Spatial Sigma-Delta Modulation for Massive MIMO Downlink

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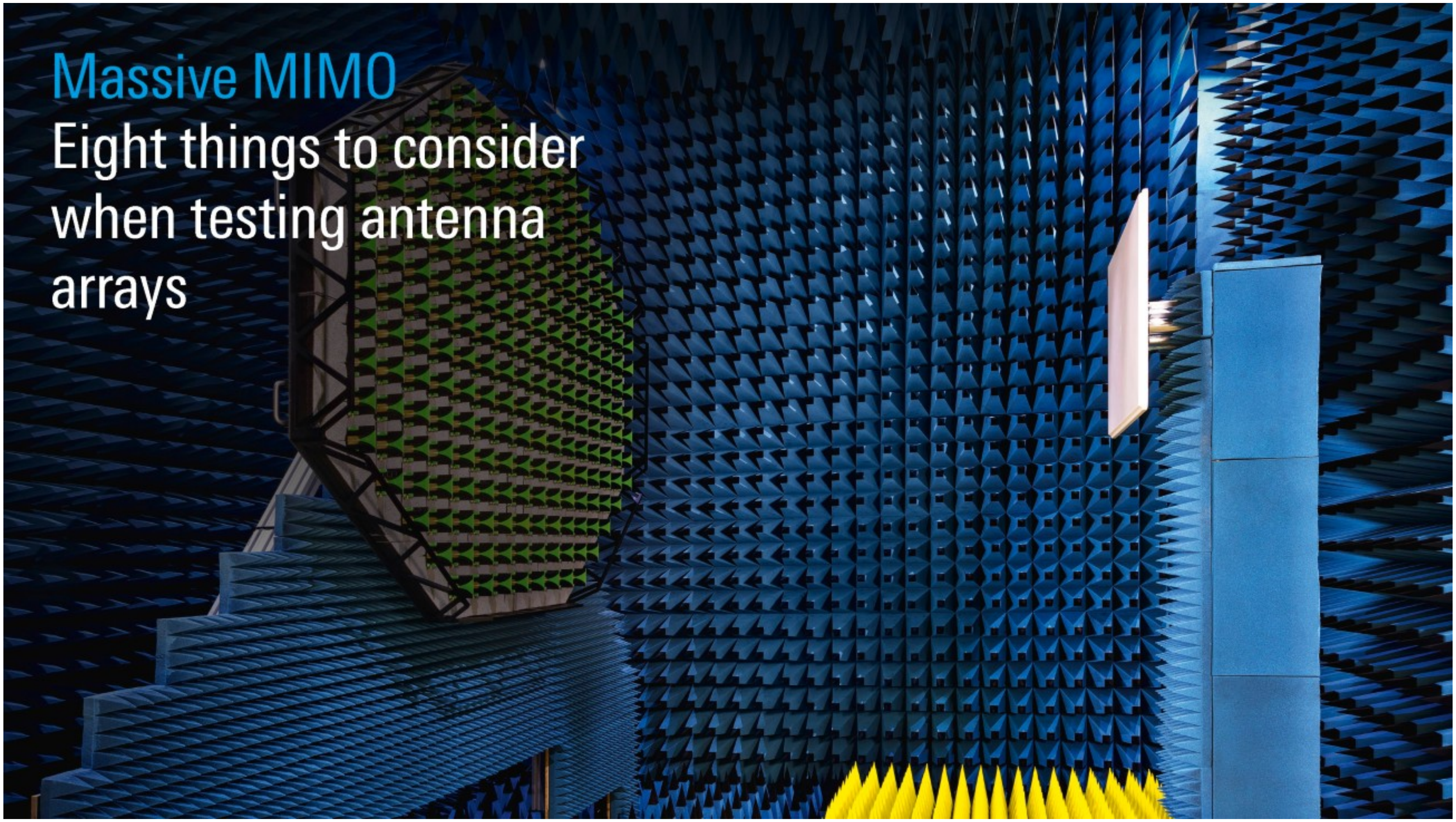
Massive MIMO

- promise many nice things



source: https://www.rohde-schwarz.com

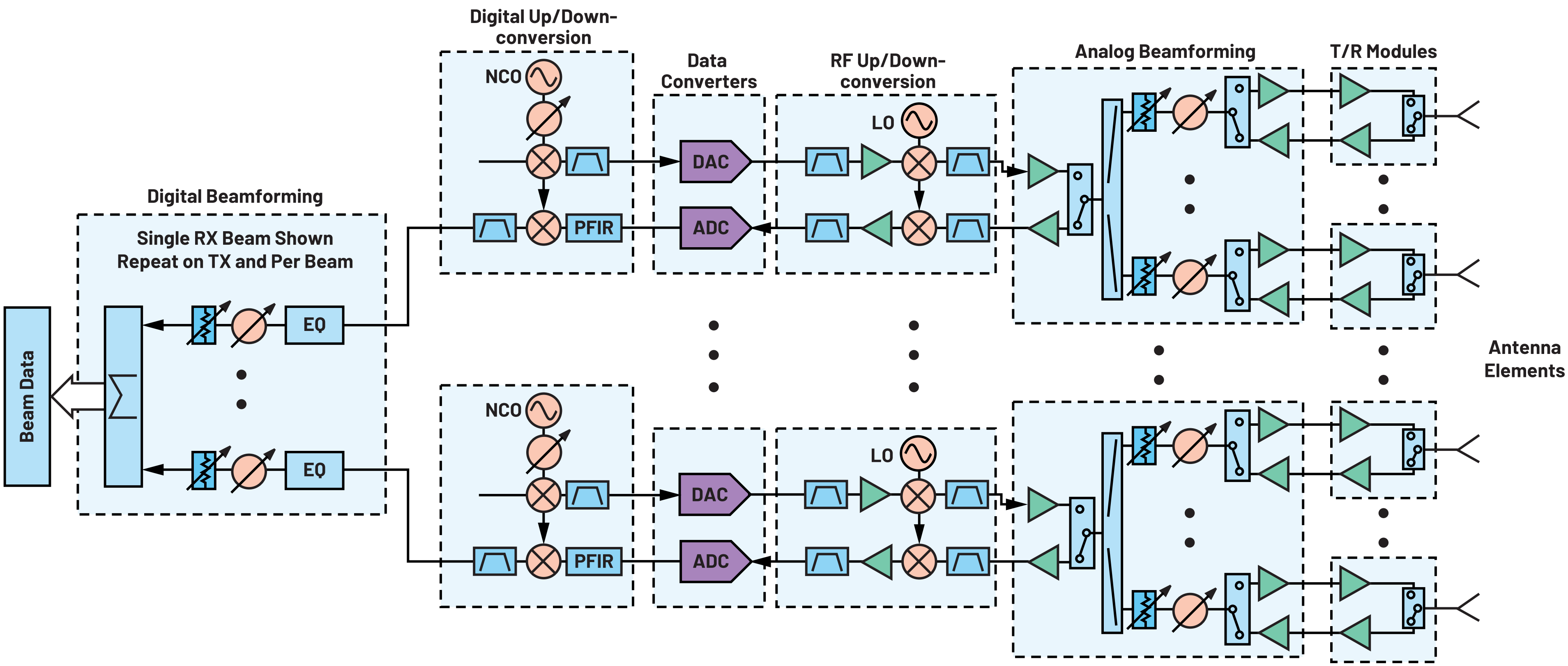
Massive MIMO

- large no. of antennas = large no. of RF chains (inc. ADCs/DACs) + large no. of power amplifiers (PAs)

- hardware cost and power consumption scale up

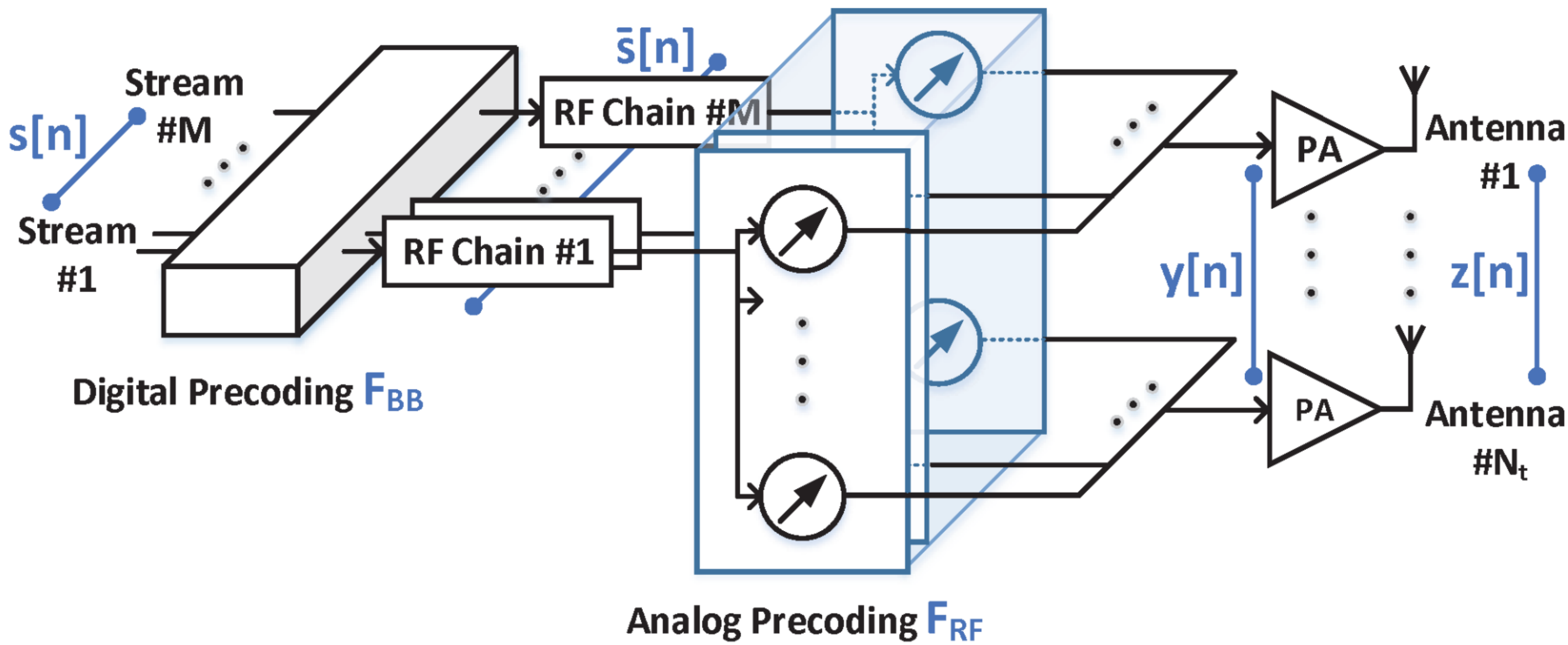
source: http://www.bristol.ac.uk/news/2017/february/massive-mimo-trials.html

Hybrid Digital-Analog MIMO



Source: P. Delos, S. Ringwood, and M. Jones, "Hybrid Beamforming Receiver Dynamic Range Theory to Practice," Technical Article, Analog Devices, 2022. Available at https://www.analog.com/en/technical-articles/hybrid-beamforming-receiver-dynamic-range.html

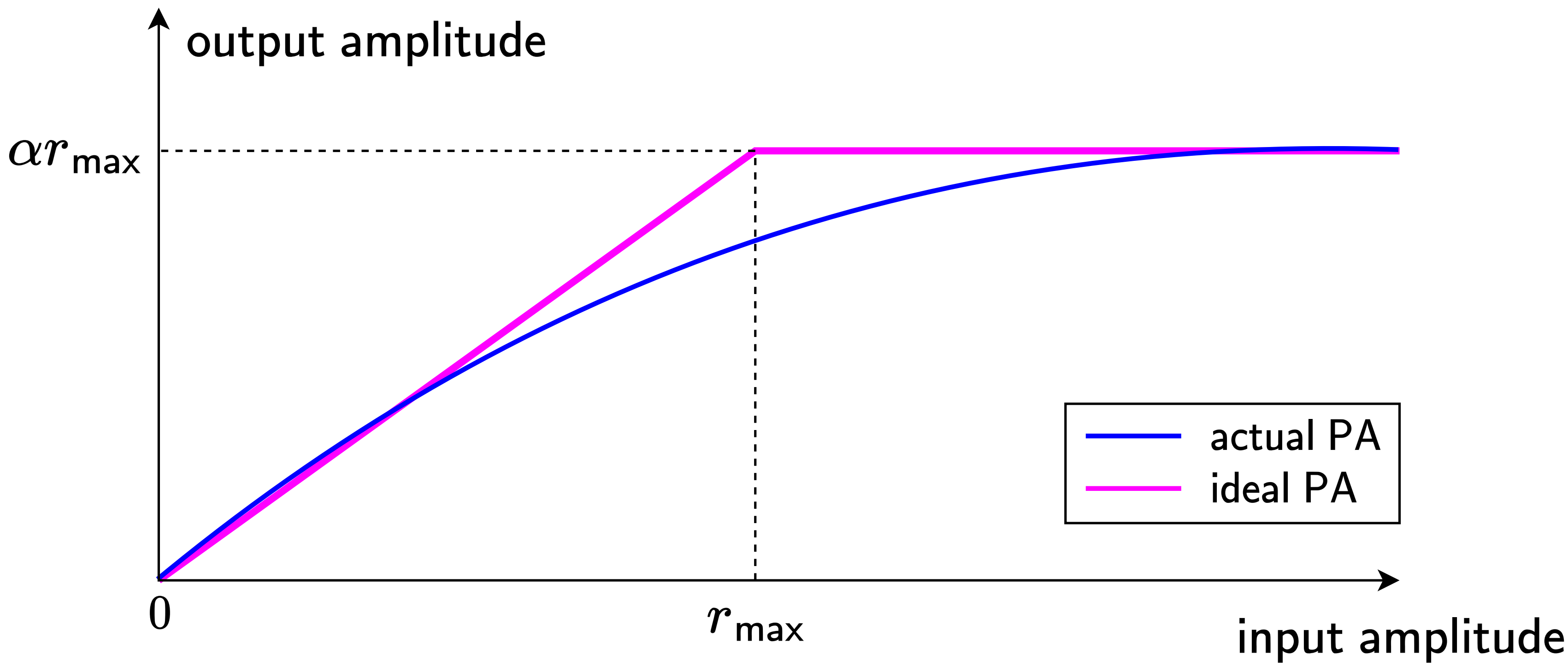
Hybrid Digital-Analog MIMO



Source: H. Yan and D. Cabric, "Digital predistortion for hybrid precoding architecture in millimeter-wave massive MIMO systems," ICASSP 2017.

Nonlinear PA Effects

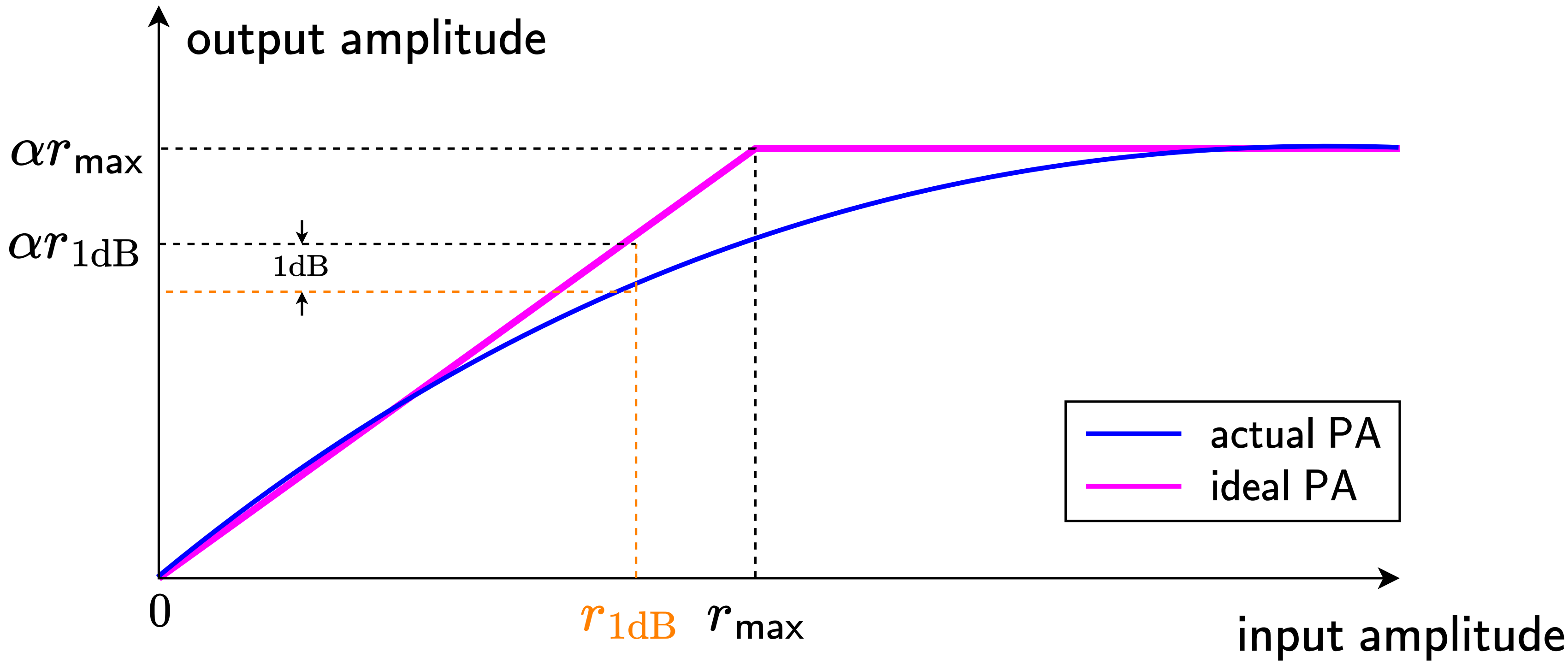
- input-output amplitude relation of PAs



- α = PA gain; r_{max} = maximum input amplitude

Nonlinear PA Effects

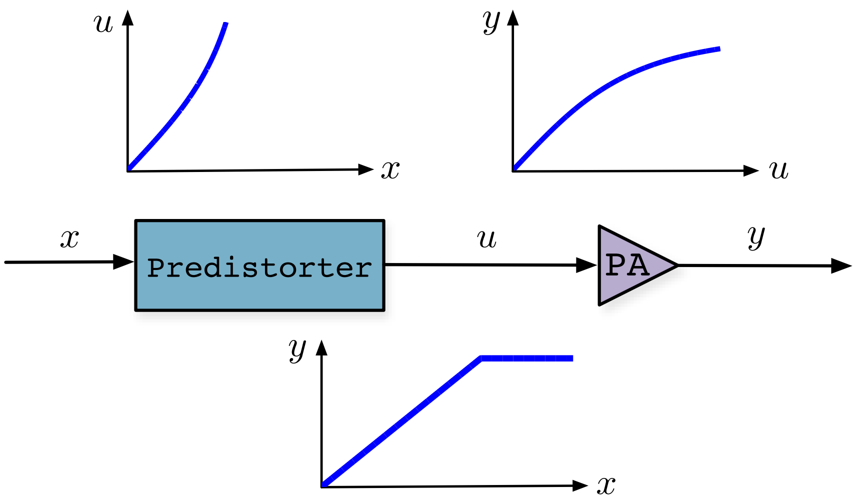
- input-output amplitude relation of PAs

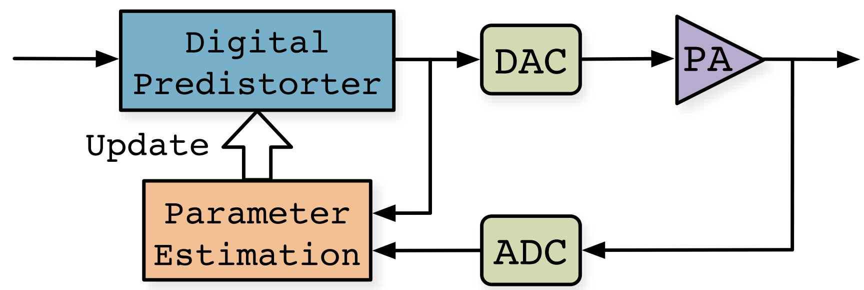


- backoff: use the linear amp. region, avoid the nonlinear region
- backoff sacrifices energy efficiency

Digital Predistortion (DPD) for Mitigating Nonlinear PA Effects

- use a nonlinear inverse mapping to equalize the nonlinear PA effects



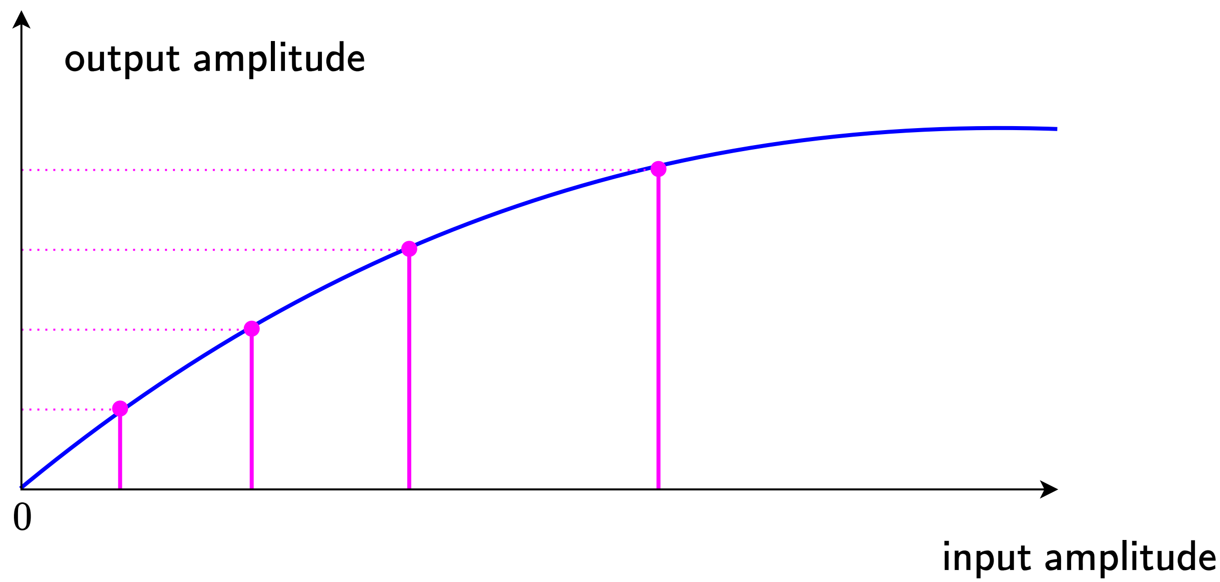


- DPD for each antenna is very expensive for massive MIMO

- the story is complicated for DPD for hybrid MIMO

MIMO with Low-Resolution ADCs/DACs

- replace high-resolution ADCs/DACs with lower-resolution ones
  - one-bit DACs (signals being -1 or 1) lead to constant envelope signals, PAs can work in low backoff mode
  - DACs with few no. of levels (say $\{\pm 1, \pm 3\}$) lead to signals that are easier to predistort

