Linear Fractional Semidefinite Relaxation Approaches to Discrete Fractional Quadratic Optimization Problems

Technical Report

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Abstract

This report considers a discrete fractional quadratic optimization problem motivated by a recent application in blind maximum-likelihood (ML) detection of higher-order QAM orthogonal space-time block codes (OSTBCs) in wireless multiple-input multiple-output (MIMO) communications. Since this discrete fractional quadratic optimization problem is NP-hard in general, we present a suboptimal approach, called linear fractional semidefinite relaxation (LFSDR), for obtaining an accurate approximate solution in polynomial complexity. Three possible relaxation possibilities are presented, namely the bounded-constrained LFSDR (BC-LFSDR), the virtually-antipodal LFSDR (VA-LFSDR), and the polynomial-inspired LFSDR (PI-LFSDR). We compare the three LFSDR methods in terms of their approximation performances and complexities. Simulation results under the scenario of blind ML higher-order QAM OSTBC detection are presented to show the performance of the three LFSDR methods as well as their computational complexities.

Keywords: Fractional quadratic function, semidefinite relaxation (SDR), orthogonal space-time block coding (OSTBC), blind maximum-likelihood (ML) detection.

1. Introduction

In the report, we consider a discrete maximization problem with a fractional quadratic objective function as follows

$$f^\star \triangleq \max \;\; \frac{\tilde{\boldsymbol{x}}^T \mathbf{A} \tilde{\boldsymbol{x}} + 2\mathbf{a}^T \tilde{\boldsymbol{x}} + a}{\tilde{\boldsymbol{x}}^T \mathbf{B} \tilde{\boldsymbol{x}} + 2\mathbf{b}^T \tilde{\boldsymbol{x}} + b} \tag{1a}$$

$$\text{s.t. } \tilde{\boldsymbol{x}} \in \{\pm 1, \pm 3, \ldots, \pm(2^q - 1)\}^{N-1}, \tag{1b}$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{(N-1)\times(N-1)}$ are symmetric, $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{N-1}$, $a, b \in \mathbb{R}$, q is an positive integer, and the optimization variables $\tilde{\boldsymbol{x}} = [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{N-1}]^T$ are discrete and are of the 2^q-ary pulse amplitude modulation (PAM) symbols. Problem (1) has been considered recently [2] in the context of blind maximum-likelihood (ML) detection of orthogonal space time block codes (OSTBCs) in wireless communications. The discrete fractional quadratic problem (1) is in general very difficult to solve. In fact, it has been shown that problem (1) is NP-hard in general, which means that it is unlikely to obtain the global optimum solution of (1) in polynomial complexity time for all possible problem instances. Therefore, it would be desirable to consider suboptimal but effective optimization methods for (1).

In the ensuing sections, three suboptimal approaches based on linear fractional semidefinite relaxation (LFSDR) [2] are presented. They are the bounded-constrained LFSDR (BC-LFSDR), the virtually-antipodal LFSDR (VA-LFSDR), and the polynomial-inspired LFSDR (PI-LFSDR). These three methods basically have the same spirit in relaxing the original fractional quadratic problem (1), but employing different ideas in dealing with the discrete constraint in (1b). Actually, the three methods are motivated respectively by the bounded-constrained SDR (BC-SDR) [8], the virtually-antipodal SDR (VA-SDR) [7], and the polynomial-inspired SDR (PI-SDR) [11] that were originally proposed for approximating the coherent ML multiple-input multiple-out (MIMO) detection problem. Following our recent developments in [5] and [6], we will show in this report that the BC-LFSDR and VA-LFSDR are equivalent to each other for any positive integer q, and the PI-LFSDR is equivalent to the BC-LFSDR and VA-LFSDR for $q = 2$. Besides, the complexity of these three methods will be compared by comparing their optimization variable sizes and their numbers of constraints. Simulation results based on the framework of blind ML OSTBC detection [2] are presented to demonstrate the performances and complexities of the three methods.

