2} \]

i.e., \[ V_x = \frac{V_o}{R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \]

\[ V_x = \left( \frac{R_1}{R_1 + R_2} \right) V_o \]  \hspace{1cm} (3) \hspace{1cm} (also \ from \ voltage \ divider \ principle)

\[ V_o = \left( 1 + \frac{R_2}{R_1} \right) V_x \]  \hspace{1cm} (4)

From (2), we have

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} \right) - \frac{V_o}{R} = \frac{V_s}{R} \]

and using (4), we get

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} \right) - \frac{1}{R} \left( 1 + \frac{R_2}{R_1} \right) V_x = \frac{V_s}{R} \]

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} - \frac{1}{R} - \left( \frac{R_2}{R_1} \right) \frac{1}{R} \right) V_x = \frac{V_s}{R} \]

which reduces to

\[ i_L = V_x \left( \frac{1}{R_1} \right) = \frac{V_s}{R} \hspace{1cm} \text{if} \hspace{1cm} R_1 = R_2 \]
2.6 Non-ideal Operational Amplifier

• Various error terms arise in practical operational amplifiers due to non-ideal behavior.

• Some of the non-ideal characteristics include:
  
  – Finite open-loop gain that causes gain error
  – Nonzero output resistance
  – Finite input resistance
  – Finite Common-Mode Rejection Ratio (CMRR)
  – Common-mode input resistance
  – DC error sources
  – Output voltage and current limits
  – Frequency Response and bandwidth of Op Amp.
  – Large Signal Limitations – Slew Rate and Full-Power Bandwidth
2.6 Non-ideal Operational Amplifier

2.6.1 Finite Open-loop Gain

A real Op provides a large but non infinite gain. Op amps are commercially available with minimum open-loop gains of 80 dB to over 120 dB.

In the figure, the output voltage of the amplifier is given by

\[ v_o = A v_{id} = A(v_s - v_i) = A(v_s - \beta v_o) \]

\[ A_v = \frac{v_o}{v_s} = \frac{A}{1 + A \beta} \]

\( A \beta \) is called loop gain.

For \( A \beta \gg 1 \),

\[ A_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1} \]

the voltage \( v_{id} \) across the input is given by

\[ v_{id} = v_s - v_i = v_s - \beta v_o = v_s - \frac{A \beta}{1 + A \beta} v_s = \frac{v_s}{1 + A \beta} \]

is called **feedback** and is no longer zero, \( v_{id} \) is small for large \( A \beta \).
2.6  Non-ideal Operational Amplifier  

2.6.2  Gain Error

In many applications it is important to know, or to control by design, just how far the actual gain deviates from the ideal gain.

- Gain Error (GE) is defined by

\[ GE = \frac{1}{\beta} - \frac{A}{1 + A\beta} = \frac{1}{\beta(1 + A\beta)} \]

- Gain error is also expressed as a fractional or percentage error.

\[ FGE = \frac{1}{\beta} - \frac{A}{1 + A\beta} = \frac{1}{1 + A\beta} \approx \frac{1}{A\beta} \quad \text{For } A\beta \gg 1, \]
2.6 Non-ideal Operational Amplifier

2.6.3 Nonzero Output Resistance

Circuits for determining output resistances of the inverting and non-inverting amplifiers.

Output terminal is driven by test source $v_x$ and current $i_x$ (Thevnin equivalent) is calculated to determine output resistance (all independent sources are turned off). The equivalent circuit is same for both inverting and non-inverting amplifiers.

$$R_{out} = \frac{v_x}{i_x}$$
2.6.3 Nonzero Output Resistance (cont.)

Analysis:  \[ i_x = i_o + i_2, \quad i_o = \frac{v_x - A v_{id}}{R_o}, \quad i_2 = \frac{v_x}{R_1 + R_2} \]

Also, \( v_{id} = -v_1 \) and

\[ v_1 = \frac{R}{R_1 + R_2} v_x = \beta v_x \quad \therefore \quad \frac{1}{R_{out}} = \frac{i_x}{v_x} = \frac{1 + A \beta}{R_o} + \frac{1}{R_1 + R_2} \]

\[ \therefore R_{out} = \frac{R_o}{1 + A \beta} \left( R_1 + R_2 \right) \]

Thus, shunt feedback at output reduces \( R_{out} \).

Since, \( R_o/(1 + A \beta) << (R_1 + R_2) \),

\[ R_{out} \approx \frac{R_o}{1 + A \beta} \]