A Linear Fractional Semidefinite Relaxation Approach to Maximum-Likelihood Detection of Higher-Order QAM OSTBC in Unknown Channels§

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***Abstract*—This paper considers the blind maximum-likelihood (ML) detection problem for orthogonal space-time block codes (OSTBCs) in multiple-input multiple-output flat-fading channels. While the blind ML detection problem for general space-time codes is difficult to solve, it has been shown that for OSTBCs with constant modulus constellations, the blind ML detection problem can be formulated as a discrete quadratic program, and then handled by a powerful convex approximation technique known as semidefinite relaxation (SDR). In this paper, we turn our attention to the case of higher-order QAM OSTBCs. Due to the nonconstant modulus nature of higher-order QAM signals, the blind ML detection problem turns out to be a discrete Rayleigh quotient maximization problem, and as a result the current SDR technique is no longer directly applicable. We propose a linear fractional SDR (LFSDR) approach to this problem. This approach first relaxes the higher-order QAM blind ML detection problem into a quasiconvex problem, followed by a simple solution approximation procedure. In general, quasiconvex problems are computationally more complex to solve than convex problems, but we show that an optimum solution of our quasiconvex problem can be efficiently obtained by solving a convex semidefinite program. The approximation accuracy of the proposed approach relative to other possible relaxation approaches is also studied. Simulation results are presented to demonstrate that the proposed LFSDR-based blind ML detector outperforms some existing suboptimal detectors and can yield promising performance even with a small to moderate number of code blocks.**

Index terms– Orthogonal space-time block coding (OSTBC), maximum-likelihood (ML) detection, noncoherent detection, blind detection, semidefinite relaxation.

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I. Introduction

The orthogonal space-time block codes (OSTBCs) have been of great interest because they can achieve the full transmit diversity by a simple symbol-by-symbol coherent maximum-likelihood (ML) detector. For blind or noncoherent data detection and channel estimation techniques, the OSTBCs are also attractive because, compared to other space-time codes, they have a much simpler blind receiver structure [1]. For instance, blind OSTBC channel estimators based on second-order statistics or signal subspace [2]–[4] have been found to yield simple closed-form solutions and they may achieve near-coherent performance provided that the channel is static for a large number of code blocks. For channels that are static only for two code blocks, the differential OSTBC scheme [5] can be applied with a symbol-by-symbol ML detector at the receiver. It however suffers from a 3 dB performance loss in signal-to-noise ratio (SNR) compared to the coherent ML detector. By contrast, the blind ML detector [6]–[10] based on the deterministic blind ML criterion [7], [11] has been shown to be able to provide near-coherent performance even for a small to moderate number of code blocks (say, 8-20 code blocks). The blind ML detection problem is a computationally difficult optimization problem. For BPSK/QPSK constellations, it has been shown [6] that the blind ML detection problem can be simplified to a Boolean quadratic program (BQP). To solve the BQP, the sphere decoding methods originally developed for integer least squares (LS) problems [12] can be used [6]. It is empirically found that sphere decoding is computationally very efficient in solving BQP problems of small size; however its complexity quickly becomes unaffordable for problems of moderate to large size. Alternatively, it has been found that the BQP can be efficiently (in polynomial time) and accurately approximated by a semidefinite relaxation (SDR) method [6], [13]. This successful endeavor has motivated some works that extend the framework to M-ary PSK (MPSK) OSTBCs [9], [10], [14], [15] and to orthogonal space-time block coded orthogonal frequency division multiplexing (OSTBC-OFDM) [16], [17].