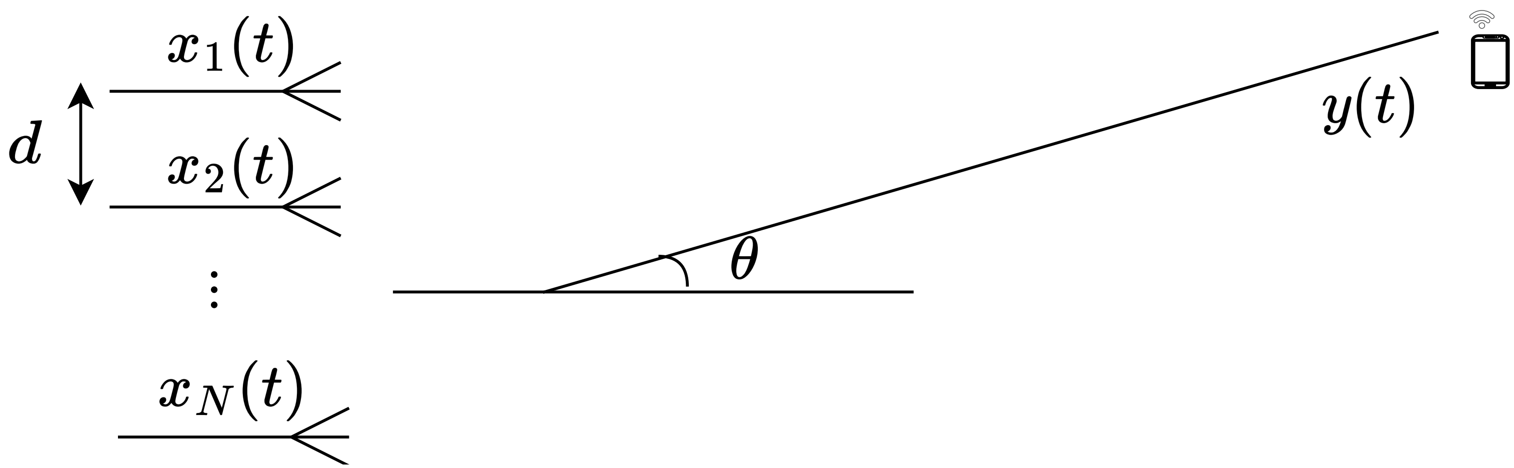


Massive MIMO with Low-Resolution ADCs/DACs

- **challenge:** significant quantization errors

- recent trend: develop SP techniques to better cope with coarse quantization effects

Some MIMO Basics



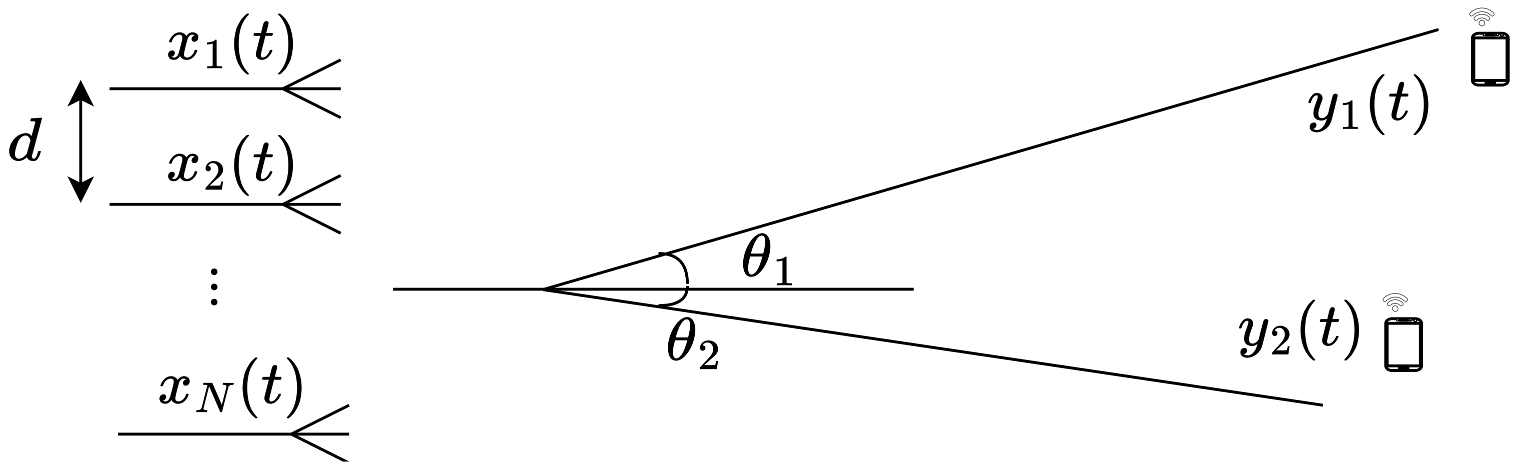
- assumption: uniform linear array, single path, far field, downlink

- model:

$$y(t) = \sum_{n=1}^{N} x_n(t) e^{-\text{j}\omega(n-1)} + \text{noise},$$

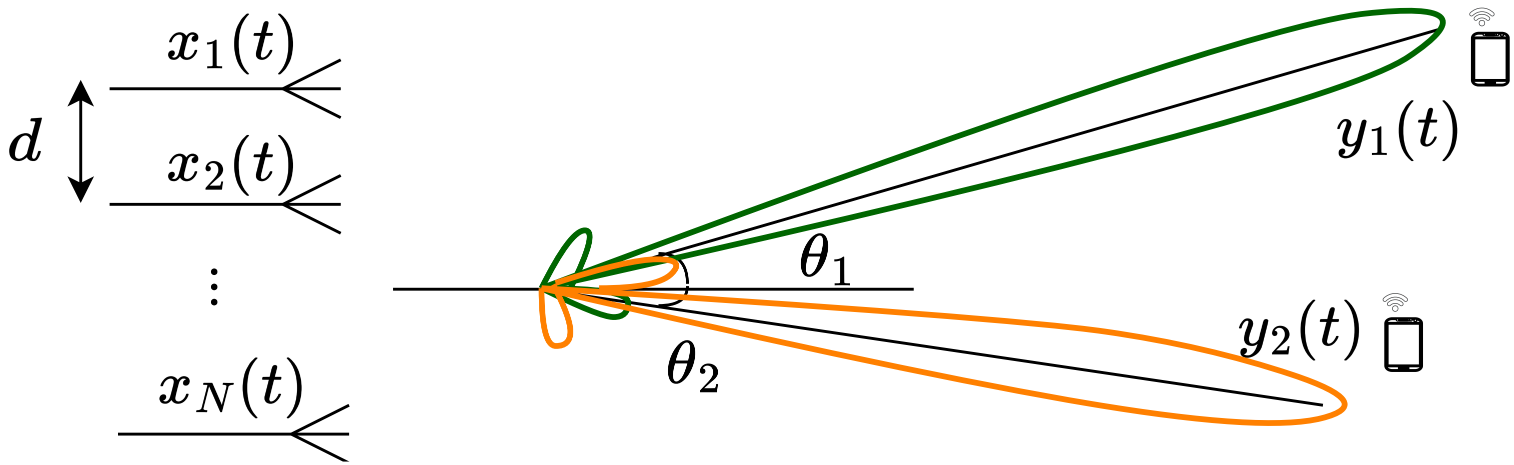
where $\omega = \frac{2\pi d}{\lambda}\sin(\theta)$, λ is the carrier wavelength, $\theta \in (-90^\circ, 90^\circ)$ is the user angle

Some MIMO Basics



- precoding: design $x_1(t), \ldots, x_N(t)$ such that each user will receive its designated information signal

Some MIMO Basics



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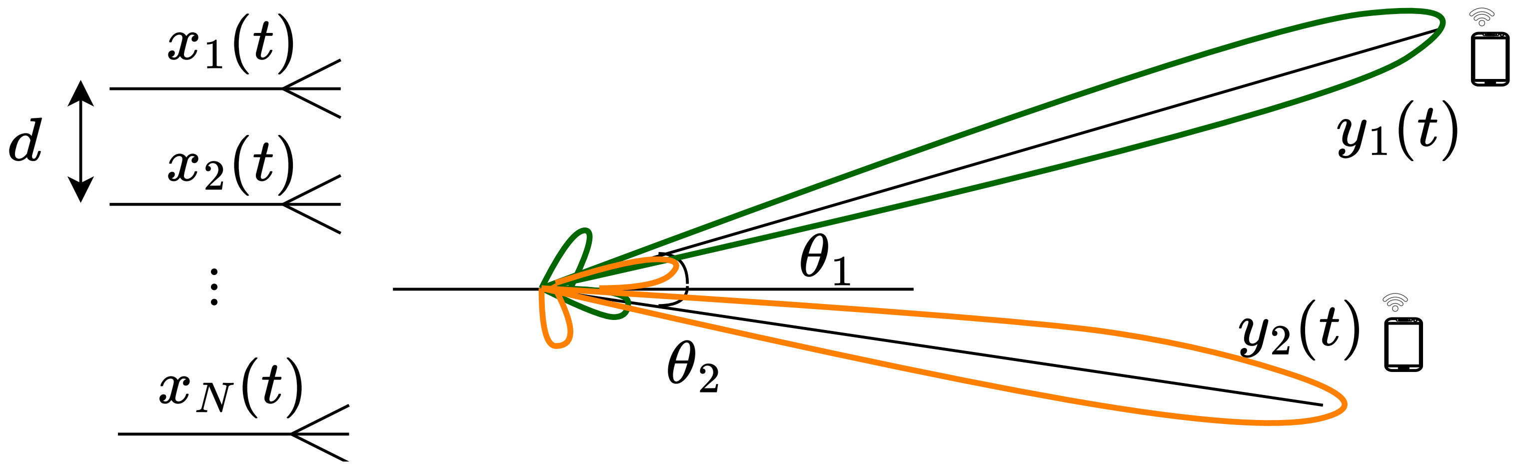
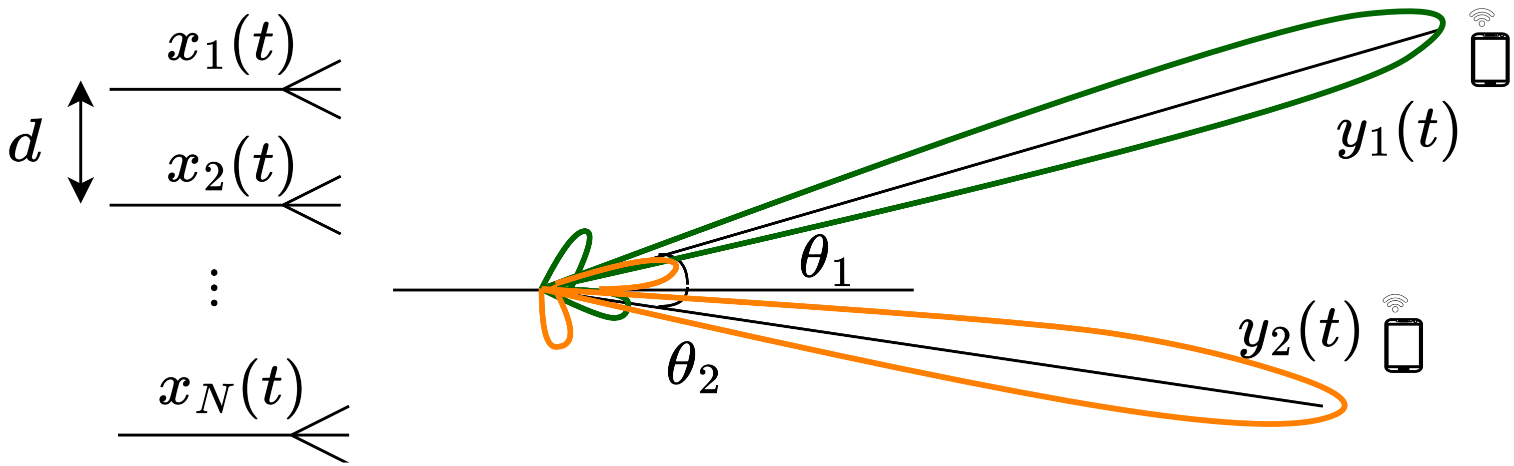


- precoding: design $x_1(t), \ldots, x_N(t)$ such that each user will receive its designated information signal

- "easy" if $x_1(t), \ldots, x_N(t)$ are continuous valued (high DAC res.)
  - precoding has been studied for more than two decades

Some MIMO Basics



- precoding: design $x_1(t), \ldots, x_N(t)$ such that each user will receive its designated information signal

- hard if $x_1(t), \ldots, x_N(t)$ are discrete valued (low DAC resolution)

- SOTA: optimize discrete variables, say, $x_n(t) \in \{\pm 1\}$
  - massive-scale discrete optimization, difficult

TSP Hardcopies in the 1990's

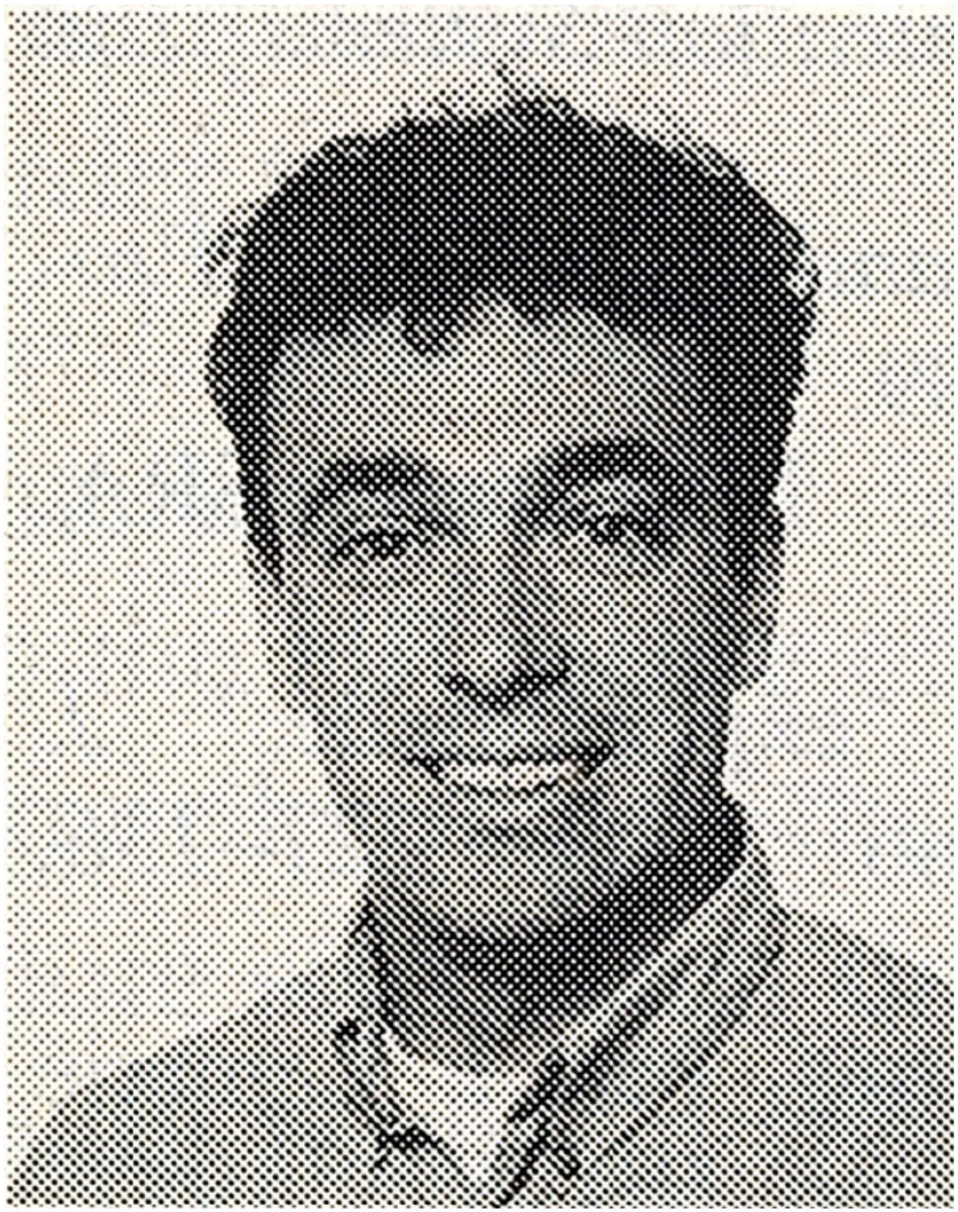
I Found This (1997)

Tim N. Davidson (M'96) received the B.E. (Hons. I) degree in electronic engineering from The University of Western Australia (UWA), Perth, in 1991 and the D.Phil. degree in engineering science from the The University of Oxford, Oxford, U.K., in 1995.

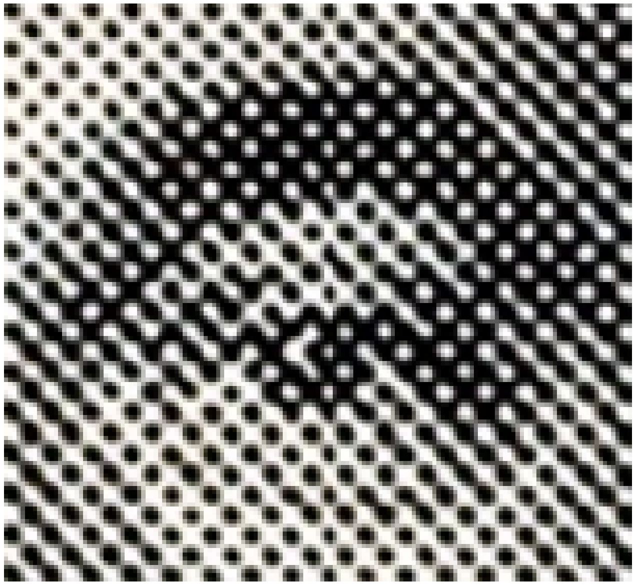
He is a post-doctoral research fellow and a part-time lecturer at the Communications Research Laboratory, McMaster University, Hamilton, Ont., Canada. His research interests are in signal processing and control, with current activity focussed on signal processing applications in digital communication systems. He has held research positions at the Adaptive Signal Processing Laboratory at UWA and the Australian Telecommunications Research Institute, Curtin University of Technology, Brisbane, Australia, and brief visiting appointments at QPSX Communications and the Digital Signal Processing Laboratory at The Chinese University of Hong Kong. He has also held project engineering positions at several mine sites in Western Australia.

Dr. Davidson was awarded the 1991 J. A. Wood Memorial Prize for "the most outstanding [UWA] graduate" in the pure and applied sciences and the 1991 Rhodes Scholarship for Western Australia.

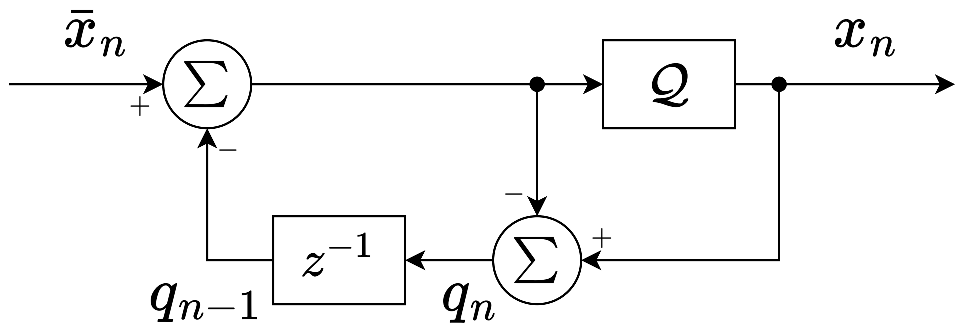
As We Take a Closer Look



It's Binary—Image Halftoning

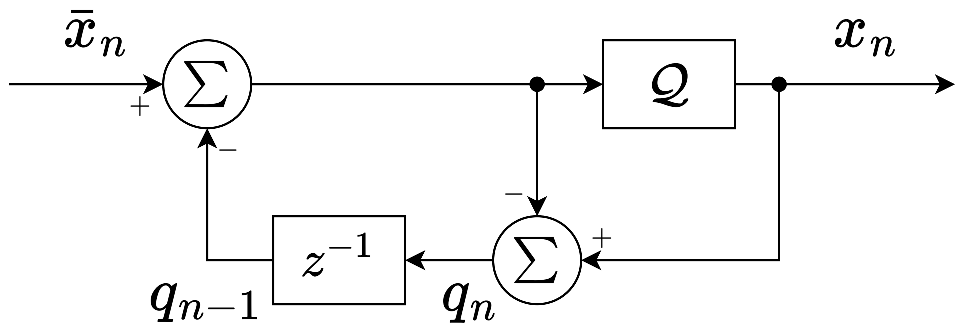


Temporal ΣΔ Modulation



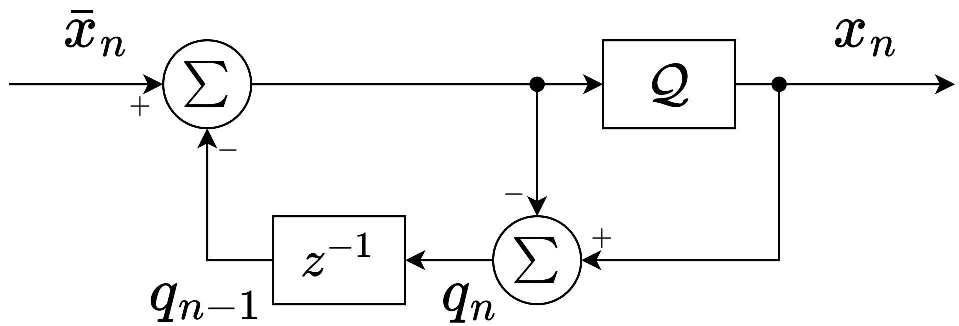
- widely used in temporal DACs/ADCs
- commercial "high-resolution" ADCs/DACs (say, 16 bits) may use a small number of signal levels (say, 5 to 7 levels) and $\Sigma\Delta$ mod.
- image halftoning is the 2D version of $\Sigma\Delta$ modulation

Temporal ΣΔ Modulation



- $\mathcal{Q}$ is a quantizer; e.g., $\mathcal{Q} = \mathrm{sgn}$ for the one-bit case

Temporal $\Sigma\Delta$ Modulation



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- **postulate:** $\mathcal{Q}(x) = x + q$, q is random & independent

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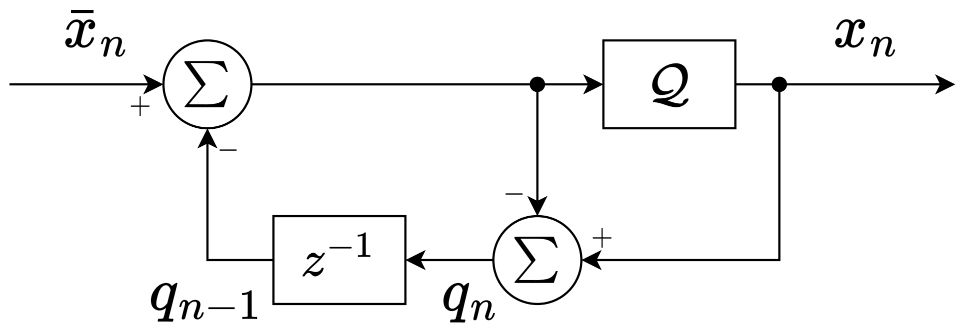
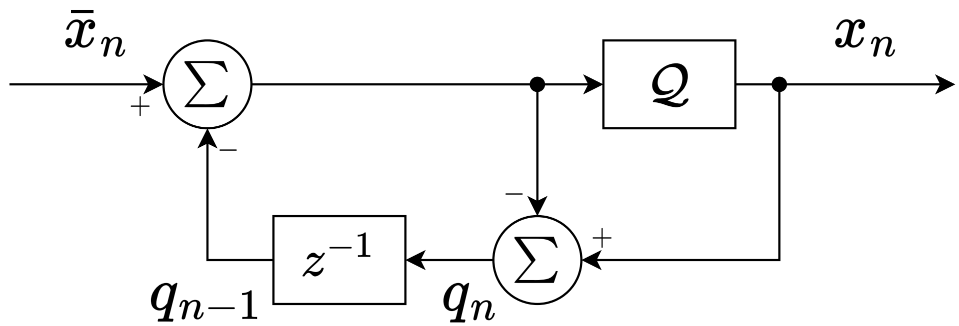


- $\mathcal{Q}$ is a quantizer; e.g., $\mathcal{Q} = \mathrm{sgn}$ for the one-bit case