2. Linear Fractional SDR Approach

Before presenting the LFSDR approach, we require to reformulate (1) into a homogeneous fractional quadratic problem. In particular, by defining

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{a} \\ \mathbf{a}^T & a \end{bmatrix}, \;\; \mathbf{D} = \begin{bmatrix} \mathbf{B} & \mathbf{b} \\ \mathbf{b}^T & b \end{bmatrix}, \tag{2}$$

one can reformulate (1) as follows

$$\max_{\boldsymbol{x}\in\mathbb{R}^n} \frac{\boldsymbol{x}^T\mathbf{G}\boldsymbol{x}}{\boldsymbol{x}^T\mathbf{D}\boldsymbol{x}} \tag{3a}$$

$$\text{subject to (s.t.)}\ x_k \in \{\pm 1, \pm 3, \ldots, \pm(2^q-1)\},\ k=1,\ldots,n-1, \tag{3b}$$

$$x_n \in \{\pm 1\}. \tag{3c}$$

It can be shown [2] that if $\boldsymbol{x}^\star = [x_1^\star, \ldots, x_{n-1}^\star, x_n^\star]^T$ is a solution of (3), then $\tilde{\boldsymbol{x}}^\star = [x_1^\star x_n^\star, \ldots, x_{n-1}^\star x_n^\star]^T$ is a solution of (1). Based on problem (3), we present the three LFSDR approaches, BC-LFSDR, VA-LFSDR and PI-LFSDR, in the ensuing subsections.

2.1 Bounded-Constrained LFSDR

Let us now introduce the BC-LFSDR approach to (3). By defining $\mathbf{X} = \boldsymbol{x}\boldsymbol{x}^T$, one can rewrite (3) in terms of $\mathbf{X}$ as follows:

$$\max_{\mathbf{X}\in\mathbb{R}^{n\times n}} \frac{\mathrm{Tr}(\mathbf{G}\mathbf{X})}{\mathrm{Tr}(\mathbf{D}\mathbf{X})} \tag{4a}$$

$$\text{s.t. } [\mathbf{X}]_{k,k} \in \{1, 9\ldots, (2^q-1)^2\},\ k=1,\ldots,n-1, \tag{4b}$$

$$[\mathbf{X}]_{n,n} = 1, \tag{4c}$$

$$\mathbf{X} \succeq \mathbf{0}\ \ \text{(positive semidefinite (PSD))}, \tag{4d}$$

$$\mathrm{rank}(\mathbf{X}) = 1, \tag{4e}$$

where $[\mathbf{X}]_{k,k}$ denotes the kth diagonal entry of $\mathbf{X}$. In (4), constraints (4b) and (4c) are due to (3b) and (3c), respectively, and (4d) and (4e) are owing to $\mathbf{X} = \boldsymbol{x}\boldsymbol{x}^T$. It can be observed from (4) that the discrete constraints in (4b) and the rank-1 constraint in (4e) are not convex and are difficult to handle. The idea of SDR is to *approximate* problem (3) by removing the rank-1 constraint but keep the PSD constraint $\mathbf{X} \succeq \mathbf{0}$. To deal with the discrete constraint in (4b), we adopt the idea of bound-constrained SDR (BC-SDR) in coherent higher-order QAM MIMO detection [8] where the discrete set $\{1, 9, \ldots, (2^q-1)^2\}$ is relaxed to an interval $[1, (2^q-1)^2]$. We then end up with the following LFSDR problem

$$f_{\mathrm{BC-LFSDR}} \triangleq \max_{\mathbf{X}\in\mathbb{R}^{n\times n}} \frac{\mathrm{Tr}(\mathbf{G}\mathbf{X})}{\mathrm{Tr}(\mathbf{D}\mathbf{X})} \tag{5a}$$

$$\text{s.t. } 1 \le [\mathbf{X}]_{k,k} \le (2^q-1)^2,\ k=1,\ldots,n-1, \tag{5b}$$

$$[\mathbf{X}]_{n,n} = 1,\ \mathbf{X} \succeq \mathbf{0}. \tag{5c}$$

We should emphasize that problem (5) is structurally quite different from the BC-SDR problem in coherent MIMO detection [8]. In the latter, the relaxation problem is a convex SDP and can be directly solved by an interior point SDP algorithm [3,6]. By contrast, problem (5) is a quasiconvex problem. In general, this class of problems can be solved in a globally optimal fashion by the classical bisection method [1] in which a sequence of SDP feasibility problems need to be solved. Fortunately, it can be shown that a globally optimum solution to problem (5) can be obtained by solving just one SDP. The idea is to transform (5) to an SDP and the complete description can be found in [2, Proposition 1].