In this paper, we consider blind OSTBC detection techniques for higher-order QAM signaling. The detection problems in this case can be quite different compared with their BPSK/QPSK and MPSK counterparts. First, we show in the paper that the higher-order QAM blind ML OSTBC detection problem is equivalent to a discrete optimization problem with a Rayleigh quotient objective function. This problem is much more difficult to handle than the BQP encountered in the BPSK/QPSK case: Not only the former has more complex objective and constraint structures, but the standard SDR and sphere decoding approaches used in the previous works [6], [9], [10], [12] are no longer directly applicable. The modified sphere decoder by Cui and Tellambura [9] can be used to search for the global blind ML solution in this nonconstant modulus case. However, our simulation results will show that for higher-order QAM OSTBCs the complexity of the modified sphere decoder increases very rapidly with the problem size (in an exponential fashion). Another work particularly worth mentioning is that by Xu *et al.* [18] who proposed a more efficient optimal detector for nonconstant modulus blind ML single-input multiple-output (SIMO) detection. This very recently proposed work has not considered the OSTBC scenario so far.

In this paper, we propose a *linear fractional SDR (LFSDR) approach* to efficient approximation of the higher-order QAM blind ML OSTBC detection problem. In this approach, we first apply an SDR idea similar to the bound-constrained SDR (BC-SDR) in higher-order QAM coherent multiple-input multiple-output (MIMO) detection [19]. However, unlike the work in [19], we will be faced with a relaxation problem that is quasiconvex due to its linear fractional objective structure. Though a quasiconvex problem can be optimally solved using the bisection method [20], it is generally argued that solving a quasiconvex problem would be more complex than solving a convex problem. We will show that the optimum solution of our quasiconvex problem can be obtained by simply solving a convex semidefinite program (SDP). Hence, the proposed LFSDR approach can be efficiently implemented, like the previous SDR method for BPSK/QPSK OSTBCs [6]. For the LFSDR approach, two more contributions are provided in this paper. First, we provide a specialized interior-point algorithm (IPA) for the proposed LFSDR, in order to improve the computational efficiency in implementations. Simulation results will show that the specialized IPA is much faster than general-purpose SDP solvers such as SeDuMi [21]. Second, we study the relationship of the proposed LFSDR with other relaxation methods. For instance, we will show that the approximation accuracy of the proposed LFSDR is at least no worse than a simple norm relaxation method.

The rest of this paper is organized as follows. The higher-order QAM blind ML OSTBC detection problem and the associated background are described in Section II. In Section III, the proposed LFSDR-based approximation method is presented. The relationship of the proposed LFSDR method with some other relaxation methods is also investigated in that section. Performance advantages of the proposed LFSDR approach over existing suboptimal methods are demonstrated in Section IV by simulation results. Finally, we give the conclusions in Section V.

II. Problem Statement and Background

We consider an MIMO OSTBC system with N_t transmit antennas and N_r receive antennas. It is assumed that the channel is frequency flat and it remains static for a number of P consecutive code blocks. The respective received signal model is given by

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{u}_p) + \mathbf{W}_p, \qquad p = 1, \dots, P. \tag{1}$$

Here,

$\mathbf{Y}_p \in \mathbb{C}^{N_r \times T}$	received code matrix at block p, with T being the block length of the OSTBCs;
$\mathbf{u}_p \in \mathcal{U}^K$	transmitted symbol vector at block p, with $\mathcal{U} \subset \mathbb{C}$ being the symbol constellation set and K being the number of symbols per block;
$\mathbf{C} : \mathbb{C}^K \to \mathbb{C}^{N_t \times T}$	function that maps the given symbol vector to an OSTBC block;
$\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$	MIMO channel matrix;
$\mathbf{W}_p \in \mathbb{C}^{N_r \times T}$	additive white Gaussian noise matrix with the average power per entry given by σ_w^2.

An OSTBC mapping function $\mathbf{C}(\cdot)$ can always be expressed in a linear dispersion form as [22], [23]

$$\mathbf{C}(\mathbf{u}_p) = \sum_{k=1}^{K} \mathrm{Re}(u_{p,k})\mathbf{A}_k + j\sum_{k=1}^{K} \mathrm{Im}(u_{p,k})\mathbf{B}_k \tag{2}$$

where $j = \sqrt{-1}$ and $\mathbf{A}_k, \mathbf{B}_k \in \mathbb{R}^{N_t \times T}$ are the code basis matrices. The basis matrices are specially designed such that, for any $\mathbf{u}_p \in \mathbb{C}^K$, the orthogonal condition is satisfied:

$$\mathbf{C}(\mathbf{u}_p)\mathbf{C}^H(\mathbf{u}_p) = \|\mathbf{u}_p\|^2 \mathbf{I}_{N_t}, \tag{3}$$

where $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix.