- **postulate:** $\mathcal{Q}(x) = x + q$, q is random & independent

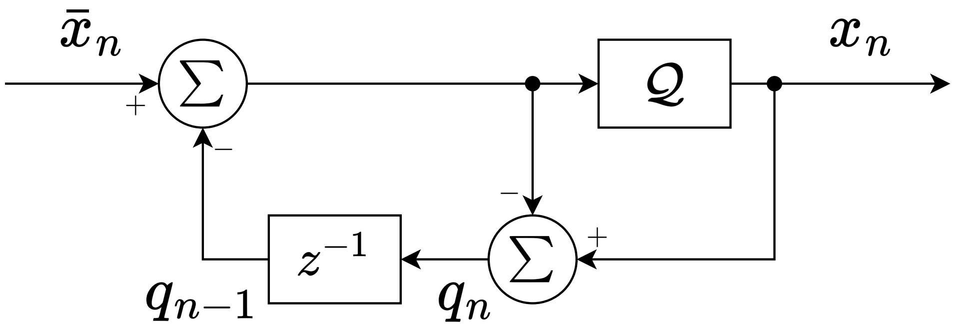
- $x_n = \mathcal{Q}(\bar{x}_n - q_{n-1}) = (\bar{x}_n - q_{n-1}) + q_n$, $n = 0, 1, \ldots$

Temporal ΣΔ Modulation

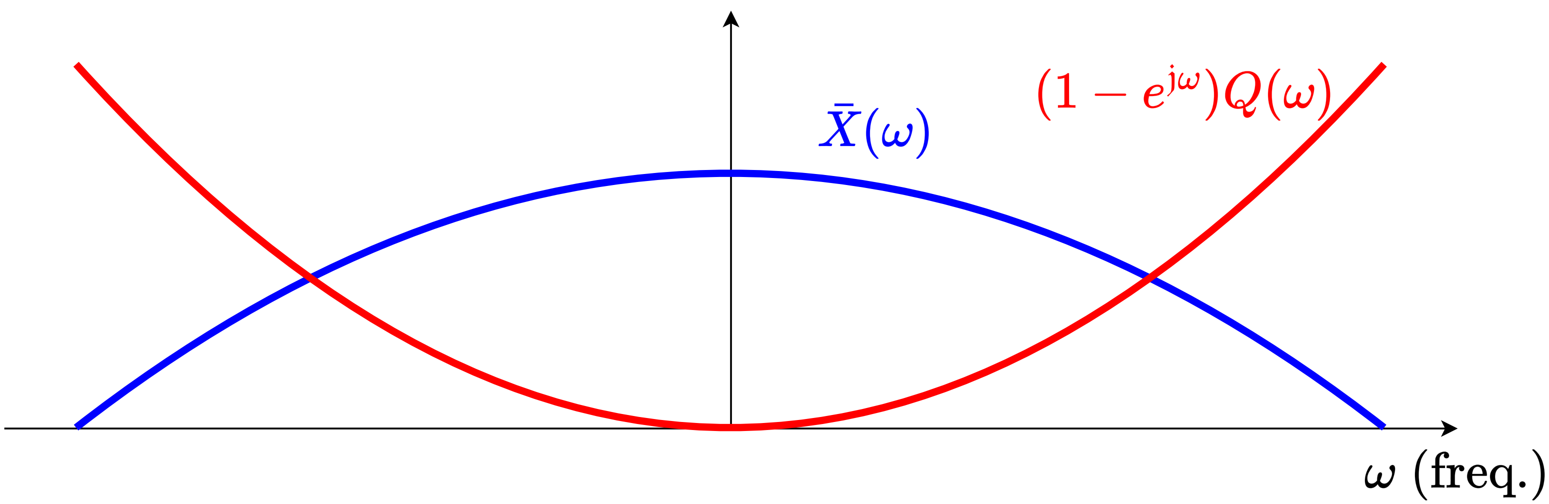


- $\mathcal{Q}$ is a quantizer; e.g., $\mathcal{Q} = \mathrm{sgn}$ for the one-bit case
- **postulate:** $\mathcal{Q}(x) = x + q$, q is random & independent
- $x_n = \mathcal{Q}(\bar{x}_n - q_{n-1}) = (\bar{x}_n - q_{n-1}) + q_n$, $n = 0, 1, \ldots$
- Fourier transform: $X(\omega) = \bar{X}(\omega) + (1 - e^{-\mathrm{j}\omega})Q(\omega)$

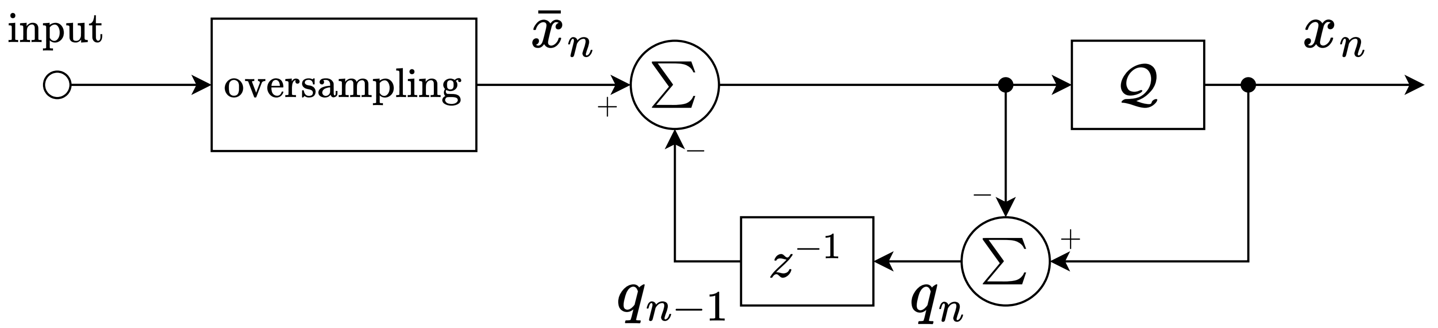
Temporal ΣΔ Modulation



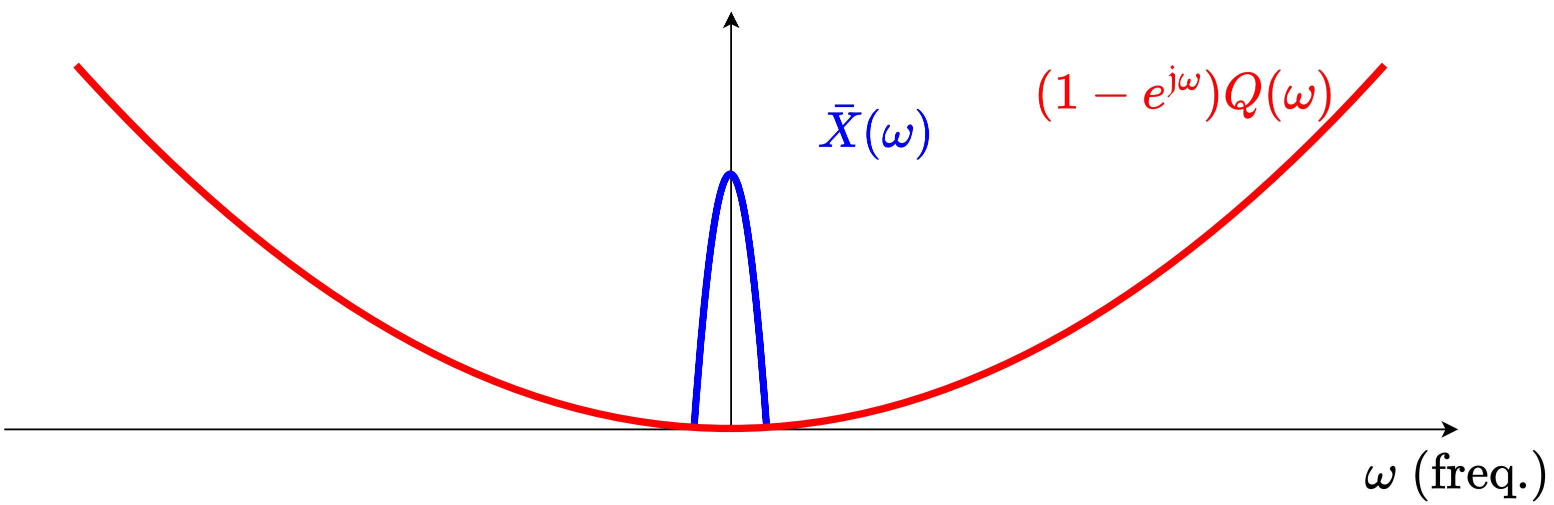
- Fourier transform: $X(\omega) = \bar{X}(\omega) + \underbrace{(1 - e^{-\mathrm{j}\omega})}_{\textbf{high pass!}} Q(\omega)$



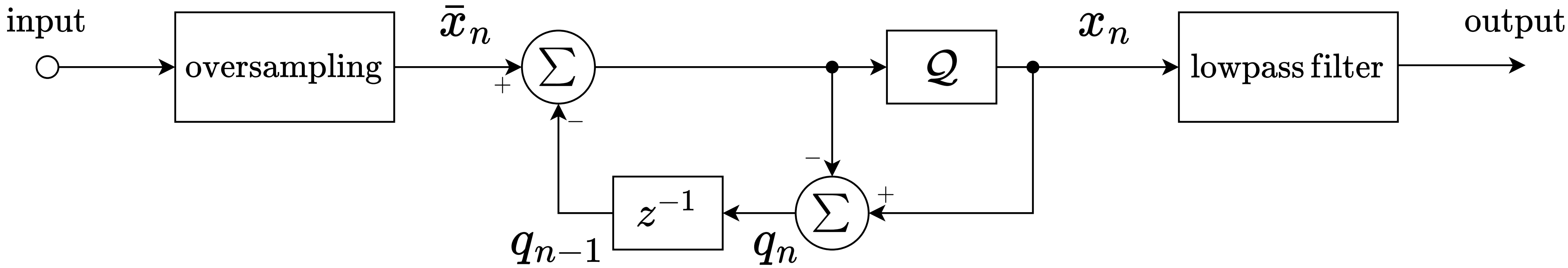
Temporal ΣΔ Modulation



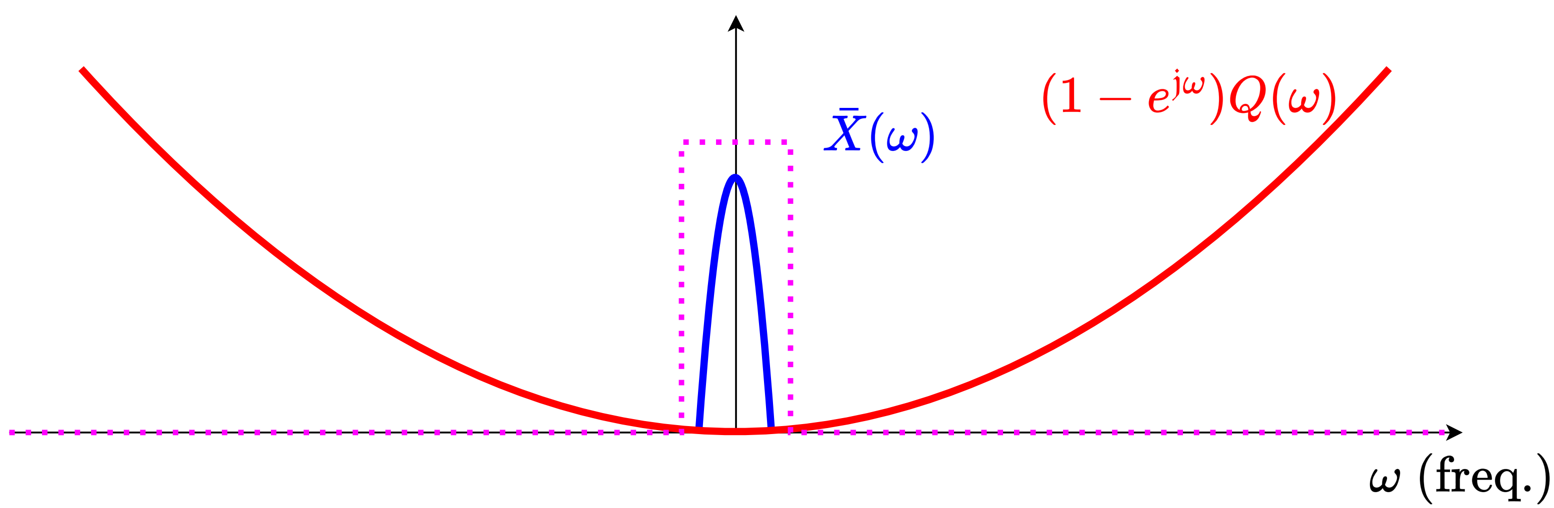
- oversampling: make $\bar{x}_n$ lowpass, avoid the high frequency region



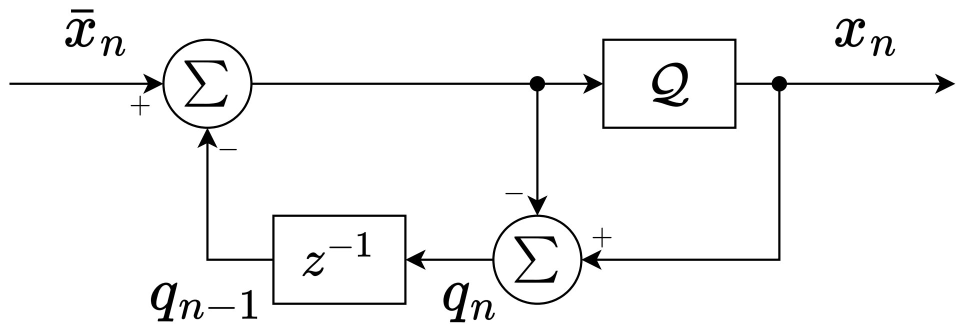
Temporal ΣΔ Modulation



- x_n is converted to analog via a lowpass filter, which removes much of the q. noise



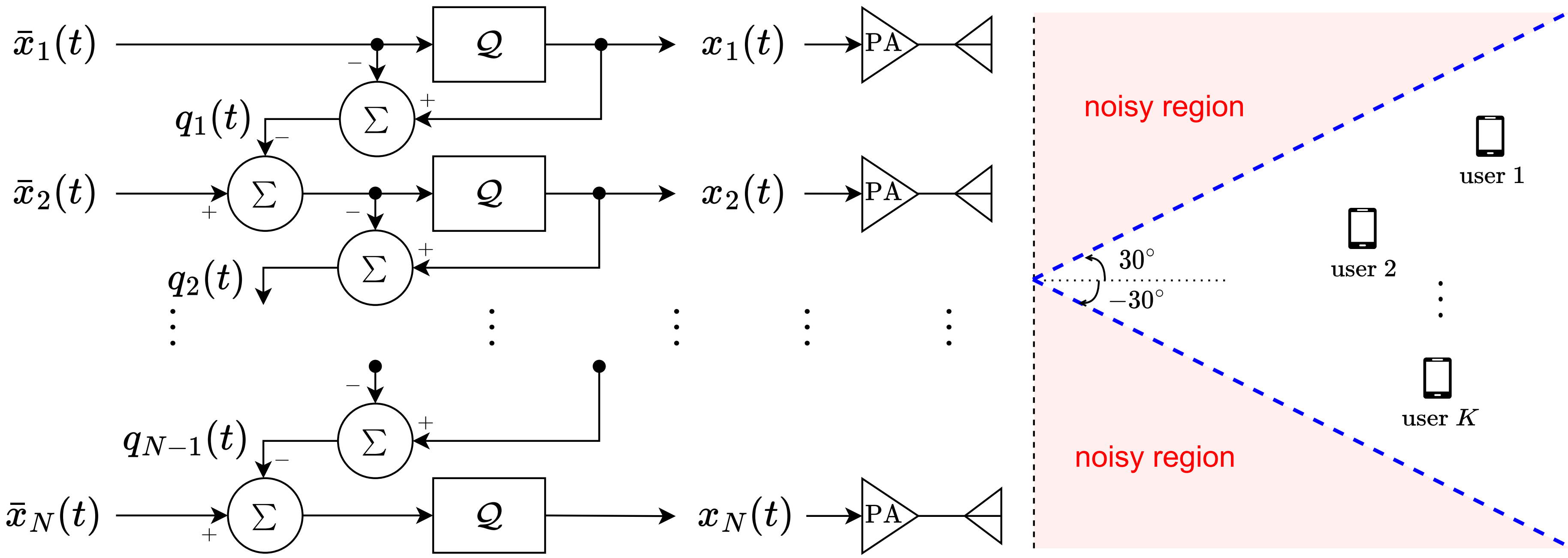
Temporal $\Sigma\Delta$ Modulation



- can have large $|q_n|$, or $|q_n| \to \infty$, if we don't constrain $\{\bar{x}_n\}$
- Let $\mathcal{Q}$ be the rounding function for $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$, where M is the no. of signal levels.
- no-overload condition: Let A be the maximum amplitude of $\{\bar{x}_n\}$, or $|\bar{x}_n| \le A$ for all n.

$$A \le M - 1 \quad \Longrightarrow \quad |q_n| \le 1 \text{ for all } n.$$

Spatial ΣΔ Modulation for Few-Bit Massive MIMO



- **idea:** ΣΔ in space
  - shape q. noise to high spatial frequencies
  - serve users in low spatial frequencies
  - precoding: same as the traditional (simply speaking)