Since the relaxation problem (5) in general yields a matrix solution with rank greater than one, a rank-one feasible solution to the original problem (3) can be obtained through a Gaussian randomization procedure. Readers are referred to [2] for the details. Next, we present the VA-LFSDR and the PI-LFSDR which use quite different ideas from the BC-LFSDR in handling the 2^q-ary PAM constraint.

2.2 Virtually-Antipodal LFSDR

The idea of VA-SDR is to represent each 2^q-PAM symbol by a linear combination of q binary symbols [7], that is,

$$\begin{aligned} x_k \in \{\pm 1, \pm 3, \ldots, \pm(2^q - 1)\} \Longleftrightarrow \; & x_k = b_{1,k} + 2b_{2,k} + \cdots + (2^{q-1})b_{q,k}, \\ & b_{1,k}, b_{2,k}, \ldots, b_{q,k} \in \{\pm 1\}. \end{aligned} \tag{6}$$

Let us define $\mathbf{b} \triangleq [b_1, b_2, \ldots, b_{q(n-1)+1}]^T = [\,\mathbf{b}_1^T,\ \mathbf{b}_2^T, \cdots, \mathbf{b}_q^T,\ b_{q(n-1)+1}]^T$ where $\mathbf{b}_i = [b_{i,1}, \ldots, b_{i,n-1}]^T \in \{\pm 1\}^{n-1}$ for $i = 1, 2$, and $b_{2n-1} \in \{\pm 1\}$. Then one can express $\boldsymbol{x}$ as

$$\boldsymbol{x} \triangleq \mathbf{T}\mathbf{b} = \begin{bmatrix} \mathbf{I}_{n-1} & 2\mathbf{I}_{n-1} & \cdots & 2^{q-1}\mathbf{I}_{n-1} & 0 \\ \mathbf{0}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T & 1 \end{bmatrix} \mathbf{b}. \tag{7}$$

By substituting (7) into (3), we obtain an equivalent formulation of problem (3):

$$\max_{\mathbf{b} \in \mathbb{R}^{2n-1}} \frac{\mathbf{b}^T \mathbf{T}^T \mathbf{G} \mathbf{T} \mathbf{b}}{\mathbf{b}^T \mathbf{T}^T \mathbf{D} \mathbf{T} \mathbf{b}} \tag{8a}$$

$$\text{s.t.}\ \ b_i \in \{\pm 1\},\ \ i = 1, ..., q(n-1) + 1. \tag{8b}$$

Again, applying the standard SDR principle to (8) gives rise to the VA-LFSDR

$$f_{\rm VA-LFSDR} \triangleq \max_{\mathbf{B} \in \mathbb{R}^{q(n-1)+1 \times q(n-1)+1}} \frac{\mathrm{Tr}(\mathbf{T}^T \mathbf{G} \mathbf{T} \mathbf{B})}{\mathrm{Tr}(\mathbf{T}^T \mathbf{D} \mathbf{T} \mathbf{B})} \tag{9a}$$

$$\text{s.t.}\ \ [\mathbf{B}]_{k,k} = 1,\ k = 1, \ldots, q(n-1) + 1, \tag{9b}$$

$$\mathbf{B} \succeq \mathbf{0}. \tag{9c}$$

Problem (9) is a quasiconvex problem. But, like the BC-LFSDR case, a globally optimum solution to (9) can be effectively obtained by solving an SDP. The idea is again to apply an SDP transformation that follows the same spirit as that in [2, Proposition 1].