Here we are interested in detecting $\{\mathbf{u}_p\}_{p=1}^P$ from $\{\mathbf{Y}_p\}_{p=1}^P$ without knowing $\mathbf{H}$ (a.k.a. noncoherent OSTBC detection). To this end, we consider the deterministic blind ML problem [7], [11]

$$\min_{\substack{\mathbf{u}_p \in \mathcal{U}^K \\ p=1,\dots,P}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{N_r \times N_t}} \sum_{p=1}^{P} \|\mathbf{Y}_p - \mathbf{H}\mathbf{C}(\mathbf{u}_p)\|^2 \right\}, \tag{4}$$

in which the unknown data $\{\mathbf{u}_p\}_{p=1}^P$ and channel $\mathbf{H}$ are jointly detected and estimated. To see how the joint optimization problem (4) can be handled, let us define

$$\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2, \dots, \mathbf{Y}_P] \in \mathbb{C}^{N_r \times PT}, \tag{5}$$

$$\mathbf{C}(\mathbf{u}) = [\mathbf{C}(\mathbf{u}_1), \mathbf{C}(\mathbf{u}_2), \dots, \mathbf{C}(\mathbf{u}_P)] \in \mathbb{C}^{N_t \times PT}, \tag{6}$$

$$\mathbf{u} = [\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_P^T]^T \in \mathcal{U}^{PK}, \tag{7}$$

and write (4) as

$$\min_{\mathbf{u} \in \mathcal{U}^{PK}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{N_r \times N_t}} \|\mathbf{Y} - \mathbf{H}\mathbf{C}(\mathbf{u})\|_F^2 \right\}. \tag{8}$$

The solution of the inner minimization term in (8) is given by

$$\mathbf{H} = \mathbf{Y}\mathbf{C}^H(\mathbf{u}) \left[\mathbf{C}(\mathbf{u})\mathbf{C}^H(\mathbf{u})\right]^{-1}. \tag{9}$$

By substituting (9) into (8) and after some matrix manipulations, the blind ML problem (4) can be reformulated as the following maximization problem

$$\max_{\mathbf{u}\in\mathcal{U}^{PK}} \mathrm{Tr}\left(\mathbf{Y}\mathbf{C}^H(\mathbf{u})\left[\mathbf{C}(\mathbf{u})\mathbf{C}^H(\mathbf{u})\right]^{-1}\mathbf{C}(\mathbf{u})\mathbf{Y}^H\right), \quad (10)$$

in which $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix. To show how (10) can be further simplified, let us take the QPSK as an example, i.e., $\mathcal{U}=\{\pm 1 \pm j\}$. One can define

$$\mathbf{s}_p \triangleq [s_{p,1},\ldots,s_{p,2K}]^T = [\mathrm{Re}(\mathbf{u}_p^T),\ \mathrm{Im}(\mathbf{u}_p^T)]^T \in \{\pm 1\}^{2K}, \quad (11)$$

and rewrite (2) into a more convenient form as

$$\mathbf{C}(\mathbf{u}_p)=\mathbf{C}(\mathbf{s}_p)=\sum_{k=1}^{2K} s_{p,k}\mathbf{X}_k, \quad (12)$$

where $\mathbf{X}_k=\mathbf{A}_k$ and $\mathbf{X}_{k+K}=j\mathbf{B}_k$ for $k=1,\ldots,K$. Due to the constant modulus property of QPSK and the orthogonal property in (3), the term $\mathbf{C}(\mathbf{u})\mathbf{C}^H(\mathbf{u})$ in (10) can be reduced to

$$\mathbf{C}(\mathbf{u})\mathbf{C}^H(\mathbf{u})=2PK\mathbf{I}_{N_t}, \quad (13)$$