Illustration: Angular Power Spectrum

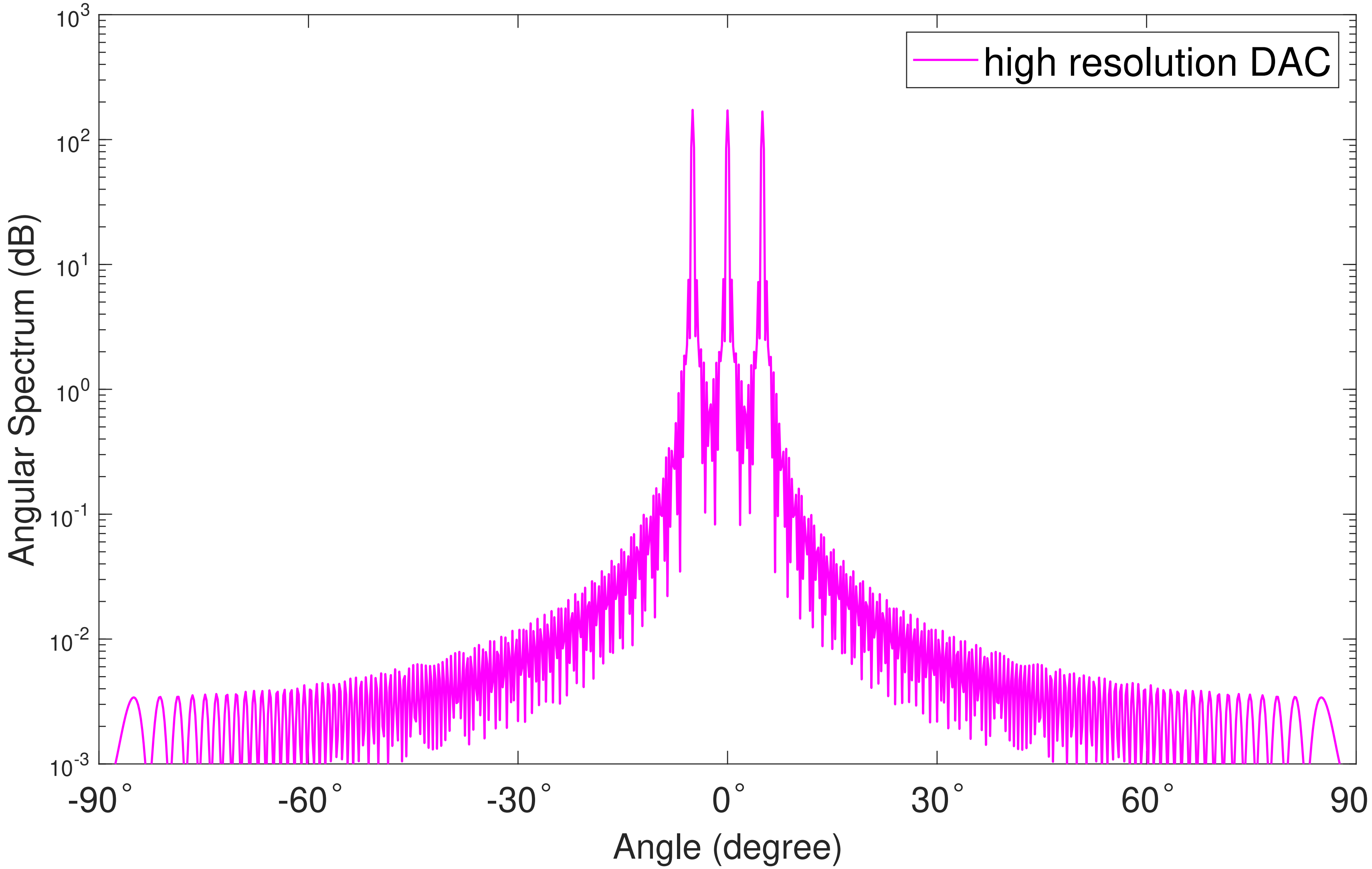


Illustration: Angular Power Spectrum

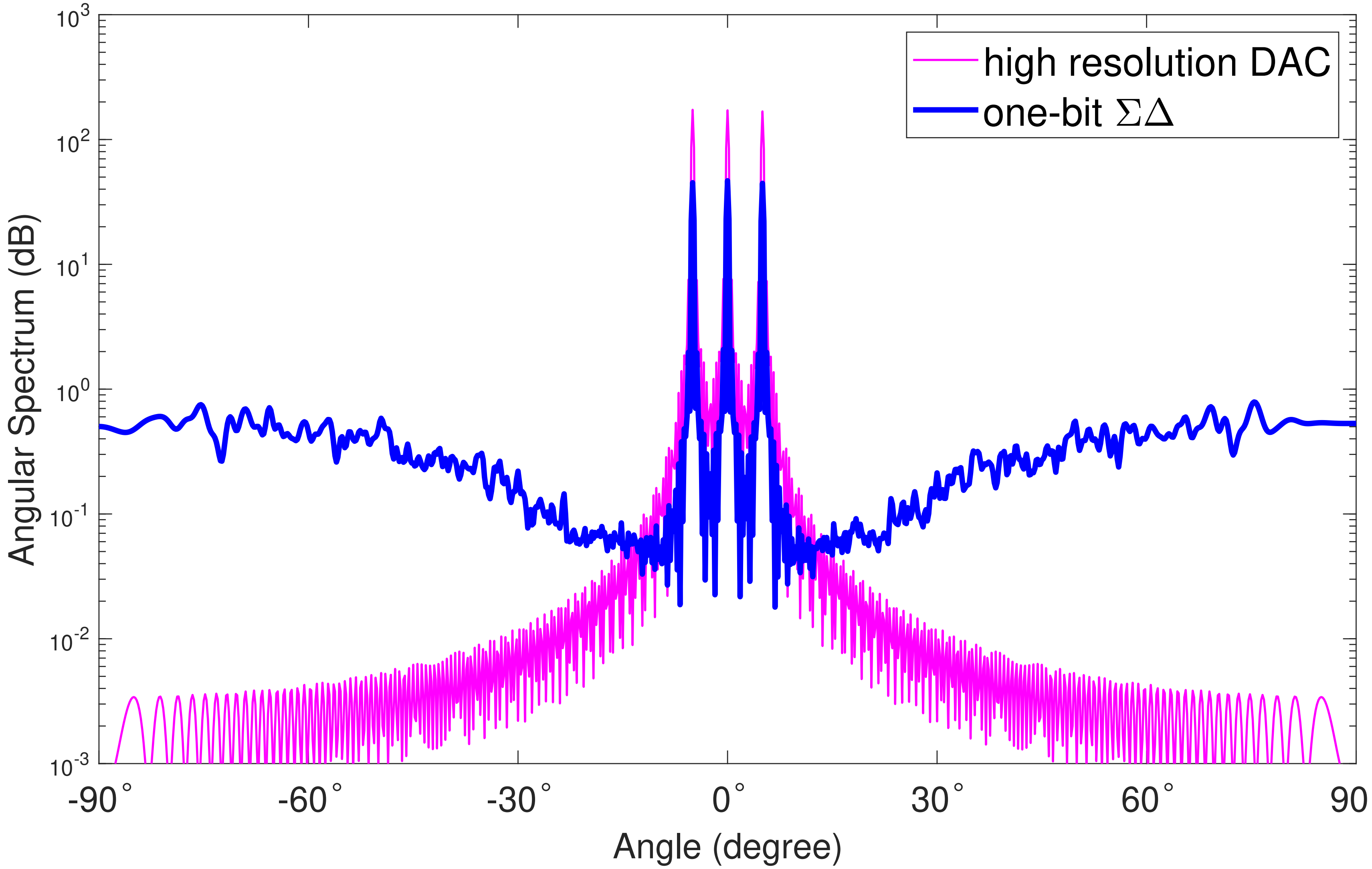
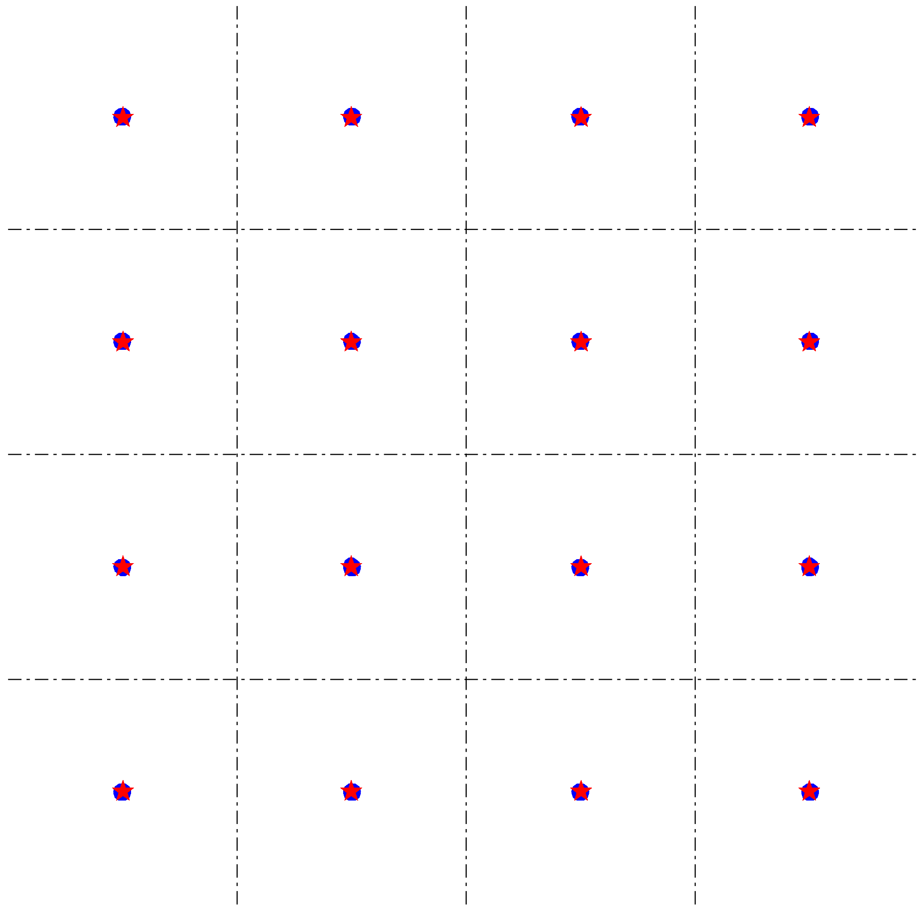
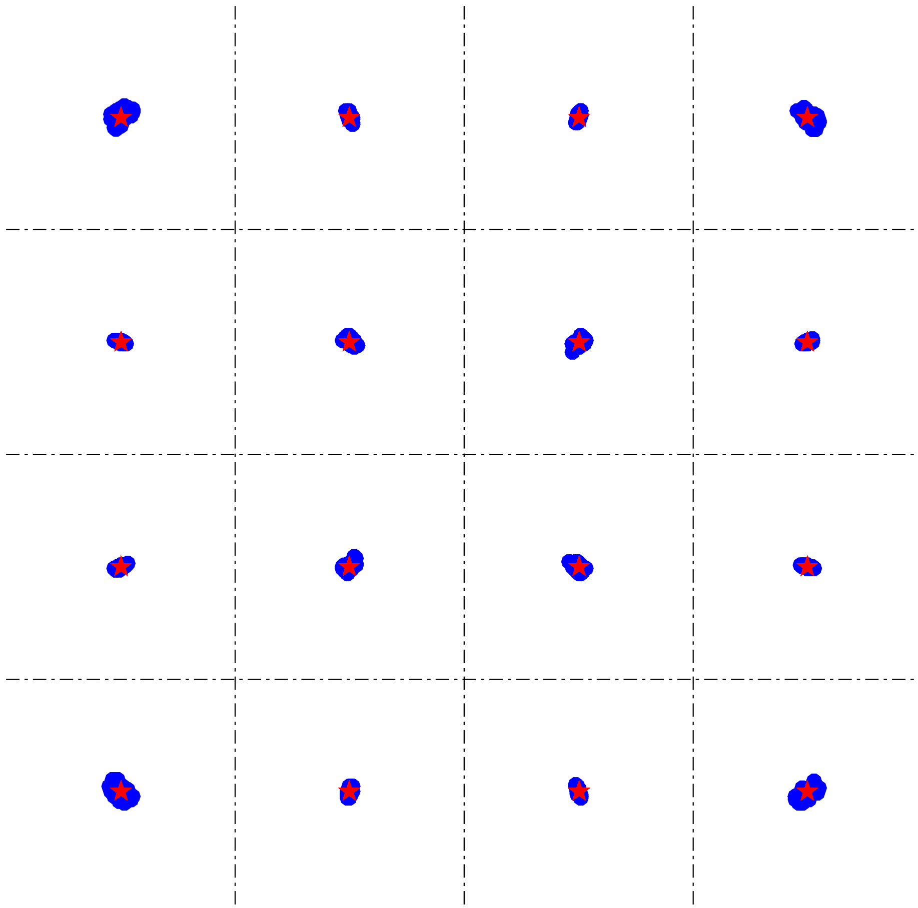


Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



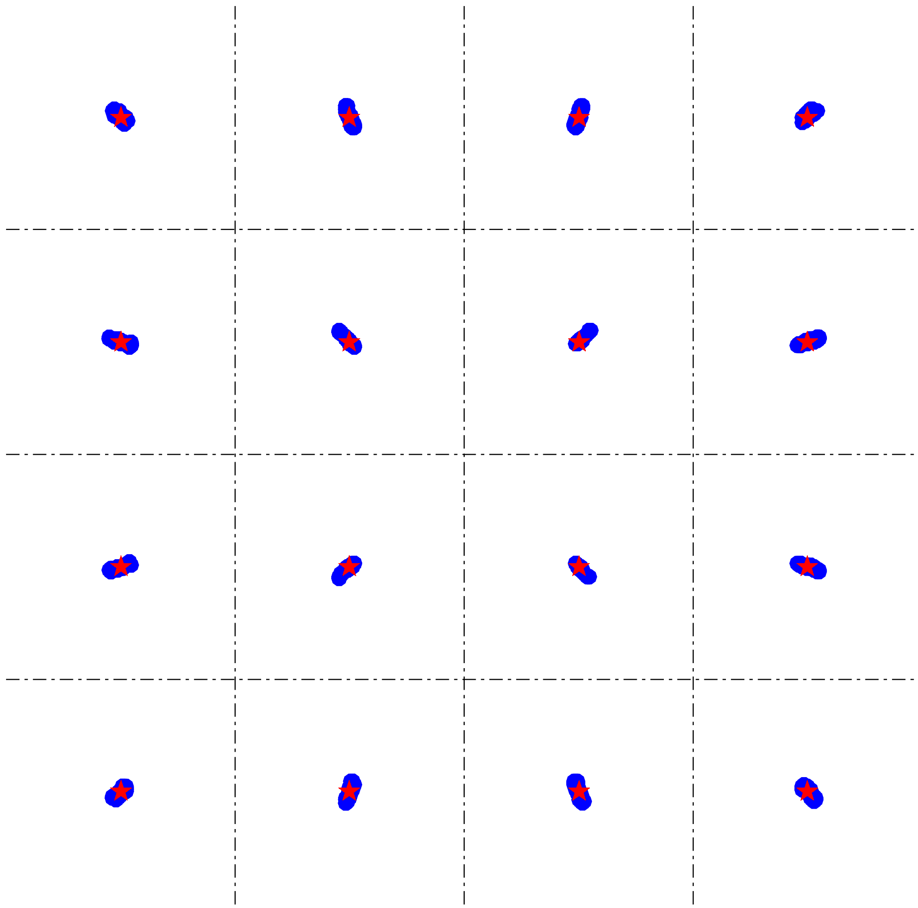
$$N = 512, d = \lambda/8, \theta = 0^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



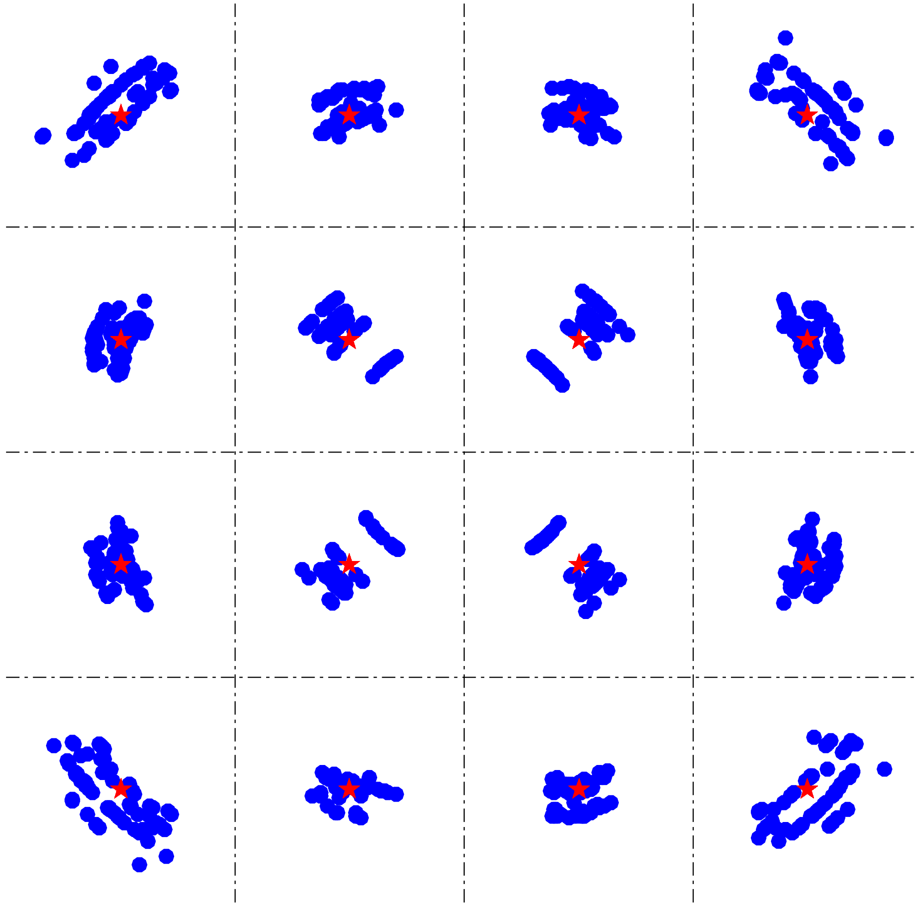
$$N = 512, d = \lambda/8, \; \theta = 10^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



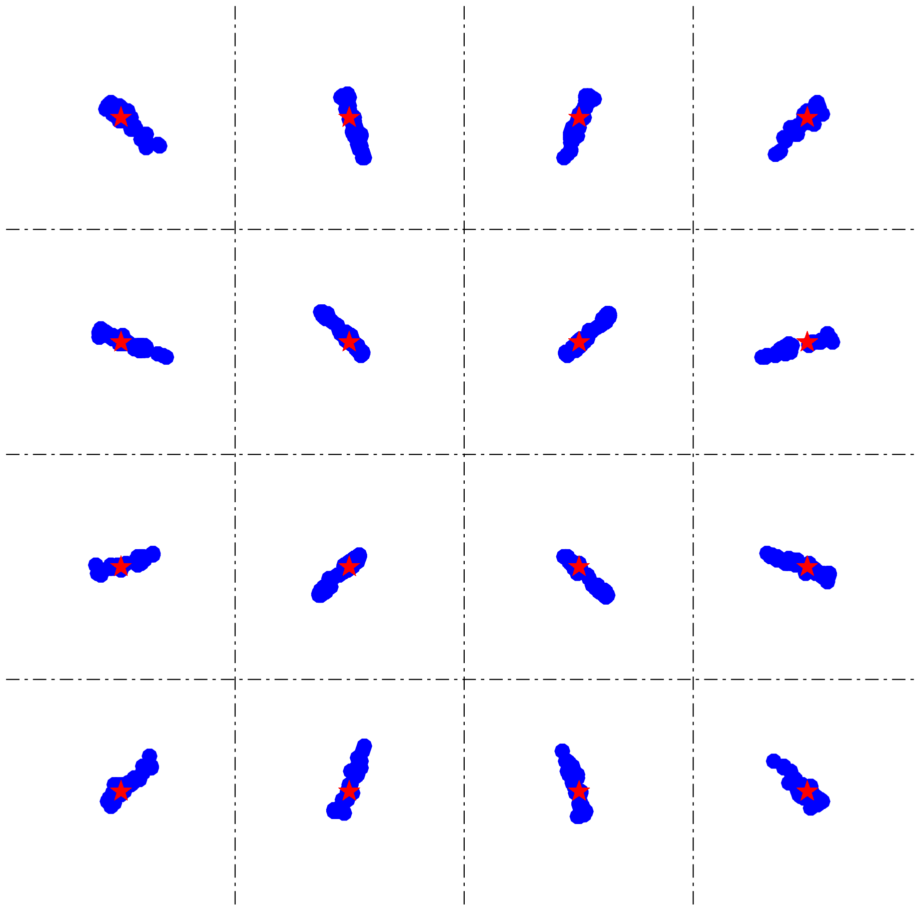
$$N = 512, d = \lambda/8, \; \theta = 20^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



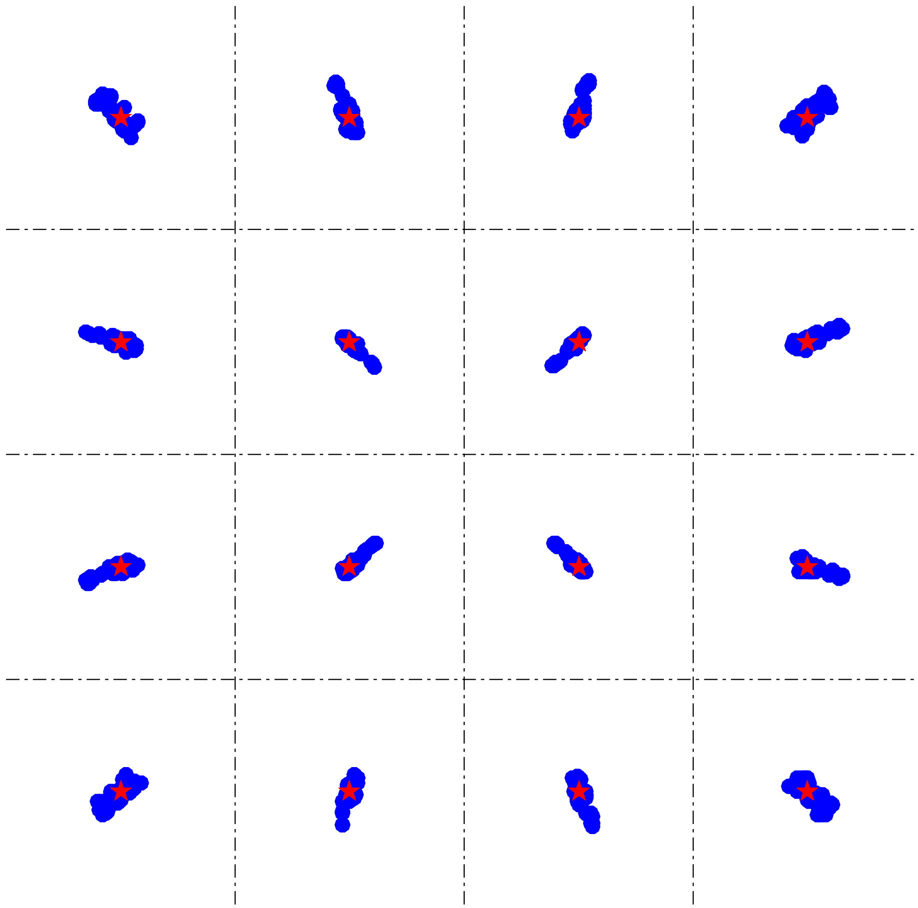
$$N = 512, d = \lambda/8, \theta = 30^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



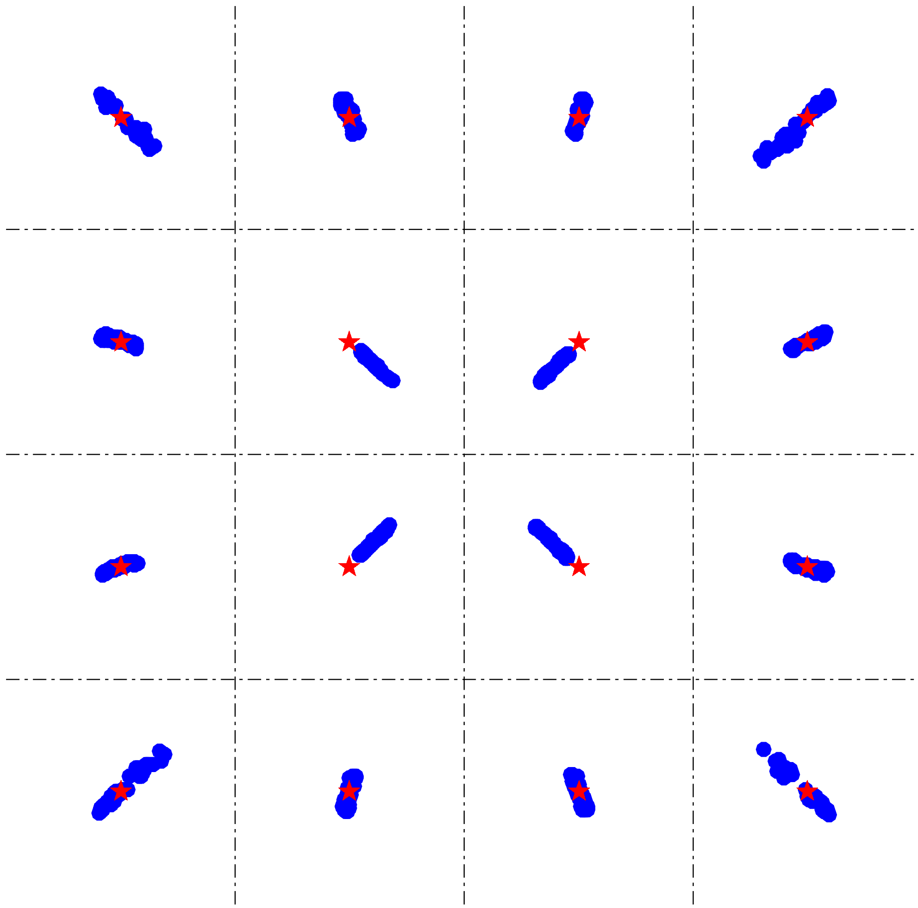
$$N = 512, d = \lambda/8, \theta = 40^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



$$N = 512, d = \lambda/8, \ \theta = 50^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



$$N = 512, d = \lambda/8, \ \theta = 60^\circ$$

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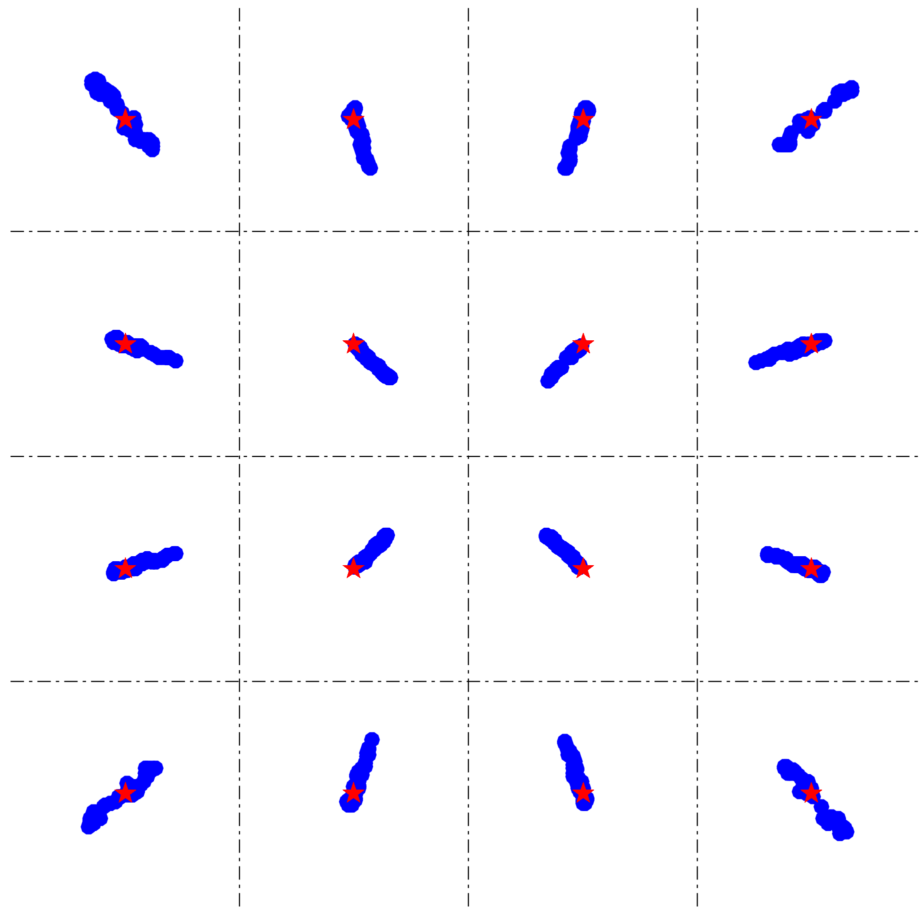
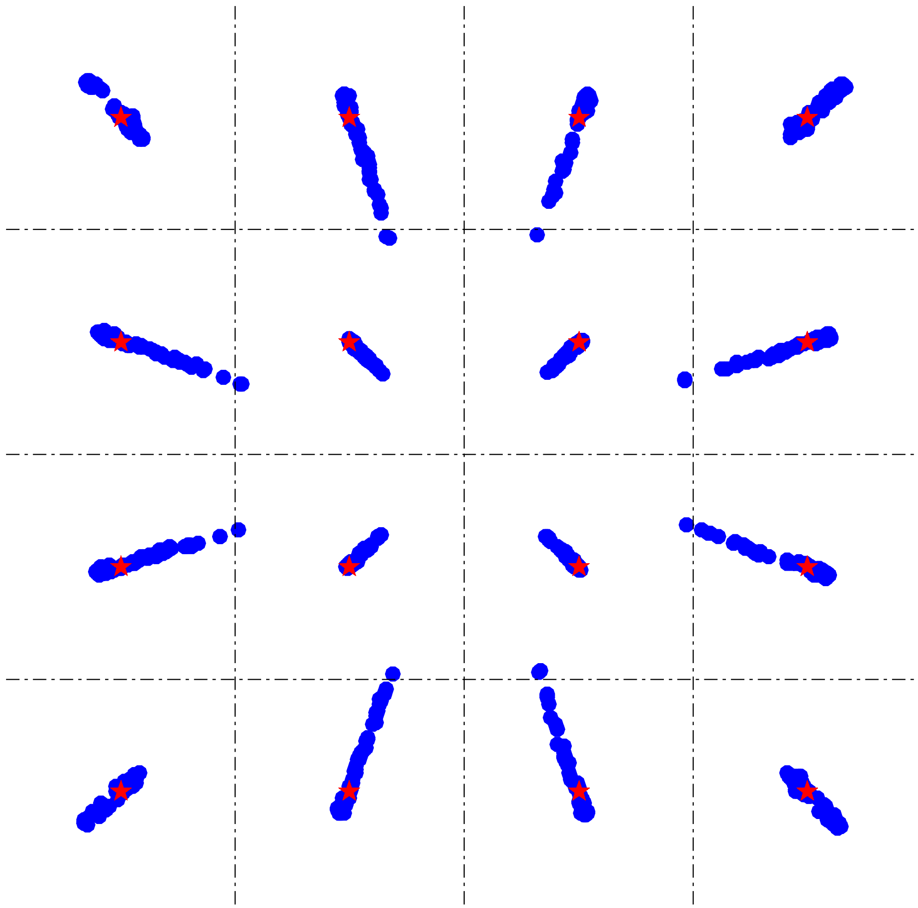