2.3 Polynomial-Inspired LFSDR

We herein consider the PI-LFSDR for the case of $q = 2$, for simplicity (extension to $q > 2$ is possible but would be complicated; e.g., see [5] for the case of $q = 4$). Let $w_k = x_k^2$, $k = 1, \ldots, n-1$. According to the following observation [11]:

$$\begin{aligned} w_k \in \{1, 9\} &\Longleftrightarrow (w_k - 1)(w_k - 9) = 0 \\ &\Longleftrightarrow w_k^2 - 10w_k + 9 = 0, \end{aligned} \tag{10}$$

problem (3) can alternatively be expressed as

$$\max_{\boldsymbol{w} \in \mathbb{R}^{n-1}, \boldsymbol{x} \in \mathbb{R}^n} \frac{\boldsymbol{x}^T \mathbf{G} \boldsymbol{x}}{\boldsymbol{x}^T \mathbf{D} \boldsymbol{x}} \tag{11a}$$

$$\text{s.t. } x_k^2 - w_k = 0, \tag{11b}$$

$$w_k^2 - 10w_k + 9 = 0, \; k = 1, \ldots, n-1, \tag{11c}$$

$$x_n \in \{\pm 1\}, \tag{11d}$$

where $\boldsymbol{w} = [w_1, \ldots, w_{n-1}]^T$. By following the SDR principle where we write $\mathbf{X} = \boldsymbol{x}\boldsymbol{x}^T$ and $\mathbf{W} = \boldsymbol{w}\boldsymbol{w}^T$ and then relax them to $\mathbf{X} \succeq \mathbf{0}$ and $\mathbf{W} \succeq \boldsymbol{w}\boldsymbol{w}^T$, we obtain the following PI-LFSDR:

$$f_{\text{PI-LFSDR}} \triangleq \max_{\mathbf{W}, \mathbf{X}, \boldsymbol{w}} \frac{\text{Tr}(\mathbf{G}\mathbf{X})}{\text{Tr}(\mathbf{D}\mathbf{X})} \tag{12a}$$

$$\text{s.t. } [\mathbf{X}]_{k,k} - w_k = 0, \tag{12b}$$

$$[\mathbf{W}]_{k,k} - 10w_k + 9 = 0, \; k = 1, \ldots, n-1, \tag{12c}$$

$$[\mathbf{X}]_{n,n} = 1, \tag{12d}$$

$$\mathbf{X} \succeq \mathbf{0}, \tag{12e}$$

$$\mathbf{W} - \boldsymbol{w}\boldsymbol{w}^T \succeq \mathbf{0}. \tag{12f}$$

Like the BC-LFSDR and VA-LFSDR, the SDP transformation idea in [2, Proposition 1] is applicable to the PI-LFSDR problem in (12). Thus, a globally optimum solution to (12) can also be obtained by solving an SDP.

2.4 Computational Complexity and Performance Comparisons

One can observe from (5), (9) and (12) that the BC-LFSDR, the VA-LFSDR and the PI-LFSDR posses very different problem structures though all of them are quasiconvex problems. We first compare the complexities of the three LFSDRs in terms of the number of optimization variables and the number of constraints, as shown in Table 1 for the case of $q = 2$. One can see from the table that the BC-LFSDR is most favorable in terms of computational complexity.

On the other hand, our parallel development on a related subject [5, 6] has revealed that the three LFSDR methods in fact are equivalent to each other. We summarize this key result as the following proposition:

Proposition 1

1. *For any positive integer* q*, the BC-LFSDR in* (5) *and the VA-LFSDR in* (9) *are equivalent in the sense that*

$$f_{\rm BC-LFSDR} = f_{\rm VA-LFSDR}, \tag{13}$$

and that the optimum solution of one of the LFSDRs can be used to construct that of another LFSDR.