which is constant and does not depend on $\{\mathbf{u}_p\}_{p=1}^P$. By utilizing the linear dispersion property in (12) and by (13), it can be shown [6] that problem (10) can be expressed as a Boolean quadratic program (BQP) as follows

$$\mathbf{s}^\star = \arg\max_{\mathbf{s}\in\{\pm 1\}^{2PK}} \mathbf{s}^T\mathbf{F}\mathbf{s}, \quad (14)$$

where $\mathbf{s}=[\mathbf{s}_1^T,\ldots,\mathbf{s}_P^T]^T\in\{\pm 1\}^{2PK}$ and

$$\mathbf{F}=\begin{bmatrix}\mathbf{F}_{1,1} & \cdots & \mathbf{F}_{1,P}\\ \vdots & \ddots & \vdots \\ \mathbf{F}_{P,1} & \cdots & \mathbf{F}_{P,P}\end{bmatrix}\in\mathbb{R}^{2PK\times 2PK},$$

$$[\mathbf{F}_{p,q}]_{k,\ell}=\mathrm{Re}\{\mathrm{Tr}\{\mathbf{Y}_p\mathbf{X}_k^H\mathbf{X}_\ell\mathbf{Y}_q^H\}\}.$$

Though the BQP in (14) appears to be simple, it in essence is an NP-hard problem, which indicates that the BQP is unlikely to be solved in polynomial time. Fortunately, recent developments [6] have shown that an approximation method based on semidefinite relaxation (SDR) is able to provide a near-optimal solution of (14) with a polynomial-time worst-case complexity of $O((2PK)^{3.5})$. Alternatively, the BQP can be optimally solved by a standard sphere decoding algorithm. Readers are referred to [6] for the details; also to [9], [10], [15], [24] and [25] for the extension of the SDR method and the sphere decoding algorithm to MPSK signals.

In this paper, we investigate the blind ML OSTBC detection problem (4) with nonconstant modulus signal constellations. Specifically, we focus on the case of higher-order QAM signaling (e.g., 16-QAM and 64-QAM). Mathematically, a 4^q-QAM constellation set (where $q>1$ is a positive integer) can be represented by

$$\mathcal{U}=\{\ u=u_R+j\ u_I \mid u_R,u_I\in\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}\ \}.$$

Since a QAM symbol is composed of two independent pulse amplitude modulated (PAM) symbols, the $\mathbf{s}_p$ in (11) in this case is a 2^q-PAM vector, i.e.,

$$\mathbf{s}_p \triangleq [s_{p,1},\ldots,s_{p,2K}]^T\in\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}^{2K}. \quad (15)$$

Because QAM signals are not constant modulus, we instead have (13) as

$$\mathbf{C}(\mathbf{u})\mathbf{C}^H(\mathbf{u})=\|\mathbf{u}\|^2\mathbf{I}_{N_t}=\|\mathbf{s}\|^2\mathbf{I}_{N_t}. \quad (16)$$

By following the same reformulation idea as for BPSK and QPSK constellations [6], but considering (16), one can show that the blind ML OSTBC detection problem [viz., Problem (10)] for the 4^q-QAM signaling case can be simplified to a discrete maximization problem as follows

$$\mathbf{s}^\star=\arg\max_{\mathbf{s}\in\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}^{2PK}} \frac{\mathbf{s}^T\mathbf{F}\mathbf{s}}{\mathbf{s}^T\mathbf{s}}. \quad (17)$$

Comparing (17) with the BQP in (14), one can observe that the former has a Rayleigh quotient objective function. As a result, one would find that the standard SDR method and sphere decoding algorithm for the QPSK constellation cannot be applied to the higher-order QAM case.

III. Linear Fractional SDR Approach

In this section, we present the main results of this paper, namely the LFSDR approach to the approximation of the higher-order QAM blind ML OSTBC detection problem.

A. Linear Fractional Semidefinite Relaxation (LFSDR)

In the higher-order QAM blind ML OSTBC detection problem in (17), one can see that the optimal symbol decision suffers from ambiguity up to a scalar of $\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}$. To fix this problem, we assume that one of the 2^q-PAM symbols in $\mathbf{s}$ is known to the receiver; e.g., through the use of one pilot symbol. Without loss of generality, $s_{1,1}$ is assumed to be the known symbol.