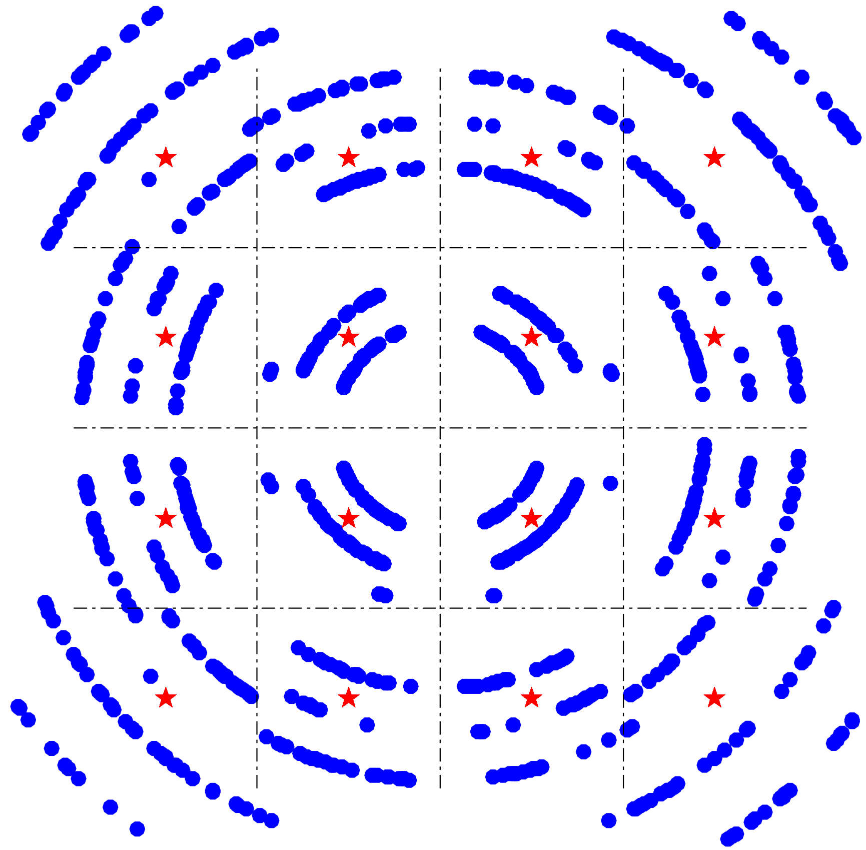
$$N = 512, d = \lambda/8, \ \theta = 70^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User



$$N = 512, d = \lambda/8, \theta = 80^\circ$$

Illustration: Scatter Plot of One-Bit $\Sigma\Delta$, One User

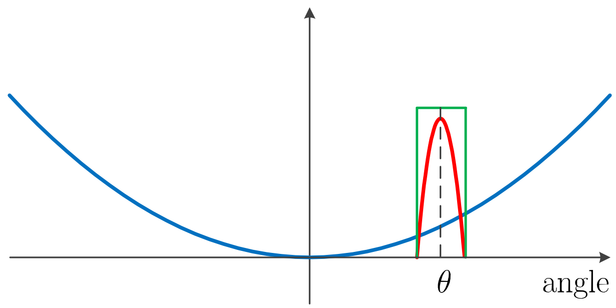


$$N = 512, d = \lambda/8, \theta = 90^\circ$$

Analysis for the One-User Case

- the effective SNR:

$$\mathsf{SNR}_{\mathsf{eff}} = \frac{|\alpha|^2 PN}{\underbrace{\frac{8|\alpha|^2 P}{3} \left|\sin\left(\frac{\omega}{2}\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$



where $\omega = \frac{2\pi d}{\lambda}\sin(\theta)$.

- model:

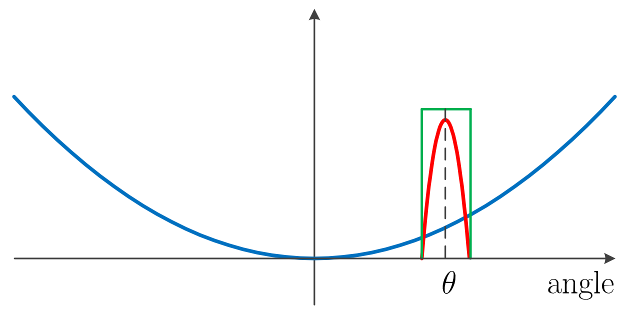
$$\text{received signal} = \sqrt{\frac{P}{2N}}\alpha \boldsymbol{a}(\theta)^T \boldsymbol{x}(t) + \text{noise},$$

P = tx power; N = no. of antennas; α = path gain; σ_v^2 = noise power; $\boldsymbol{a}(\theta) = (1, e^{-j\omega}, \ldots, e^{-j(N-1)\omega})$; λ= carrier wavelength; d = inter-antenna spacing; 1-bit; precoder = MRT (max. ratio tx)

Analysis for the One-User Case

- the effective SNR:

$$\mathrm{SNR}_{\mathrm{eff}} = \frac{|\alpha|^2 P N}{\underbrace{\frac{8|\alpha|^2 P}{3} \left|\sin\left(\frac{\omega}{2}\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$



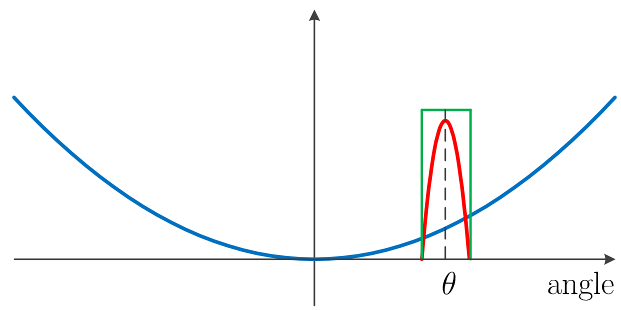
where $\omega = \frac{2\pi d}{\lambda} \sin(\theta)$.

- implications:
  - increasing tx power P does *not* reduce the q. noise power
  - increasing the no. of antennas N increases the effective SNR
    - ∗ favorable: massive antennas, small per-antenna power $\frac{P}{N}$

Analysis for the One-User Case

- the effective SNR:

$$\text{SNR}_{\text{eff}} = \frac{|\alpha|^2 PN}{\underbrace{\frac{8|\alpha|^2 P}{3}\left|\sin\left(\frac{\omega}{2}\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$



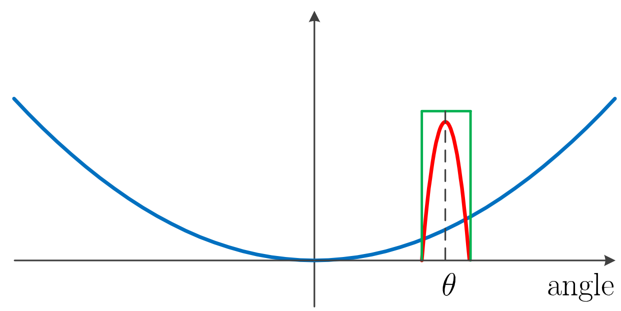
where $\omega = \frac{2\pi d}{\lambda}\sin(\theta)$.

- implications:
  - smaller inter-antenna spacing $d \Longrightarrow$ smaller q. noise power
    - $*$ identical to over-sampling in temporal $\Sigma\Delta$ modulation
    - $*$ mutual coupling prohibits us from making d too small

Analysis for the One-User Case

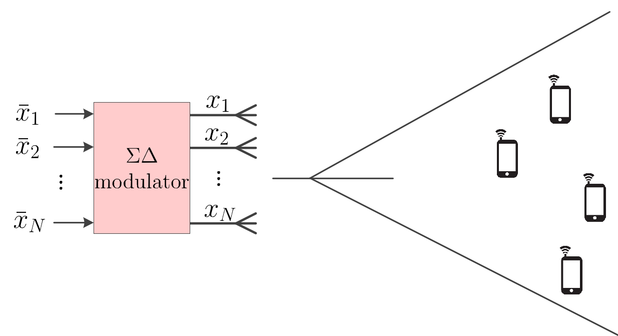
- the effective SNR:

$$\mathsf{SNR}_{\text{eff}} = \frac{|\alpha|^2 PN}{\underbrace{\frac{8|\alpha|^2 P}{3} \left|\sin\left(\frac{\omega}{2}\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$

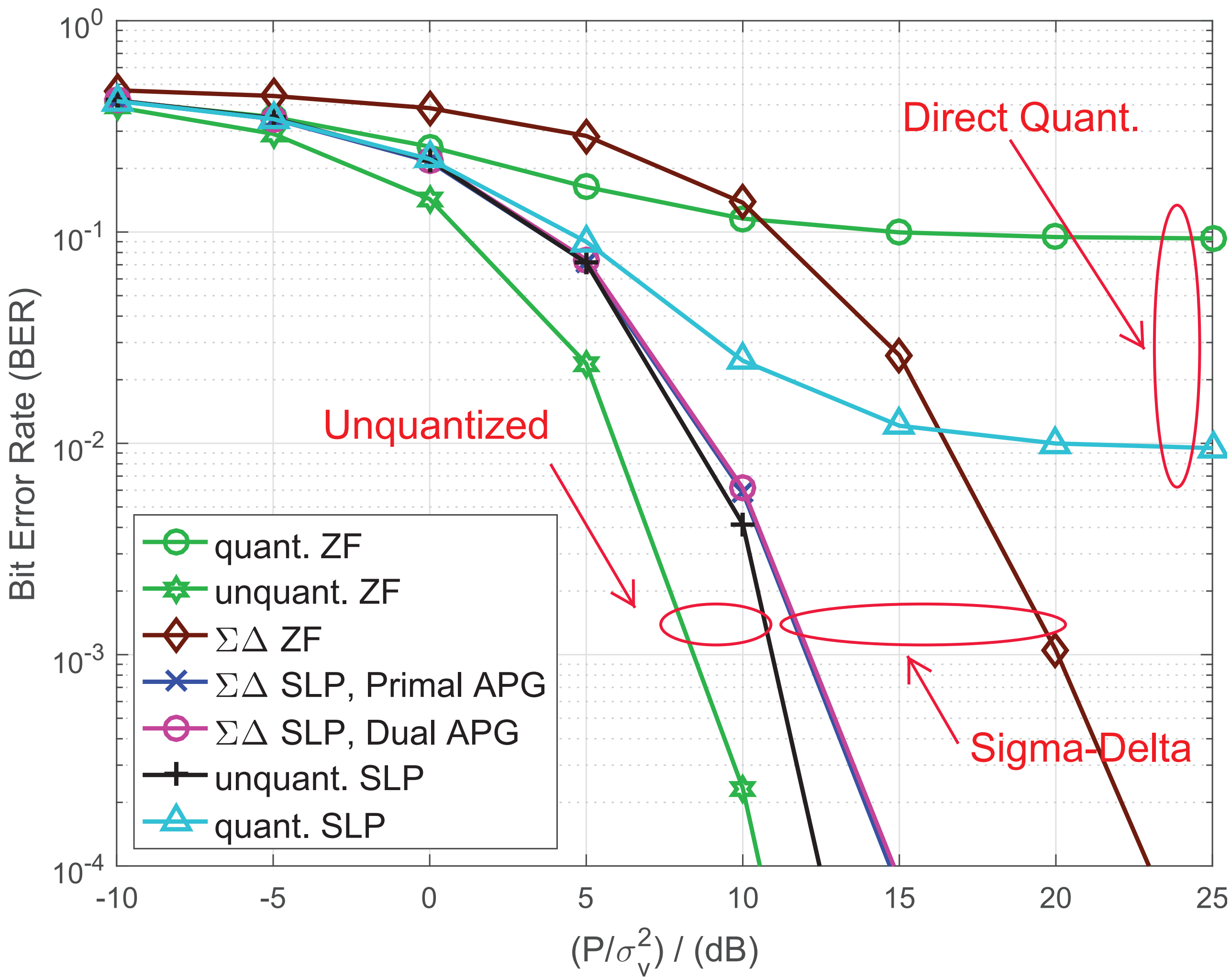


where $\omega = \frac{2\pi d}{\lambda}\sin(\theta)$.

- implications:
  - larger $|\theta| \Longrightarrow$ larger q. noise power
  - we can serve an angle sector, say $[-30^\circ, 30^\circ]$



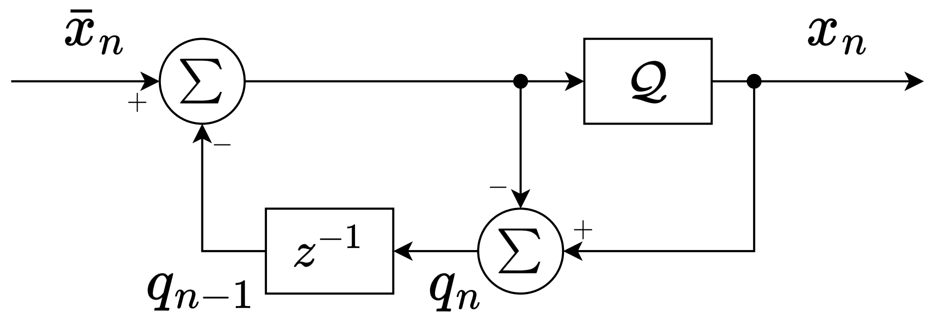
Simulation Result: Multiuser One-Bit MIMO



- number of antennas $N = 256$, angle sector $= [-30°, 30°]$, no. of users $K = 24$, $d = \lambda/8$, 8-ary PSK

Second-Order ΣΔ Modulator

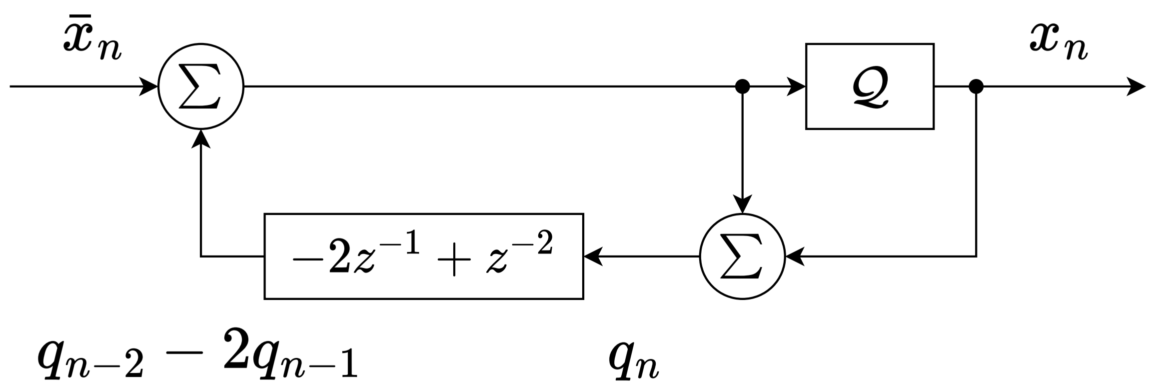
- recall the first-order ΣΔ modulator:



  - Fourier transform: $X(\omega) = \bar{X}(\omega) + \underbrace{(1 - e^{-j\omega})}_{\text{highpass}} Q(\omega)$
  - no-overload condition: $A \leq M - 1$

Second-Order ΣΔ Modulator

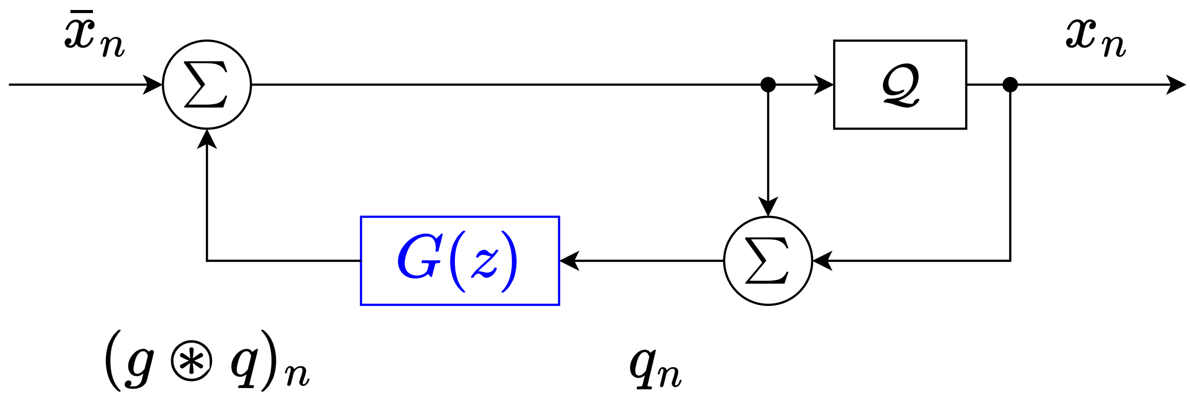
- second-order ΣΔ modulator:



  - Fourier transform: $X(\omega) = \bar{X}(\omega) + \underbrace{(1 - e^{-j\omega})^2}_{\text{stronger highpass!}} Q(\omega)$

  - no-overload condition: $A \leq M - 3$

General $\Sigma\Delta$ Modulator Structure

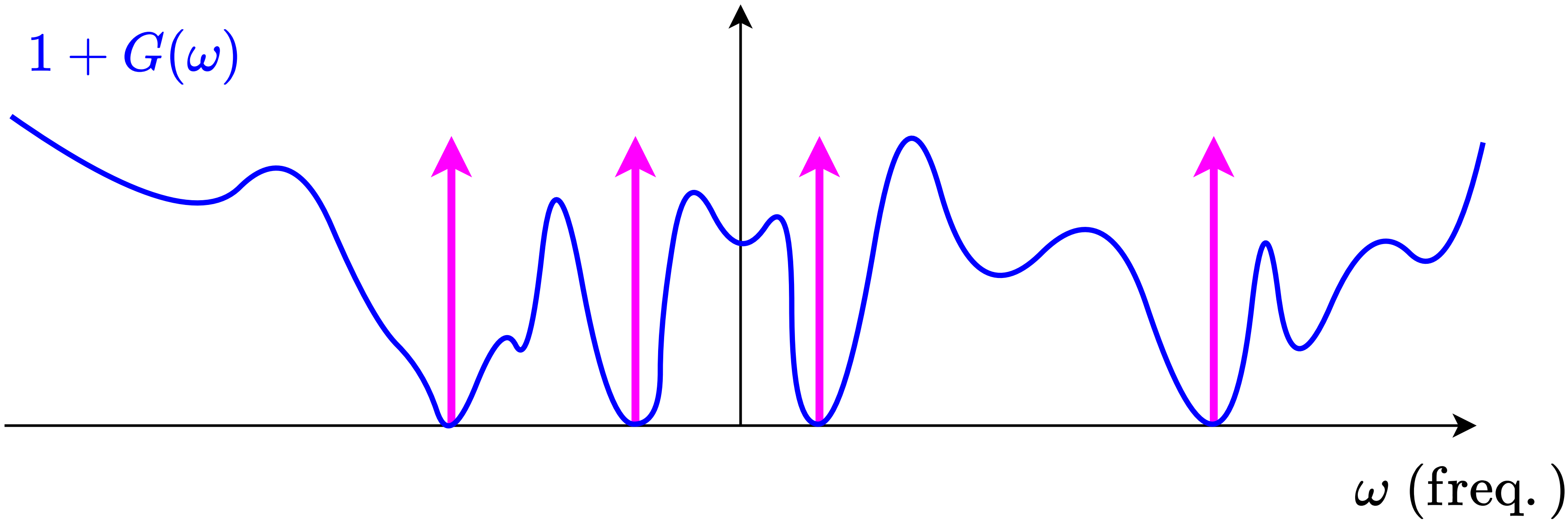


- $(g \circledast q)_n = \sum_{l=1}^{L} g_l q_{n-l}$: general higher-order filter

- $X(\omega) = \bar{X}(\omega) + \underbrace{(1 + G(\omega))}_{\text{flexible to design}} Q(\omega)$

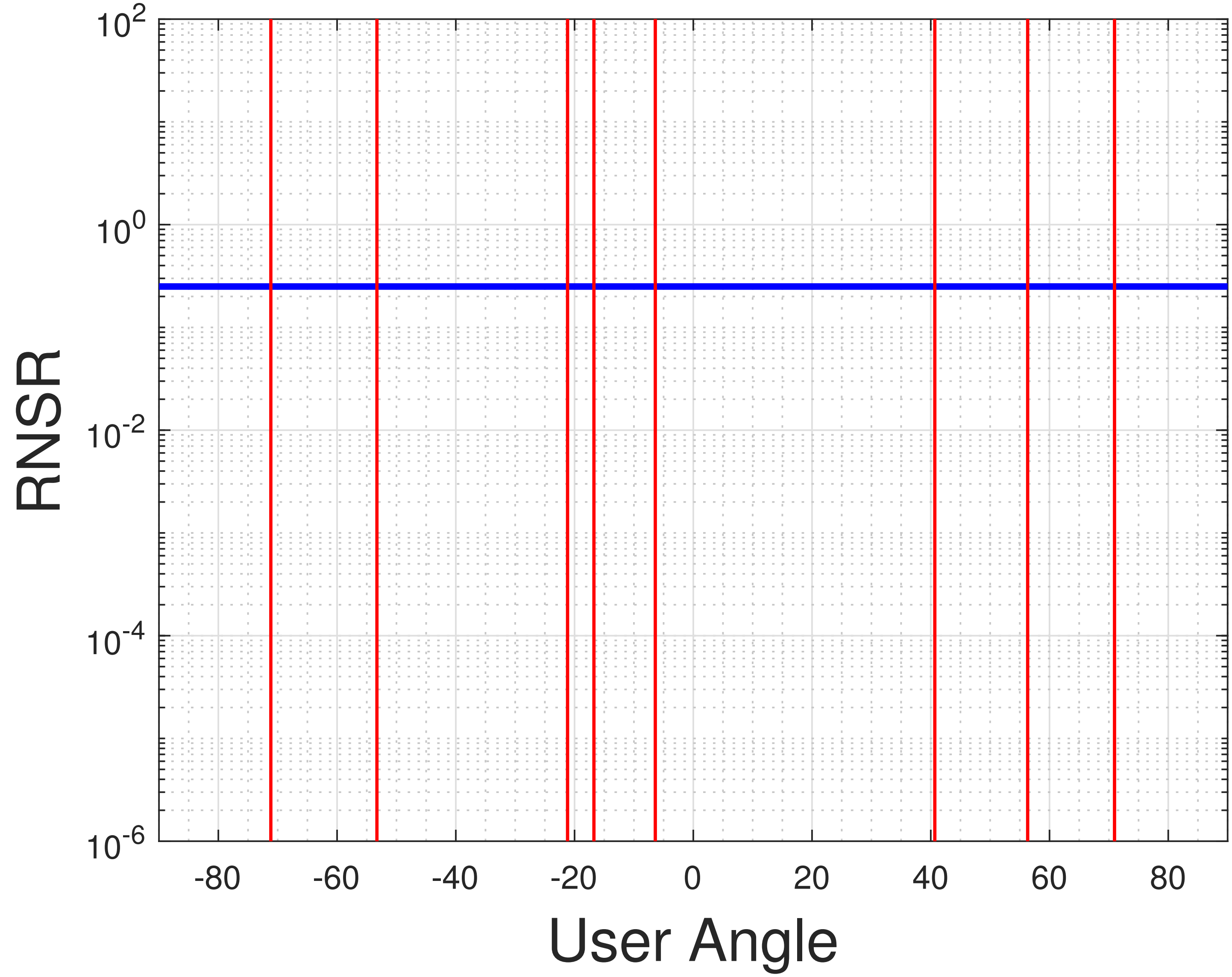
- no-overload condition: $A \leq M - (\sum_{l=1}^{L} |\Re(g_l)| + |\Im(g_l)|) \implies |\Re(q_n)| \leq 1, |\Im(q_n)| \leq 1$ for all n