2. *For the case of* $q = 2$*, the PI-LFSDR in* (12) *and the BC-LFSDR in* (5) *are equivalent in the sense that*

$$f_{\rm BC-LFSDR} = f_{\rm PI-LFSDR}, \tag{14}$$

and that the optimum solution of one of the LFSDRs can be used to construct that of another LFSDR. This equivalence result also holds when PI-LFSDR is extended to $q = 4$.

Readers are referred to [5, 6] for the full details. It is worthwhile to mention that the goal of [6] and [5] is to prove the equivalence of the three SDR methods, BC-SDR, PI-SDR and VA-SDR, in higher-order QAM coherent ML MIMO detection. Since the result presented there is independent of the objective structure of the problems, it is perfectly applicable to the BC-LFSDR, VA-LFSDR and the PI-LFSDR considered in the report. The equivalence of the three LFSDRs will be verified in the next section by simulations.

3. Simulation Results

In this section, we examine the performance of the BC-LFSDR, the PI-LFSDR and the VA-LFSDR by simulations. To this end, we consider the scenario of the blind ML higher-order QAM OSTBC

	Number of variables	Number of constraints
BC-LFSDR	n^2	$(2n-1)$ inequality/equality constraints and 1 PSD constraint
VA-LFSDR	$(2n-1)^2$	$(2n-1)$ inequality/equality constraints and 1 PSD constraint
PI-LFSDR	$n^2+(n-1)^2+(n-1)$	$(2n-1)$ inequality/equality constraints and 2 PSD constraints

Table 1. Comparison of number of optimization variables and constraints of BC-LFSDR, VA-LFSDR, and PI-LFSDR for $q=2$.

detection problem in wireless MIMO systems [2]. Suppose that the MIMO OSTBC system has N_t transmit antennas and N_r receive antennas. The received signal model at the receiver is given by

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{u}_p) + \mathbf{W}_p, \qquad p = 1, \ldots, P. \tag{15}$$

Here,

$\mathbf{Y}_p \in \mathbb{C}^{N_r \times T}$ — received code matrix at block p, with T being the block length of the OSTBCs;

$\mathbf{u}_p \in \mathcal{U}^K$ — transmitted symbol vector at block p, with $\mathcal{U} \subset \mathbb{C}$ being the symbol constellation set and K being the number of symbols per block;

$\mathbf{C}(\mathbf{u}_p) \in \mathbb{C}^{N_t \times T}$ — OSTBC mapping function [10] with

$$\mathbf{C}(\mathbf{u}_p) = \sum_{k=1}^{K} \mathrm{Re}(u_{p,k})\mathbf{A}_k + j\sum_{k=1}^{K} \mathrm{Im}(u_{p,k})\mathbf{B}_k$$

where $j=\sqrt{-1}$ and $\mathbf{A}_k, \mathbf{B}_k \in \mathbb{R}^{N_t \times T}$ are the code basis matrices;

$\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ — MIMO channel matrix;

$\mathbf{W}_p \in \mathbb{C}^{N_r \times T}$ — additive white Gaussian noise matrix with the average power per entry given by σ_w^2.

It is assumed that the channel is frequency flat and it remains static for a number of P consecutive code blocks. The blind ML OSTBC detection problem is to consider the following optimization problem

$$\min_{\substack{\mathbf{u}_p \in \mathcal{U}^K \\ p=1,\ldots,P}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{N_r \times N_t}} \sum_{p=1}^{P} \|\mathbf{Y}_p - \mathbf{H}\mathbf{C}(\mathbf{u}_p)\|^2 \right\}, \tag{16}$$

in which the unknown data $\{\mathbf{u}_p\}_{p=1}^{P}$ and channel $\mathbf{H}$ are jointly detected and estimated, respectively.