Let us partition

$$\mathbf{F}=\begin{bmatrix}u & \mathbf{v}^T\\ \mathbf{v} & \mathbf{R}\end{bmatrix},\quad \mathbf{s}=\begin{bmatrix}s_{1,1}\\ \tilde{\boldsymbol{x}}\end{bmatrix}, \quad (18)$$

where $u\in\mathbb{R}$, $\mathbf{v}\in\mathbb{R}^{2PK-1}$, $\mathbf{R}\in\mathbb{R}^{(2PK-1)\times(2PK-1)}$ and $\tilde{\boldsymbol{x}}\in\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}^{2PK-1}$. With $s_{1,1}$ being known, the blind ML detection problem [in (17)] is modified as

$$f_{\mathrm{ML}}\triangleq \max_{\tilde{\boldsymbol{x}}\in\{\pm 1,\pm 3,\ldots,\pm(2^q-1)\}^{2PK-1}} \frac{\tilde{\boldsymbol{x}}^T\mathbf{R}\tilde{\boldsymbol{x}}+2(s_{1,1}\mathbf{v}^T)\tilde{\boldsymbol{x}}+s_{1,1}^2u}{\tilde{\boldsymbol{x}}^T\tilde{\boldsymbol{x}}+s_{1,1}^2} \quad (19a)$$

We consider a homogeneous reformulation of (19) which is an essential procedure in applying SDR [6], [13], [19]. By following the reformulation steps for semiblind ML OSTBC detection (see Section VI in [6]), one can show that:

Fact 1 *Define* $n=2PK$, *and*

$$\mathbf{G}=\begin{bmatrix}\mathbf{R} & s_{1,1}\mathbf{v}\\ s_{1,1}\mathbf{v}^T & s_{1,1}^2u\end{bmatrix},\quad \mathbf{D}=\begin{bmatrix}\mathbf{I}_{n-1} & \mathbf{0}\\ \mathbf{0}^T & s_{1,1}^2\end{bmatrix}. \quad (20)$$

Problem (19) *can be reformulated as*

$$\max_{\boldsymbol{x}\in\mathbb{R}^n} \ \frac{\boldsymbol{x}^T\mathbf{G}\boldsymbol{x}}{\boldsymbol{x}^T\mathbf{D}\boldsymbol{x}} \tag{21a}$$

$$\text{subject to (s.t.)} \quad x_k \in \{\pm 1, \pm 3, \ldots, \pm(2^q-1)\}, \quad k=1,\ldots,n-1, \tag{21b}$$

$$x_n \in \{\pm 1\}, \tag{21c}$$

and the relationship between (21) *and* (19) *is as follows: If* $\boldsymbol{x}^\star = [x_1^\star, \ldots, x_{n-1}^\star, x_n^\star]^T$ *is a solution of* (21)*, then* $\tilde{\boldsymbol{x}}^\star = [x_1^\star x_n^\star, \ldots, x_{n-1}^\star x_n^\star]^T$ *is a solution of* (19)*.*

Let us now introduce the LFSDR approach to (21). By defining $\mathbf{X} = \boldsymbol{x}\boldsymbol{x}^T$, one can rewrite (21) in terms of $\mathbf{X}$ as follows:

$$\max_{\mathbf{X}\in\mathbb{R}^{n\times n}} \ \frac{\mathrm{Tr}(\mathbf{G}\mathbf{X})}{\mathrm{Tr}(\mathbf{D}\mathbf{X})} \tag{22a}$$

$$\text{s.t.} \quad [\mathbf{X}]_{k,k} \in \{1, 9, \ldots, (2^q-1)^2\}, \quad k=1,\ldots,n-1, \tag{22b}$$

$$[\mathbf{X}]_{n,n} = 1, \tag{22c}$$

$$\mathbf{X} \succeq \mathbf{0} \quad \text{(positive semidefinite (PSD))}, \tag{22d}$$