General ΣΔ Modulator Structure



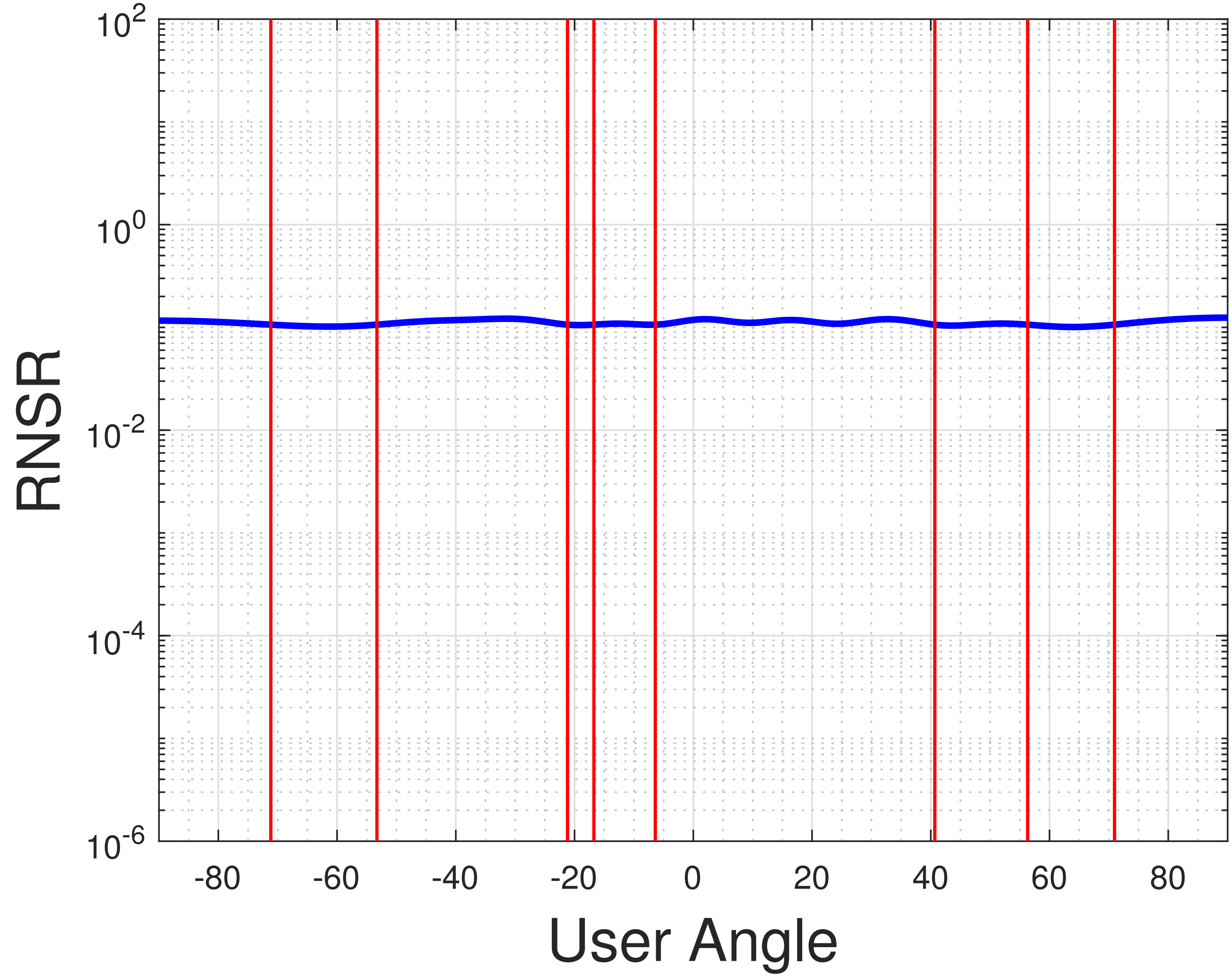
- the ΣΔ modulator needs not be highpass
- the ΣΔ modulator can be designed to have focused q. noise suppression at the users' angles θ_k's in an instantaneous manner

Numerical Result: Noise Shaping Response



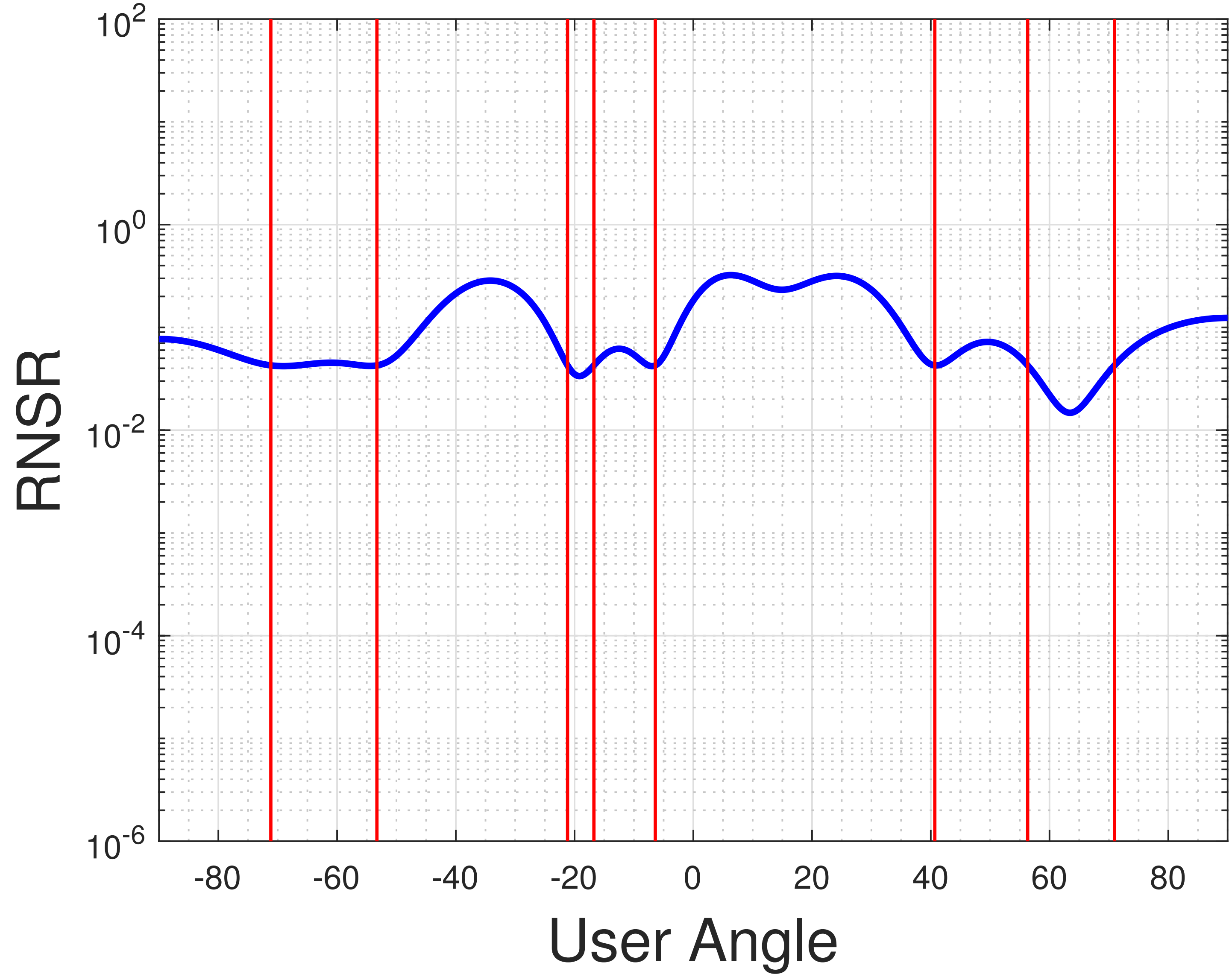
$$N = 1024, K = 8, L = 16, d = \lambda/4, \boldsymbol{M = 2}$$

Numerical Result: Noise Shaping Response



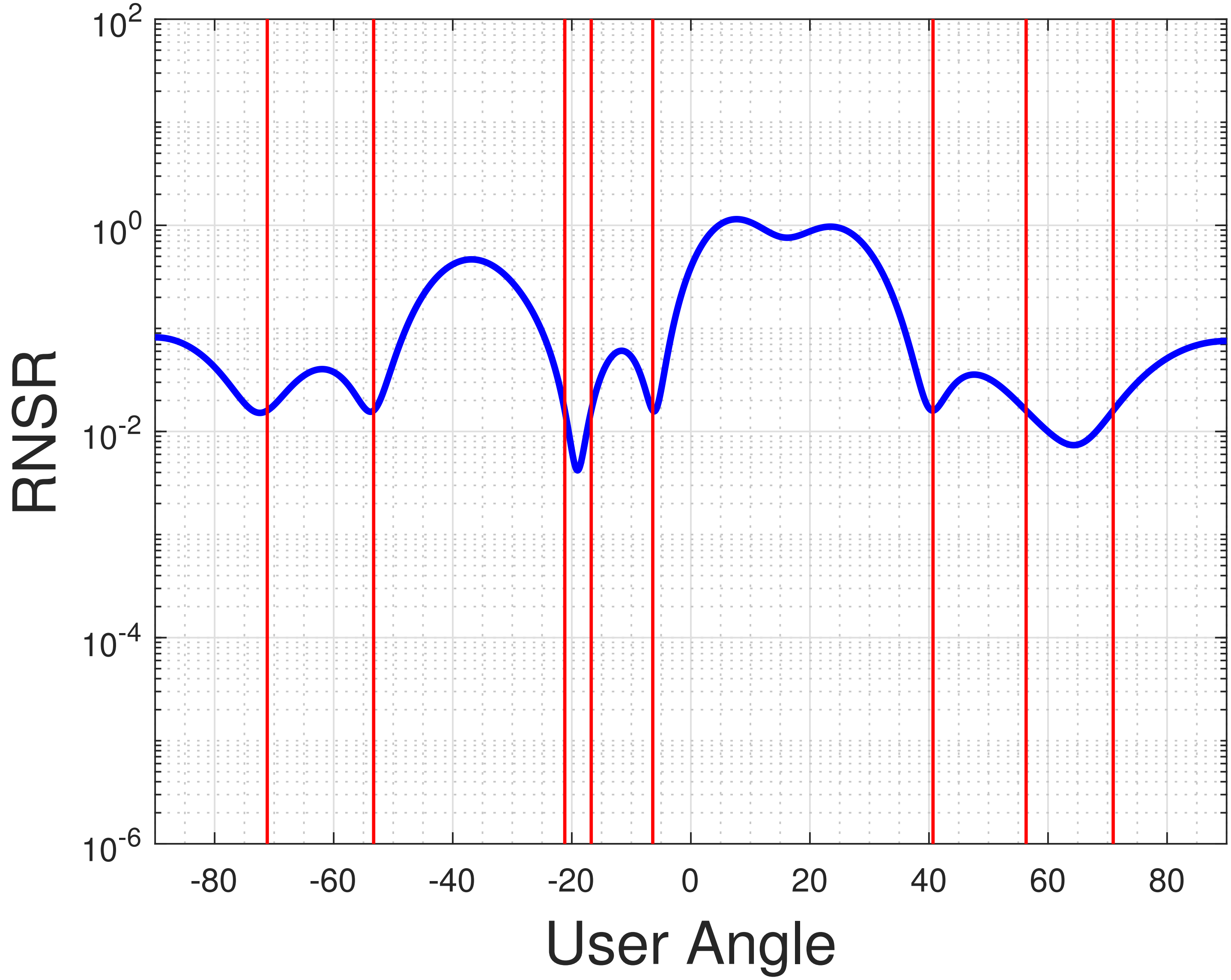
$$N = 1024, K = 8, L = 16, d = \lambda/4, \boldsymbol{M = 3}$$

Numerical Result: Noise Shaping Response



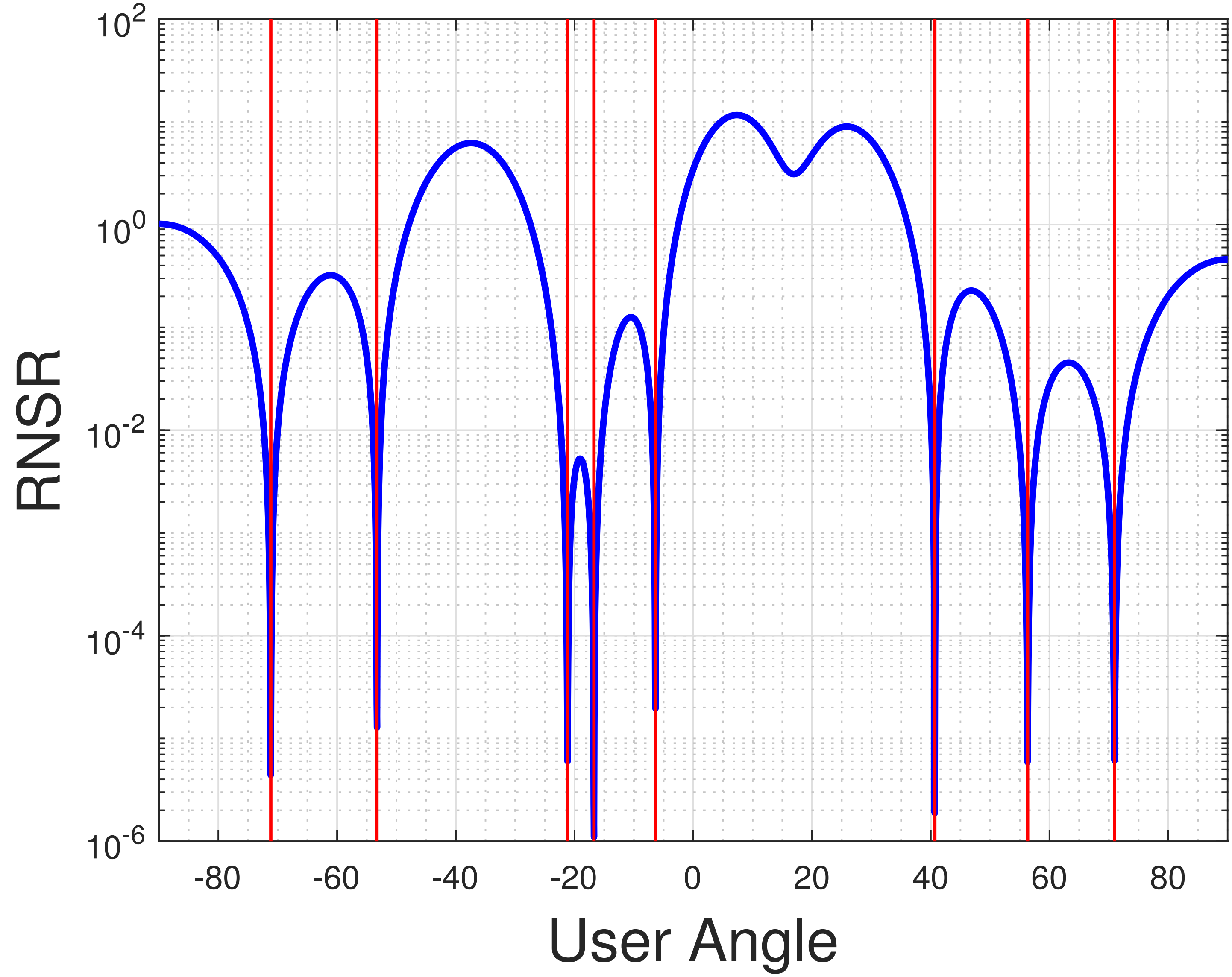
$$N = 1024, K = 8, L = 16, d = \lambda/4, \; \boldsymbol{M = 4}$$

Numerical Result: Noise Shaping Response



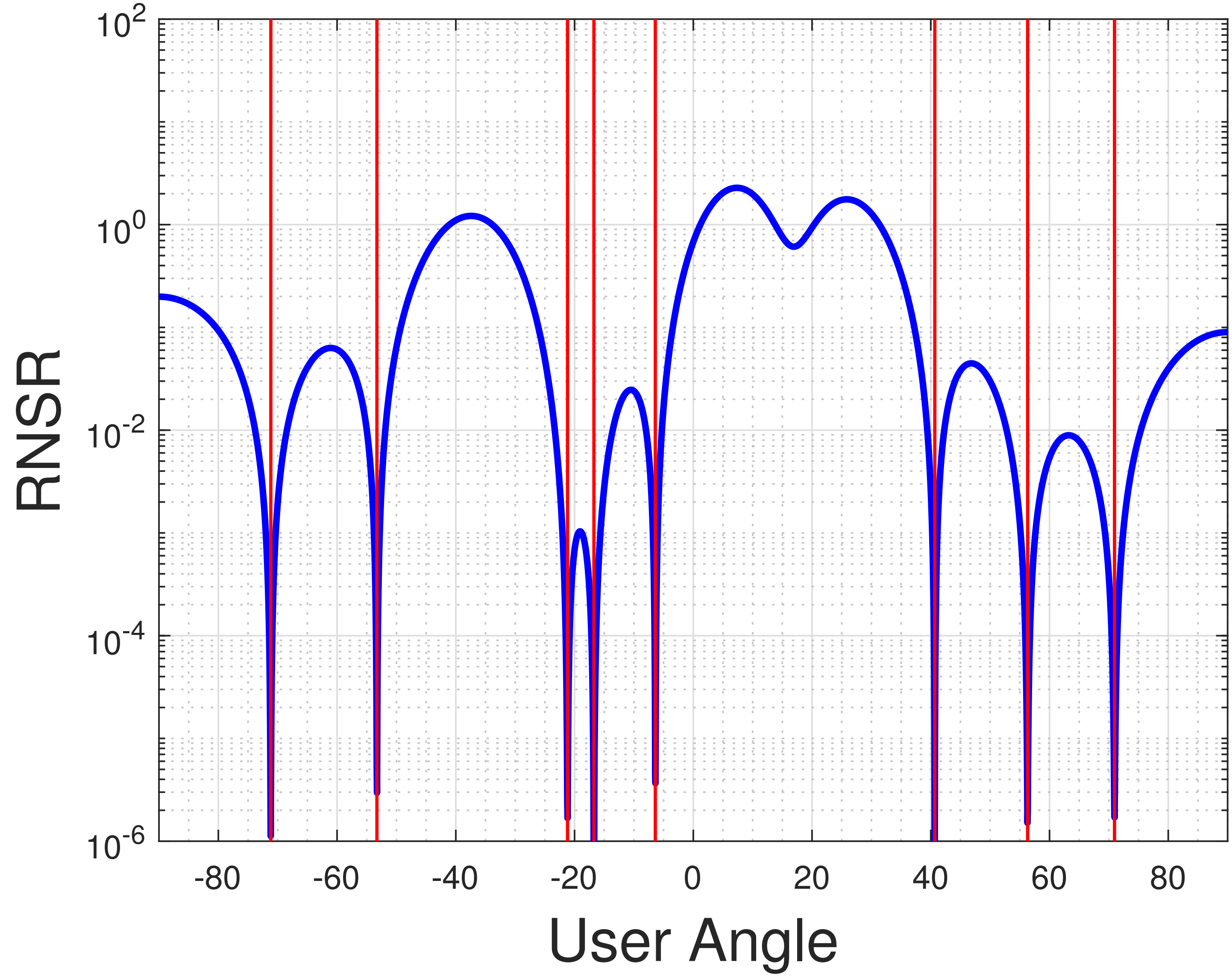
$$N = 1024, K = 8, L = 16, d = \lambda/4, \ \boldsymbol{M = 5}$$

Numerical Result: Noise Shaping Response



$$N = 1024, K = 8, L = 16, d = \lambda/4, \; \boldsymbol{M = 6}$$

Numerical Result: Noise Shaping Response



$$N = 1024, K = 8, L = 16, d = \lambda/4, \boldsymbol{M = 7}$$

SQNR Maximization Design

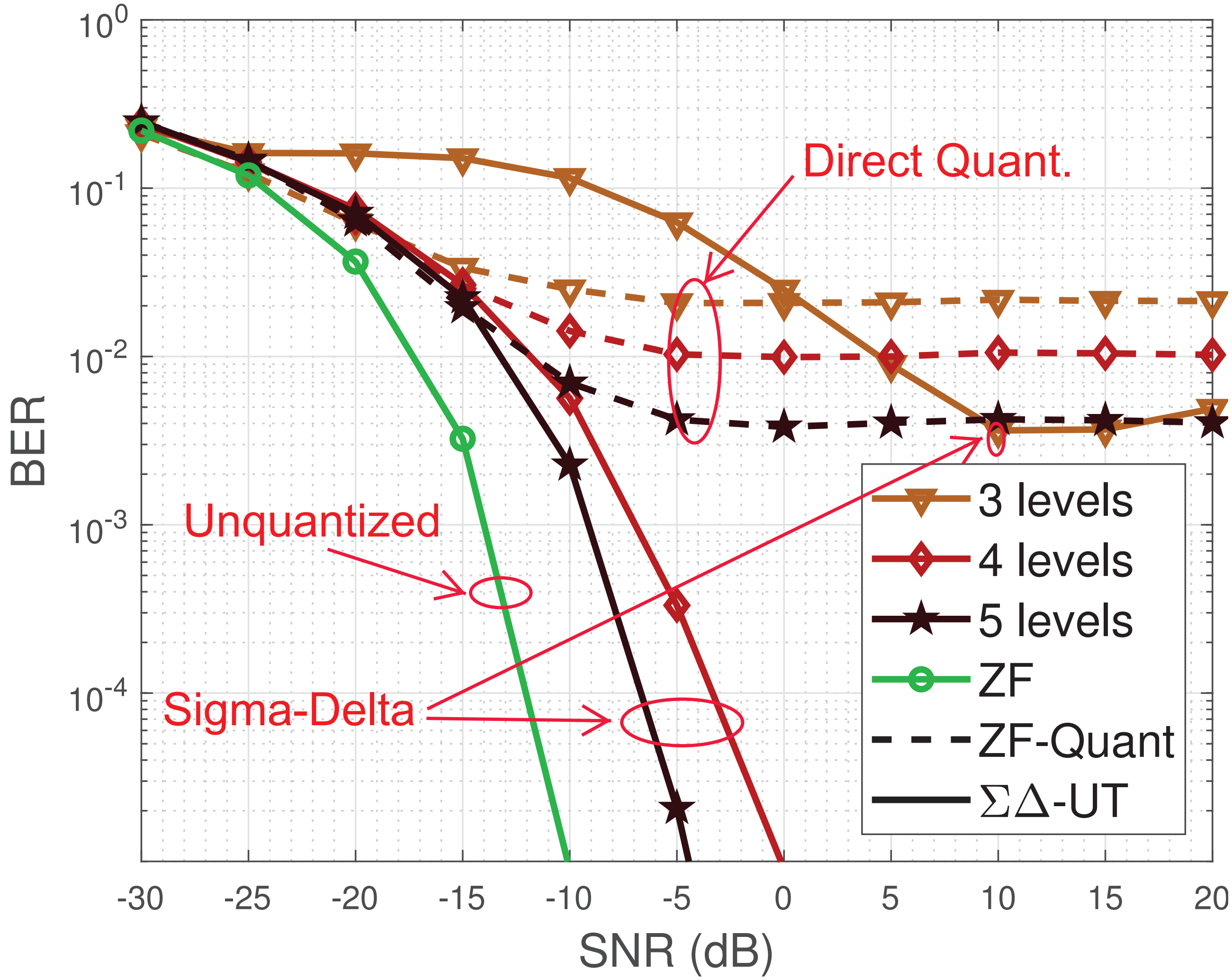
- **design:** maximize the minimum signal-to-quantization-and-noise ratio (SQNR) over all users, subject to the no-overload condition

$$\max_{A \geq 0, \boldsymbol{g} \in \mathbb{C}^L} \; \min_{k=1,\ldots,K} \mathsf{SQNR}_k$$
$$\text{s.t. } A \leq M - \|\Re(\boldsymbol{g})\|_1 - \|\Im(\boldsymbol{g})\|_1$$

where $\mathsf{SQNR}_k = \frac{\rho|\alpha_k|^2 A^2}{\frac{2N\rho|\alpha_k|^2}{3}\left|1+G\left(\frac{2\pi d}{\lambda}\sin(\theta_k)\right)\right|^2 + \sigma_v^2}$.