Suppose that the 16-QAM signals, e.g., are used, that is,

$$\mathcal{U} = \{\, u = u_R + j\, u_I \mid u_R, u_I \in \{\pm 1, \pm 3\} \,\}.$$

Define

$$\begin{aligned}\mathbf{s}_p &\triangleq [s_{p,1}, \ldots, s_{p,2K}]^T \\ &= [\mathrm{Re}(\mathbf{u}_p^T),\ \mathrm{Im}(\mathbf{u}_p^T)]^T \in \{\pm 1, \pm 3\}^{2K}, \quad (17)\\ \mathbf{s} &= [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_P^T]^T \in \{\pm 1, \pm 3\}^{2PK}. \quad (18)\end{aligned}$$

Then following the reformulation ideas in [2], one can reformulate (16) into a fractional quadratic problem as follows

$$f_{\mathrm{ML}} \triangleq \max_{\tilde{\boldsymbol{x}} \in \{\pm 1, \pm 3\}^{2PK-1}} \frac{\tilde{\boldsymbol{x}}^T \mathbf{R} \tilde{\boldsymbol{x}} + 2(s_{1,1}\mathbf{v}^T)\tilde{\boldsymbol{x}} + s_{1,1}^2 u}{\tilde{\boldsymbol{x}}^T \tilde{\boldsymbol{x}} + s_{1,1}^2}, \quad (19)$$

where $s_{1,1}$ is assumed to be the pilot PAM symbol, and

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{1,1} & \cdots & \mathbf{F}_{1,P} \\ \vdots & \ddots & \vdots \\ \mathbf{F}_{P,1} & \cdots & \mathbf{F}_{P,P} \end{bmatrix} \triangleq \begin{bmatrix} u & \mathbf{v}^T \\ \mathbf{v} & \mathbf{R} \end{bmatrix},$$

$$[\mathbf{F}_{p,q}]_{k,\ell} = \mathrm{Re}\{\mathrm{Tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_\ell \mathbf{Y}_q^H\}\},$$

and $\mathbf{X}_{2k-1} = \mathbf{A}_k$, $\mathbf{X}_{2k} = \mathbf{B}_k$ for $k = 1, \ldots, K$. It can be seen that (19) has an identical form as (1), and hence the three LFSDR methods presented in the previous section can be applied.

In the simulations, we assumed that the channel coefficients $\mathbf{H}$ were independent and identically distributed (i.i.d.) circular complex Gaussian random variables with zero mean and unit variance. The signal-to-noise ratio (SNR) was defined as

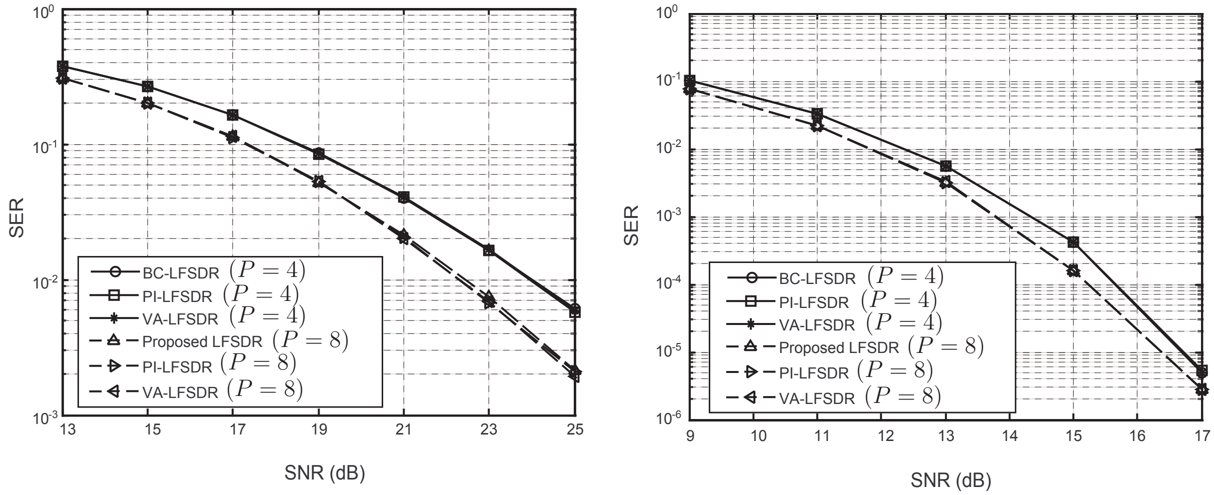
$$\mathrm{SNR} = \frac{\mathrm{E}\{\|\mathbf{H}\mathbf{C}(\mathbf{u}_p)\|_F^2\}}{\mathrm{E}\{\|\mathbf{W}_p\|_F^2\}} = \frac{10 N_t K}{T \sigma_w^2}.$$