- can be solved by convex optimization

Simulation Result: User-Targeted (UT) $\Sigma\Delta$

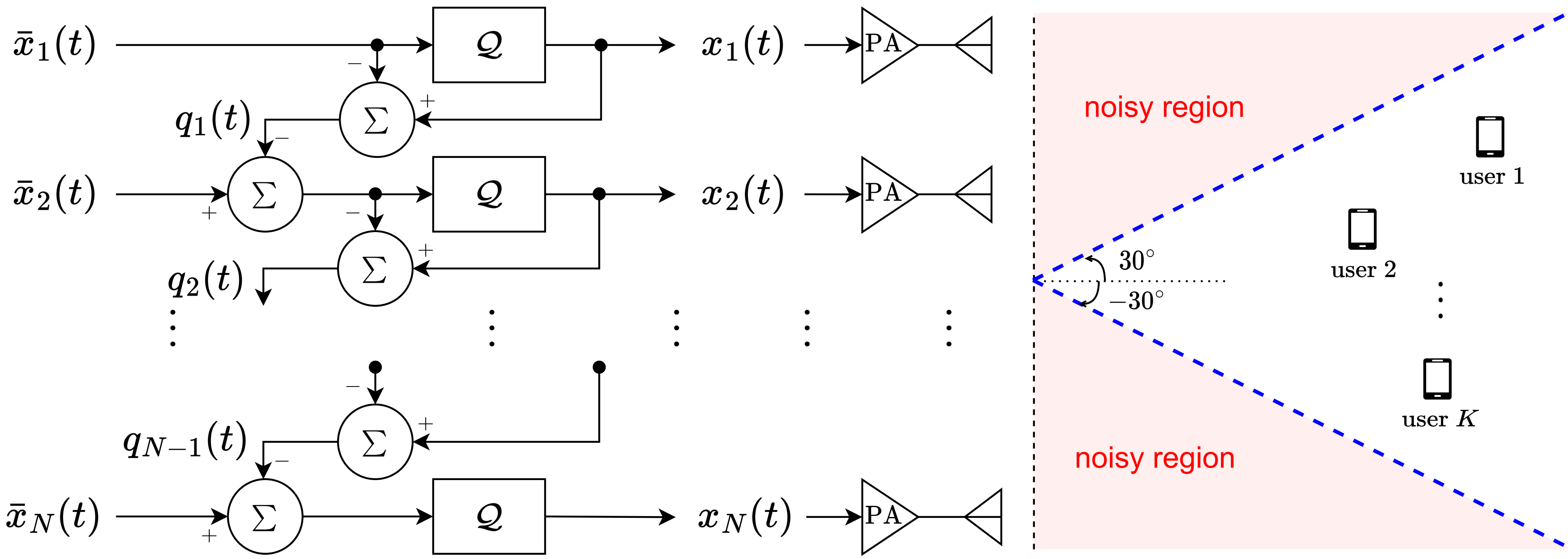


- no. of antennas $N = 1024$, $d = \lambda/2$, angle sector $= [-85^\circ, 85^\circ]$, 64-ary QAM, no. of users $K = 6$

Spatial $\Sigma\Delta$ for PA Distortion Mitigation

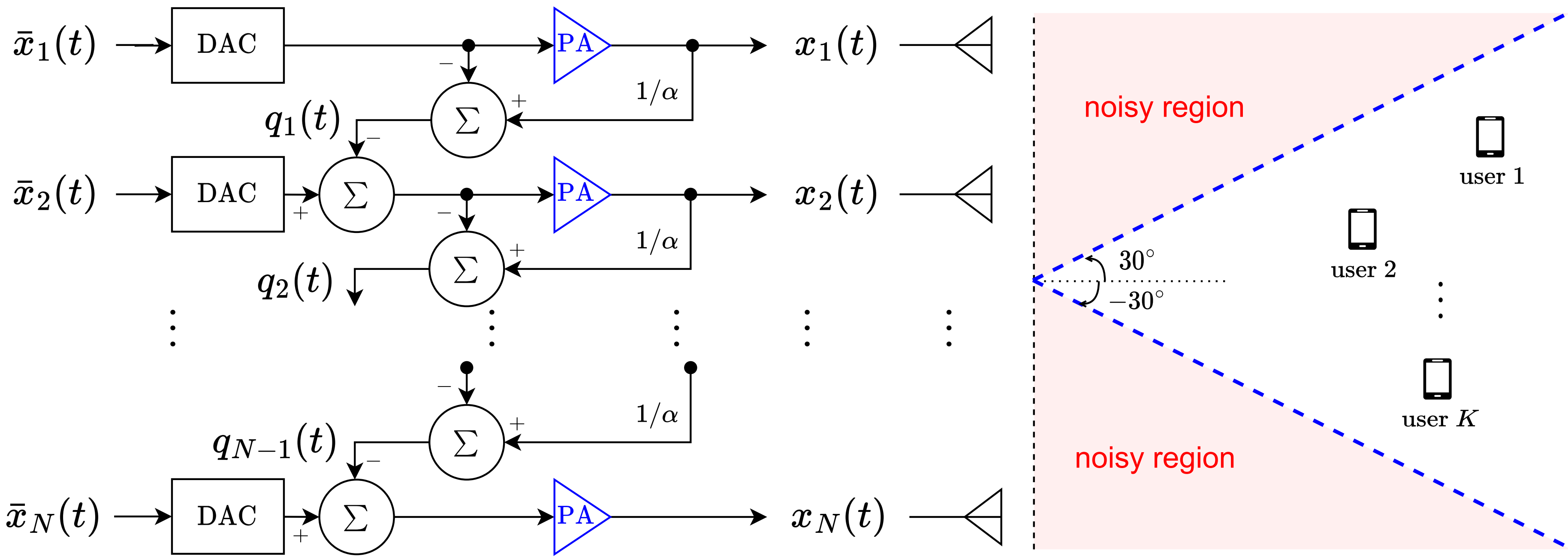
- scenario: large-scale MIMO with high-resolution DACs
- aim: PA distortion mitigation, without backoff and without DPD

Spatial ΣΔ for PA Distortion Mitigation



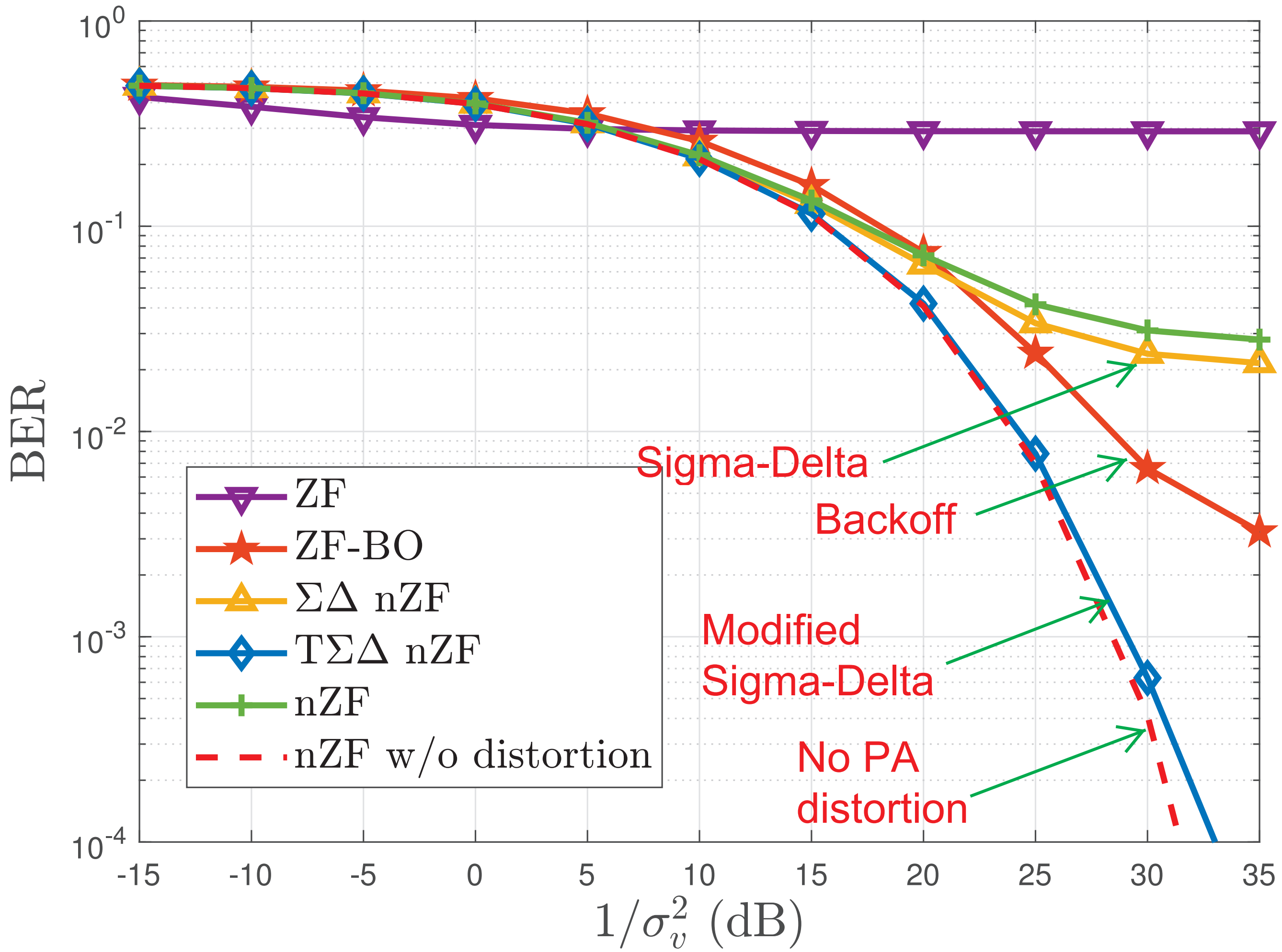
- recall spatial ΣΔ modulation for few-bit MIMO

Spatial $\Sigma\Delta$ Mod. for PA Distortion Mitigation



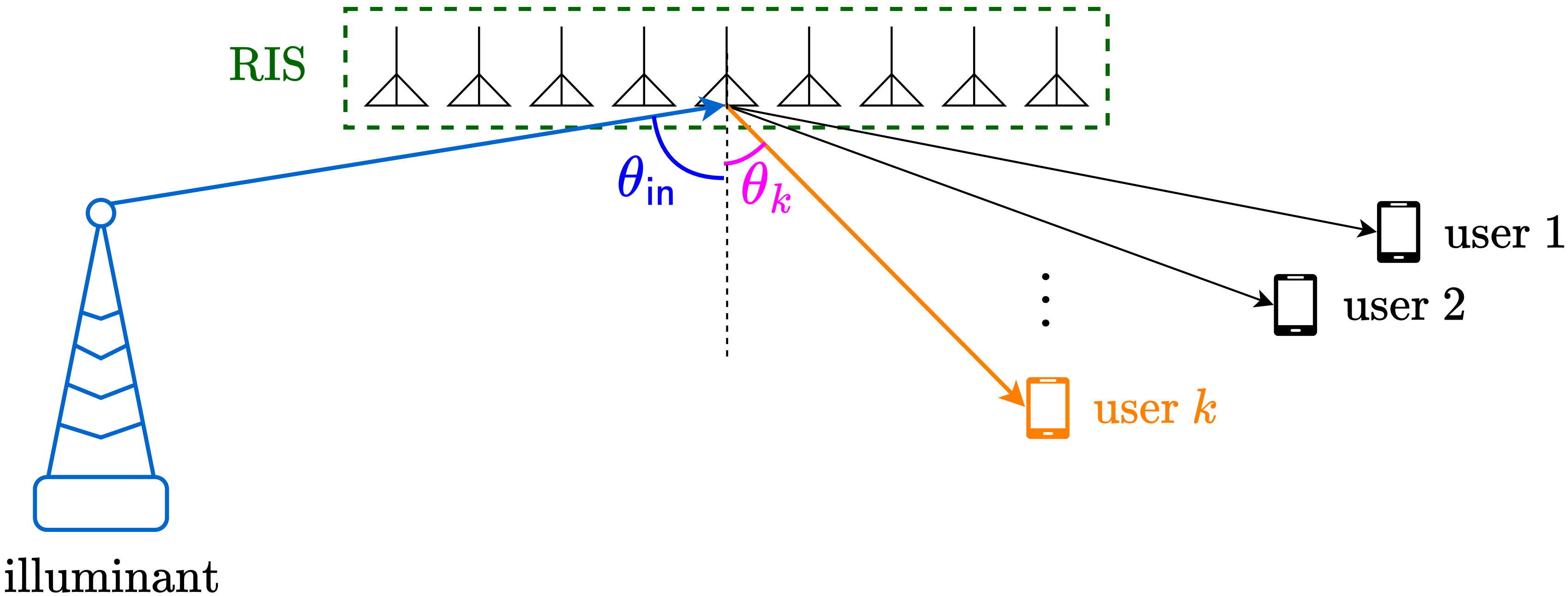
- **idea:** use $\Sigma\Delta$ mod. to shape PA distortion as highpass noise
- quantizer becomes PA, q. noise becomes PA distortion
- built by analog circuits

Simulation Result: MIMO with PA Distortion



- MIMO-OFDM, no. of antennas $N = 16$, OFDM size= 512, $d = \lambda/8$, angle sector $= [-30°, 30°]$, 64-ary QAM, no. of users $K = 4$

Spatial $\Sigma\Delta$ Mod. for RIS

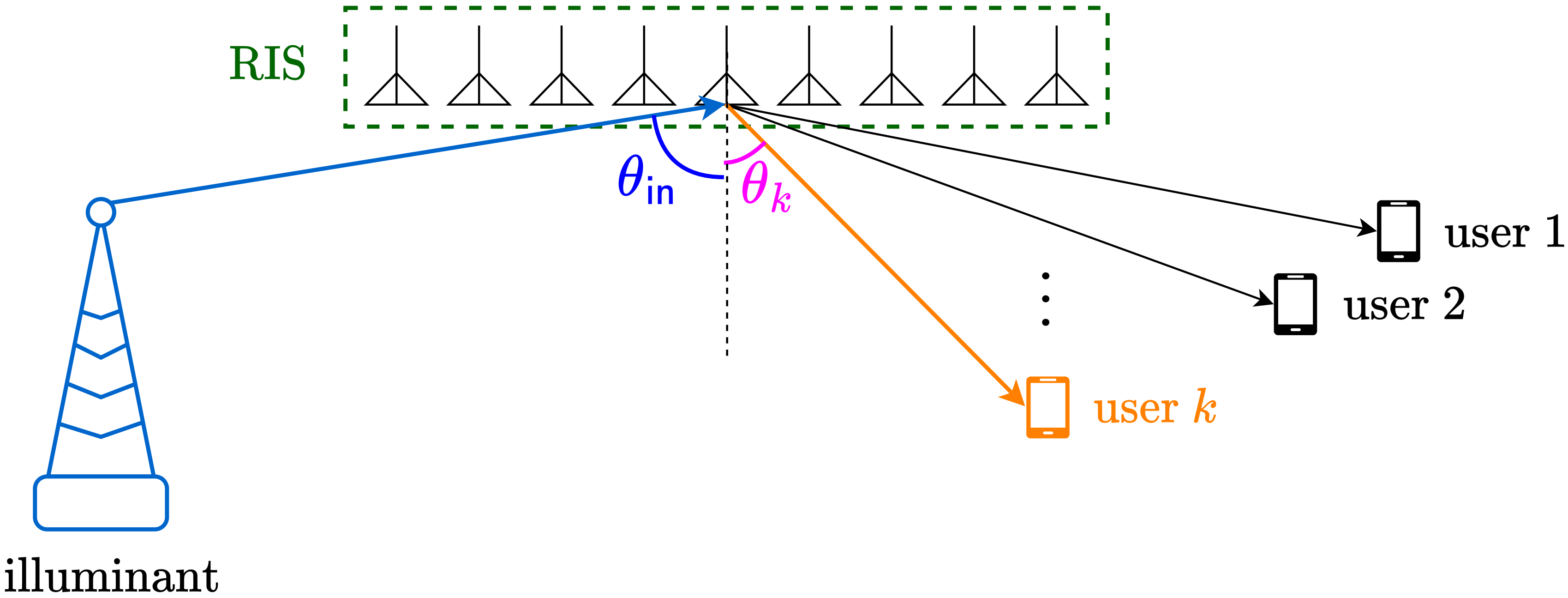


- recent research suggests that reconfigurable intelligent surface (RIS) can be used as an information source

Credit to Victor Cheng at Aarhus University, Denmark, who used this picture to explain RIS in his talk

the illuminant is the candle; the kid is the RIS; users are on the wall

Spatial $\Sigma\Delta$ Mod. for RIS



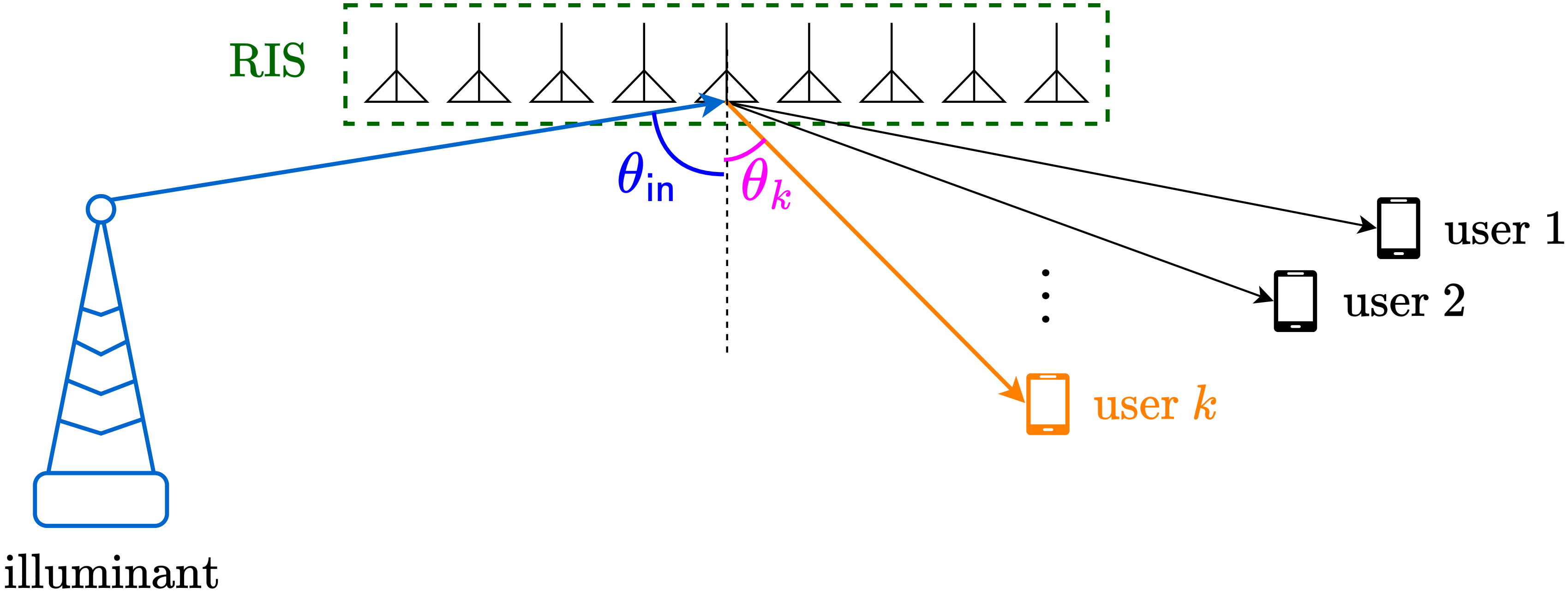
- rx signal model:

$$y_k(t) = \sum_{n=1}^{N} e^{-\mathrm{j}\omega_{\text{in}}(n-1)} \underbrace{e^{\mathrm{j}\psi_n(t)}}_{\text{phase shift of the RIS}} e^{-\mathrm{j}\omega_k(n-1)} + \text{noise}$$

$$= (\boldsymbol{a}(\theta_{\text{in}}) \circ \boldsymbol{a}(\theta_k))^\top \boldsymbol{x}(t) + \text{noise}$$

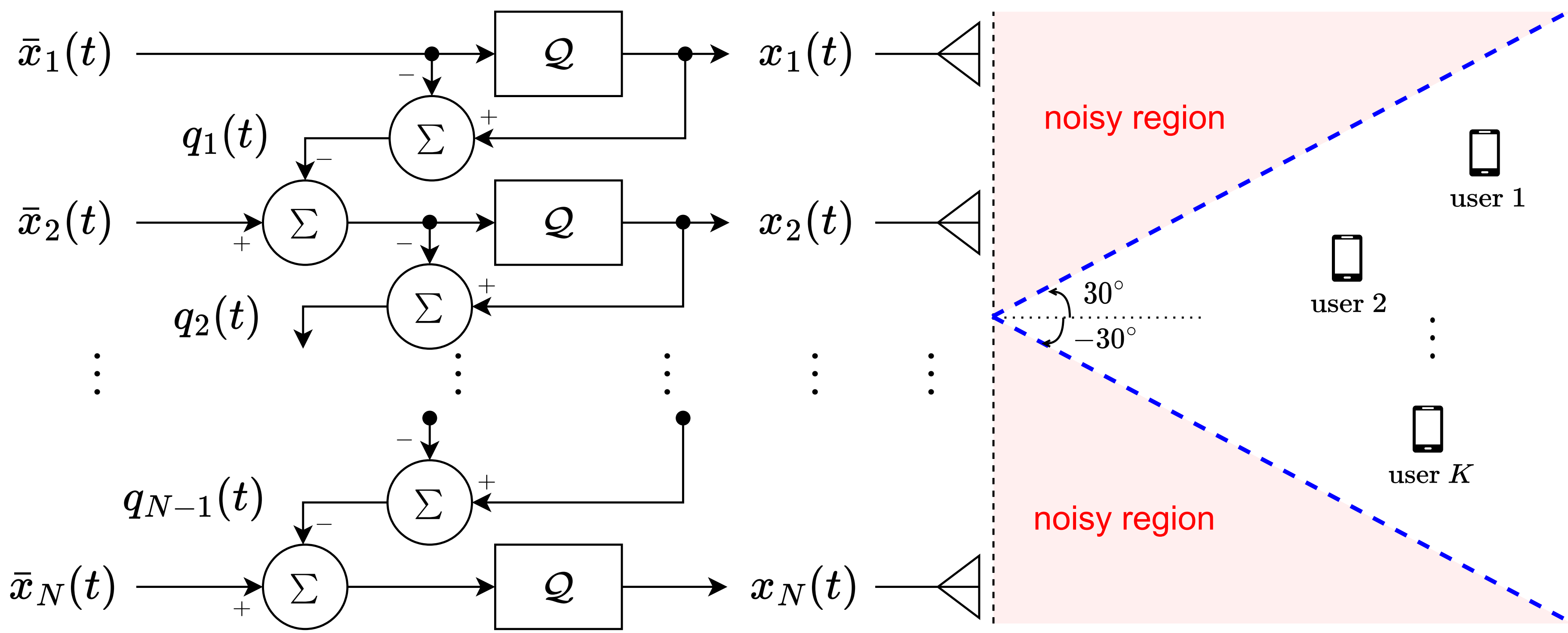
where $\boldsymbol{x}(t) = (e^{\mathrm{j}\psi_1(t)}, e^{\mathrm{j}\psi_2(t)}, \ldots, e^{\mathrm{j}\psi_N(t)})$

Spatial ΣΔ Mod. for RIS



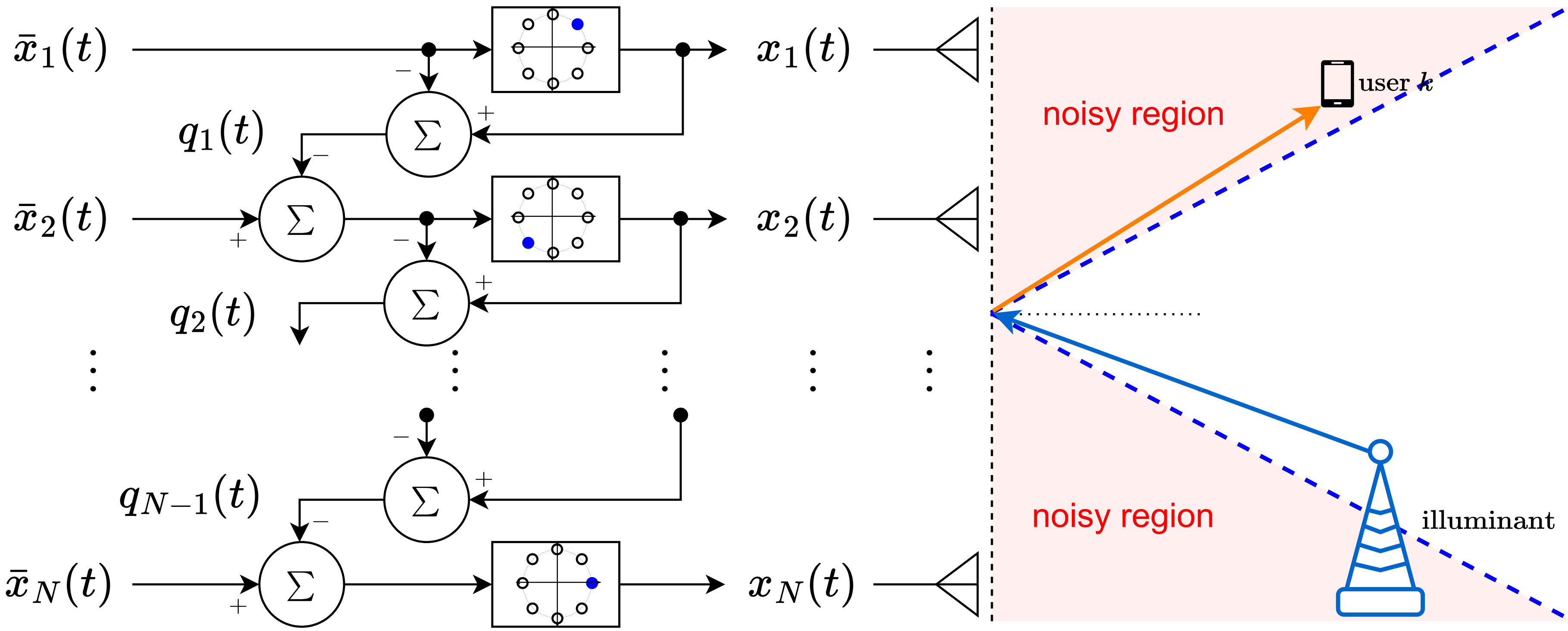
- rx signal model: $y_k(t) = (\boldsymbol{a}(\theta_{\text{in}}) \circ \boldsymbol{a}(\theta_k))^\top \boldsymbol{x}(t) + \text{noise}$
- aim: control the RIS phase vector $\boldsymbol{x}(t) = (e^{j\psi_1(t)}, e^{j\psi_2(t)}, \ldots, e^{j\psi_N(t)})$ such that users receive their designated information symbols

Spatial ΣΔ Mod. for RIS



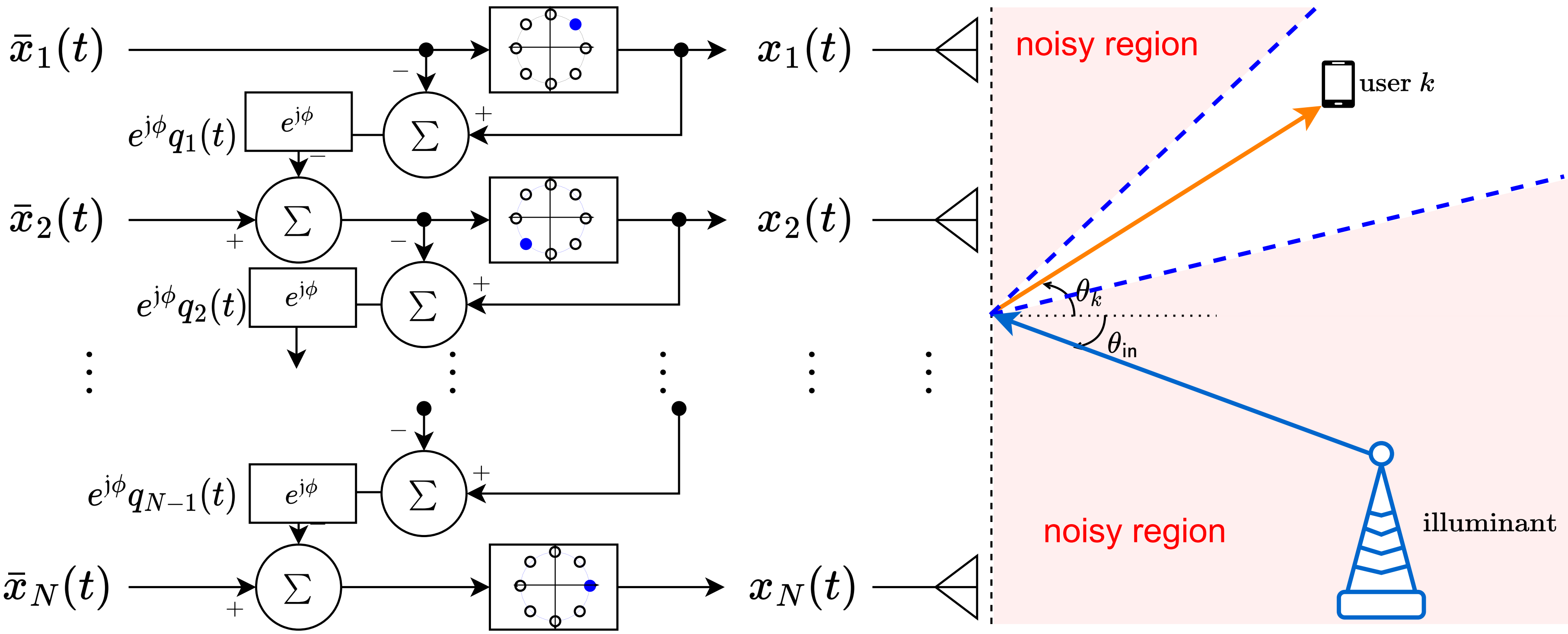
- recall spatial ΣΔ modulation for few-bit MIMO

Spatial ΣΔ Mod. for RIS



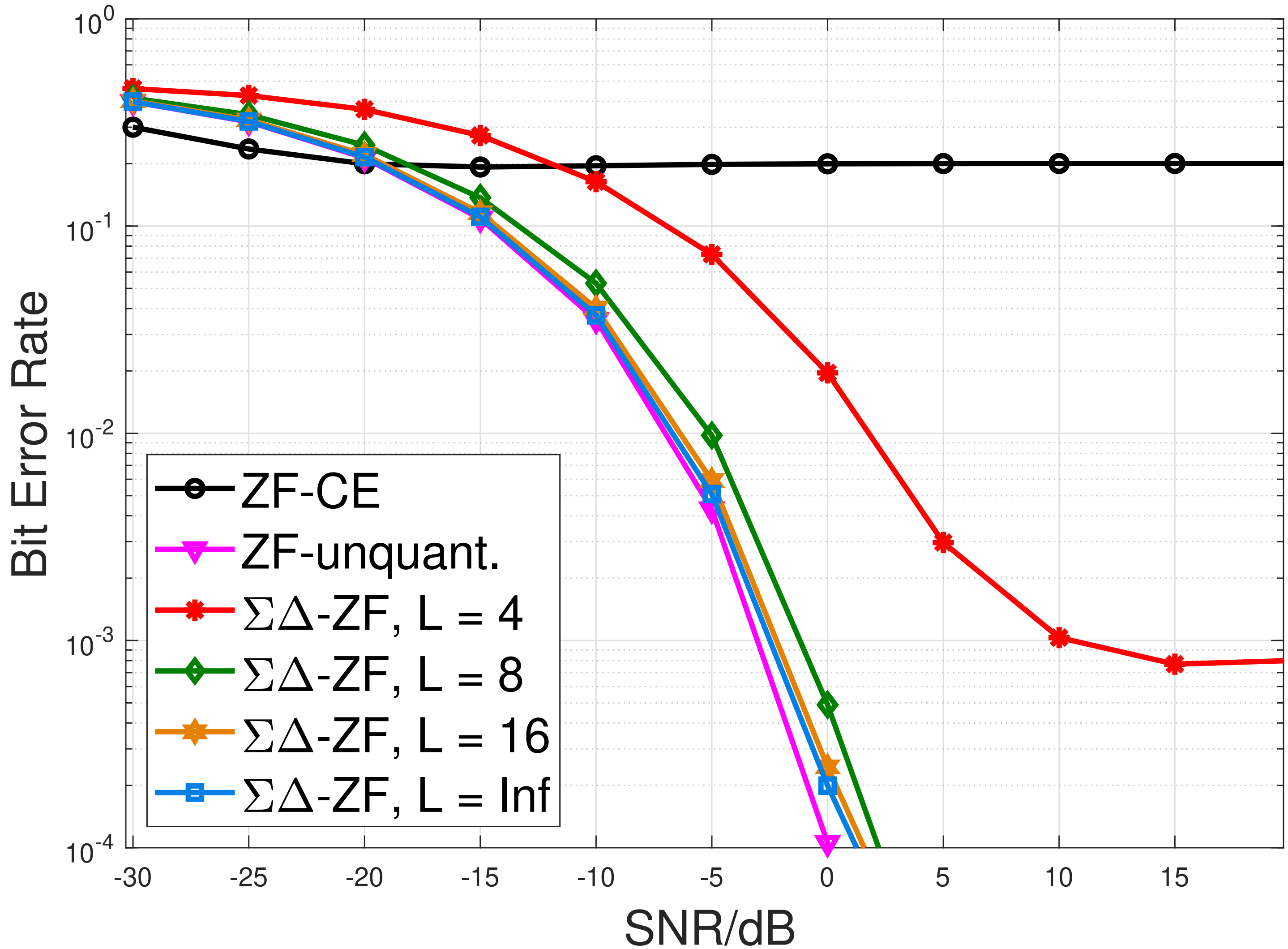
- idea: use spatial ΣΔ mod., with the quantizer being a constant-amplitude discrete-phase rounding function

Spatial ΣΔ Mod. for RIS



- idea: use spatial ΣΔ mod., with the quantizer being a constant-amplitude discrete-phase rounding function

A Bit Error Rate Simulation Result



$(N, K) = (512, 8)$, $d = \lambda/8$, $\theta_{\text{in}} = -60°$, $\theta_k \in [20°, 40°]$, 16-QAM; L is the number of discrete phases used

Precoding Design (Multiuser)

- common theme in precoding (with high resolution DACs):
  - consider linear precoding $\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{w}_k s_k(t)$, where $\boldsymbol{w}_k$ is a beamformer vector and $\{s_k(t)\}_{t=1}^{T}$ is a symbol stream for user k
  - design the precoder via

$$\max_{\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K \in \mathbb{C}^N} \ \text{performance (e.g., sum achievable rate)}$$

$$\text{s.t. } \mathbb{E}[\|\boldsymbol{x}(t)\|_2^2] \leq P \ \text{(average power constraint)}$$

- precoding for spatial $\Sigma\Delta$ modulation:

$$\max_{\boldsymbol{w}_1,\ldots,\boldsymbol{w}_K \in \mathbb{C}^N} \ \text{performance}$$

$$\text{s.t. } |\Re(\bar{x}_n(t))| \leq A, |\Im(\bar{x}_n(t))| \leq A \ \forall n, t$$

(signal amplitude constraints)

Zero-Forcing (ZF) Precoding for Spatial $\Sigma\Delta$

- for illustration, consider real-valued $\bar{\boldsymbol{x}}(t)$'s and PAM symbols (e.g., $s_k(t) \in \{\pm 1, \pm 3\}$)

- model: kth user's received signal $= \boldsymbol{h}_k^T \boldsymbol{x}(t) +$ noise, $t = 1, \ldots, T$

- ZF precoding with normalization:

$$\bar{\boldsymbol{x}}(t) = A \frac{\boldsymbol{H}^\dagger(\boldsymbol{d} \circ \boldsymbol{s}(t))}{\max\limits_{t=1,\ldots,T} \|\boldsymbol{H}^\dagger(\boldsymbol{d} \circ \boldsymbol{s}(t))\|_\infty},$$

  where $\boldsymbol{s}(t) = (s_1(t), \ldots, s_K(t))$; $\boldsymbol{d}$ is a symbol power scaling factor; $\boldsymbol{H}^\dagger$ is the pseudoinverse of $[\ \boldsymbol{h}_1, \ldots, \boldsymbol{h}_K\ ]^T$.

- the normalization makes $|\bar{x}_n(t)| \leq A \ \forall n, t$

Symbol-Level Precoding (SLP) for Spatial $\Sigma\Delta$

- linear precoding: $\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{w}_k s_k(t)$
- SLP: $\boldsymbol{x}(t)$ takes any form
- aim: shape symbols, i.e., $\boldsymbol{h}_k^T \boldsymbol{x}(t) \approx d_k s_k(t)$, at the users' side
- characteristics:
  - good control with signal amplitudes
  - exploit symbol (e.g., QAM) structures to enhance performance at the symbol level

SLP for Spatial $\Sigma\Delta$

- again, consider real-valued $\bar{\boldsymbol{x}}(t)$'s and PAM symbols

- **design:** minimize the maximum symbol-error probability (SEP) over all symbols, subject to signal amplitude constraints

$$\min_{\boldsymbol{d}\geq\boldsymbol{0},\bar{\boldsymbol{x}}(1),\ldots,\bar{\boldsymbol{x}}(T)\in\mathbb{R}^N} \quad \max_{\substack{t=1,\ldots,T,\\ k=1,\ldots,K}} \mathsf{SEP}_{i,t}$$

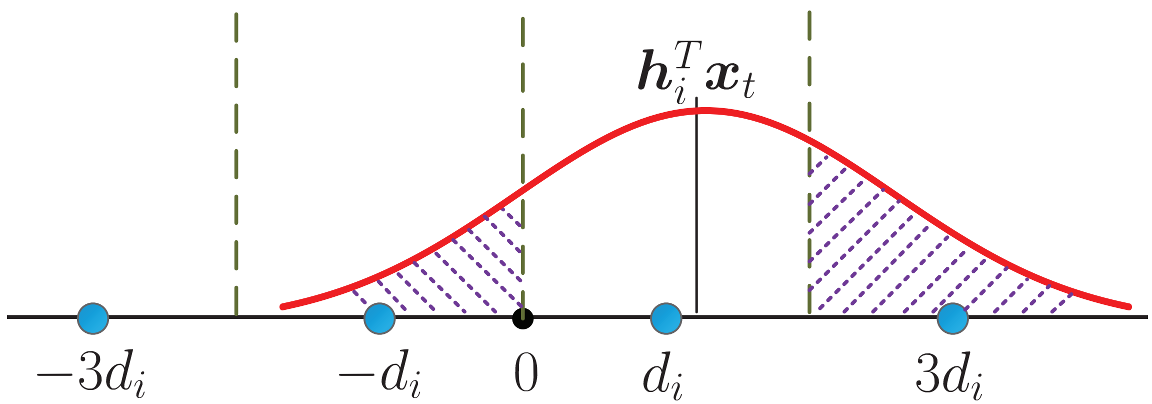
$$\text{s.t. } |\bar{x}_n(t)| \leq A, \; \forall n, t$$

SLP for Spatial $\Sigma\Delta$

- model: $y_k(t) = \boldsymbol{h}_k^T \boldsymbol{x}(t) +$ noise; detection: $\hat{s}_k(t) = \mathrm{dec}(y_k(t)/d_k)$;

$$\begin{aligned}
\mathsf{SEP}_{i,t} &:= \mathrm{Prob}(\hat{s}_k(t) \neq s_k(t)) \\
&\leq Q\left(\frac{d_k - (\boldsymbol{h}_k^T \boldsymbol{x}(t) - d_k s_k(t))}{\sigma_v/\sqrt{2}}\right) + Q\left(\frac{d_k + (\boldsymbol{h}_k^T \boldsymbol{x}(t) - d_k s_k(t))}{\sigma_v/\sqrt{2}}\right) \\
&\leq 2Q\left(\frac{d_k - |\boldsymbol{h}_k^T \boldsymbol{x}(t) - d_k s_k(t)|}{\sigma_v/\sqrt{2}}\right)
\end{aligned}$$

where $Q(x) = \int_x^\infty e^{-z^2/2}/(2\sqrt{\pi})\mathrm{d}x$.



SLP for Spatial $\Sigma\Delta$

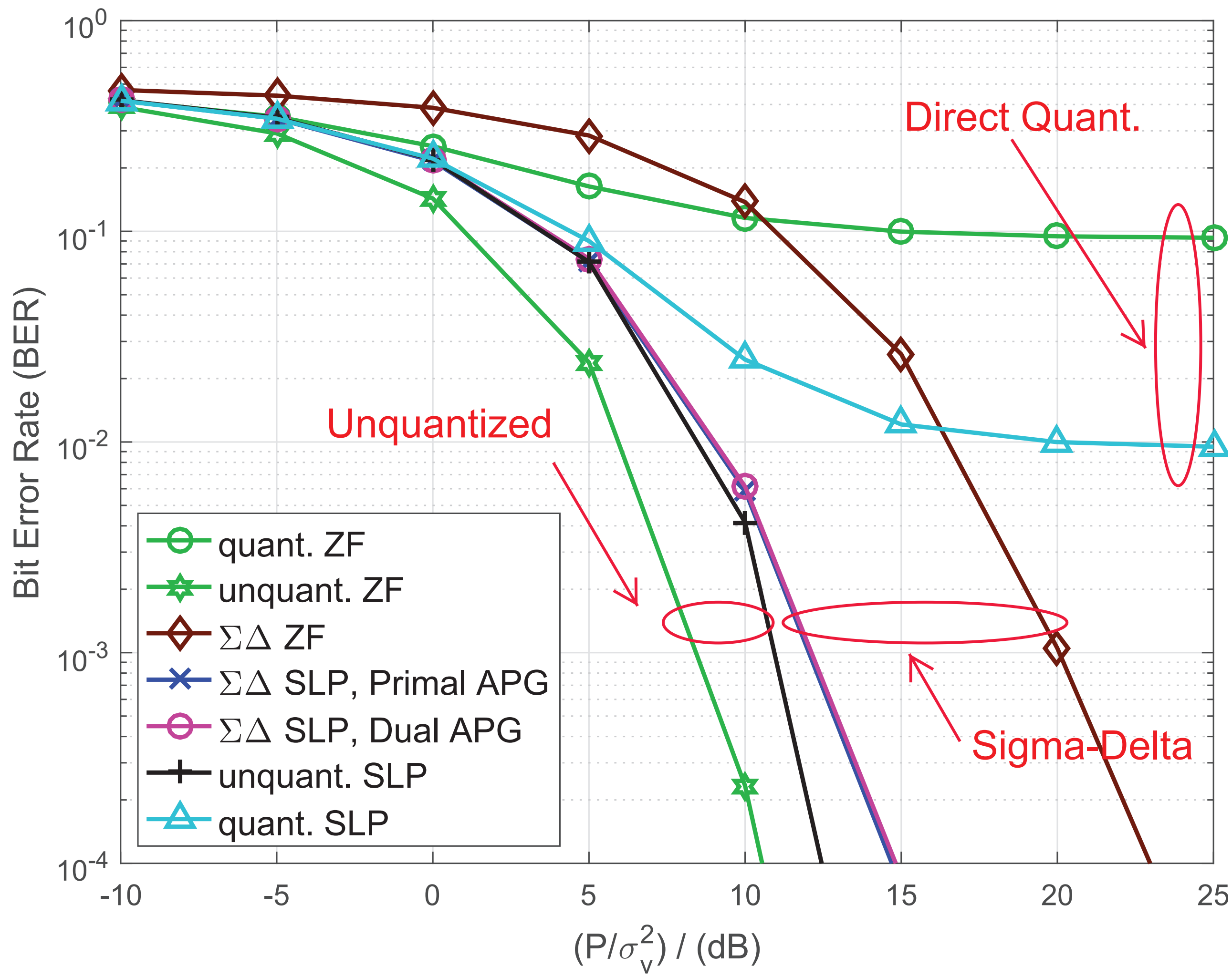
- the design can be rewritten as

$$\min_{\boldsymbol{d}\geq\mathbf{0},\bar{\boldsymbol{x}}(1),\ldots,\bar{\boldsymbol{x}}(T)\in\mathbb{R}^N} \max_{\substack{t=1,\ldots,T,\\ k=1,\ldots,K}} |\boldsymbol{h}_k^T\bar{\boldsymbol{x}}(t) - d_k s_k(t)| - d_k$$
$$\text{s.t. } |\bar{x}_n(t)| \leq A, \ \forall n, t$$

- a convex optimization problem

- our algorithm: smoothing + accelerated proximal gradient

Simulation Result: Multiuser One-Bit MIMO



- number of antennas $N = 256$, angle sector $= [-30^\circ, 30^\circ]$, no. of users $K = 24$, $d = \lambda/8$, 8-ary PSK

Conclusion and Discussion

- $\Sigma\Delta$ mod. in time is classic, dating back to as early as 1962
- its adaptation to space gives new opportunity for few-bit MIMO
- pros: simple, practical, allow us to reuse classic precoding schemes
- cons: q. noise gets to go somewhere
- spatial $\Sigma\Delta$ offers new possibilities for
  - PA distortion mitigation for large-scale MIMO
  - phase-only MIMO
  - MIMO uplink with few-bit ADCs (not covered in this talk)

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