The complex 3×4 OSTBC ($N_t = 3$, $T = 4$, $K = 3$) [4]

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 + js_2 & -s_3 + js_4 & -s_5 + js_6 & 0 \\ s_3 + js_4 & s_1 - js_2 & 0 & -s_5 + js_6 \\ s_5 + js_6 & 0 & s_1 - js_2 & s_3 - js_4 \end{bmatrix} \quad (20)$$

was used. SeDuMi [9] was employed to solve the three LFSDR problems. Each simulation result was obtained by averaging at least $10,000$ trials.

Figure 1 presents the performance comparison results for (a) $N_r = 1$ and (b) $N_r = 4$. As seen from this figure, the three different detectors exhibit almost the same performance for different numbers of P and N_r, consistent with the theoretical result in Proposition 1.



(a) $N_r = 1$ (b) $N_r = 4$

Figure 1. Performance (SER v.s. SNR) comparison results of the proposed LFSDR (BC-LFSDR), the PI-LFSDR and the VA-LFSDR blind ML detectors for the complex 3×4 OSTBC.

To compare the complexities of the three LFSDR methods, in Figure 2 we present their average running times for SNR=23 dB and $N_r = 1$. One can see from this figure that the BC-LFSDR is computationally more efficient than the PI-LFSDR and VA-LFSDR.

4. Conclusion

In conclusion, we have considered three LFSDR methods for approximating a discrete fractional quadratic optimization problem, with an application to blind ML higher-order OSTBC detection. While the three LFSDRs are rather different in appearance, they are equivalent problems as indicated by the concurrent theoretical analysis in [5]. We have used simulations to verify that the three LFSDRs indeed yield identical performance. Moreover, we have compared the numerical complexities of the three LFSDRs, and found that the BC-LFSDR is computationally most efficient among the three.

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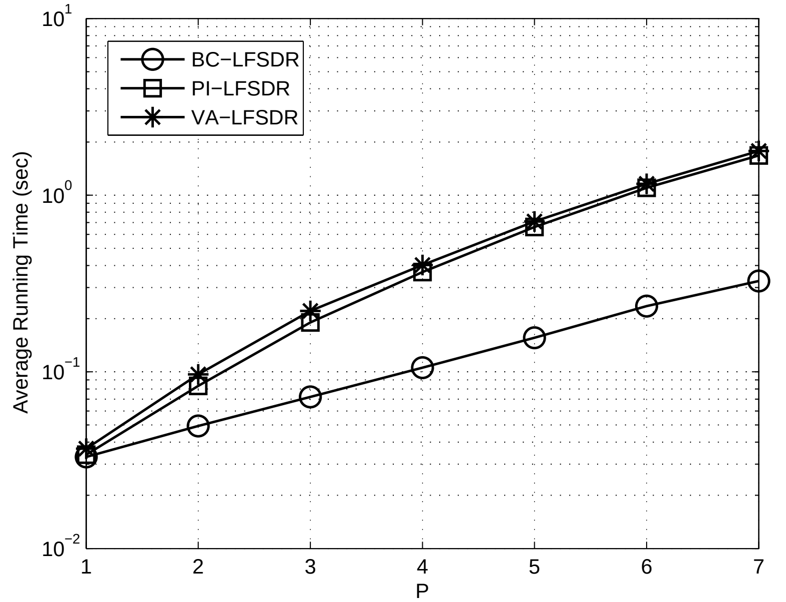

Figure 2. Computational time (in sec) comparison results of the BC-LFSDR, the PI-LFSDR and the VA-LFSDR for SNR=23dB and $N_r = 1$.

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