$$\mathrm{rank}(\mathbf{X}) = 1, \tag{22e}$$

where $[\mathbf{X}]_{k,k}$ denotes the kth diagonal entry of $\mathbf{X}$. In (22), constraints (22b) and (22c) are due to (21b) and (21c), respectively, and (22d) and (22e) are owing to $\mathbf{X} = \boldsymbol{x}\boldsymbol{x}^T$. It can be observed from (22) that the discrete constraints in (22b) and the rank-1 constraint in (22e) are not convex and are difficult to handle. The idea of SDR is to *approximate* problem (21) by removing the rank-1 constraint but keep the PSD constraint $\mathbf{X} \succeq \mathbf{0}$. To deal with the discrete constraint in (22b), we adopt the idea of bound-constrained SDR (BC-SDR) in coherent higher-order QAM MIMO detection [19] where the discrete set $\{1, 9, \ldots, (2^q-1)^2\}$ is relaxed to an interval $[1, (2^q-1)^2]$. We then end up with the following LFSDR problem

$$\mathbf{X}^\star = \arg\max_{\mathbf{X}\in\mathbb{R}^{n\times n}} \ \frac{\mathrm{Tr}(\mathbf{G}\mathbf{X})}{\mathrm{Tr}(\mathbf{D}\mathbf{X})} \tag{23a}$$

$$\text{s.t.} \quad 1 \le [\mathbf{X}]_{k,k} \le (2^q-1)^2, \quad k=1,\ldots,n-1, \tag{23b}$$

$$[\mathbf{X}]_{n,n} = 1, \tag{23c}$$

$$\mathbf{X} \succeq \mathbf{0}. \tag{23d}$$

Note that the notation $\mathbf{X}^\star$ in (23a) represents a globally optimum solution of problem (23). We should emphasize that problem (23) is structurally quite different from the BC-SDR problem in coherent MIMO detection [19]. In the latter, the relaxation problem is a convex SDP and can be directly solved by an interior point SDP algorithm [26], [27]. By contrast, problem (23) is a quasiconvex problem. In general, this class of problems can be solved in a globally optimal fashion by the classical bisection method [20] in which a sequence of SDP feasibility problems need to be solved. Fortunately, we will show in the next subsection that a globally optimum solution to problem (23) can be obtained by solving just one SDP.

B. SDP Reformulation of LFSDR and Custom-Built Interior-Point Algorithm

The quasiconvex LFSDR problem in (23) can be turned into a (convex) SDP as follows

$$\mathbf{Z}^\star = \arg\max_{\mathbf{Z}\in\mathbb{R}^{n\times n}} \ \mathrm{Tr}(\mathbf{G}\mathbf{Z}) \tag{24a}$$

$$\text{s.t.} \quad \mathrm{Tr}(\mathbf{D}\mathbf{Z}) = 1, \tag{24b}$$

$$[\mathbf{Z}]_{n,n} \le [\mathbf{Z}]_{k,k} \le (2^q-1)^2[\mathbf{Z}]_{n,n}, \quad k=1,\ldots,n-1, \tag{24c}$$

$$\mathbf{Z} \succeq \mathbf{0}, \tag{24d}$$

as stated in the following proposition.

Proposition 1 *The linear fractional quasiconvex problem* (23) *has the same optimum objective value as the SDP in* (24)*. Moreover, an optimum solution of* (23) *can be obtained from that of* (24) *through the relation*

$$\mathbf{X}^\star = \mathbf{Z}^\star/[\mathbf{Z}^\star]_{n,n}. \tag{25}$$

Proof: We first show that for any feasible $\mathbf{Z}$ of problem (24), $[\mathbf{Z}]_{n,n} \neq 0$. Suppose that $[\mathbf{Z}]_{n,n} = 0$. Then by (24c) and (24d), $\mathbf{Z} = \mathbf{0}$, which however violates (24b). Therefore, we can always define a point $\bar{\mathbf{X}} = \mathbf{Z}/[\mathbf{Z}]_{n,n}$. It is easy to show that $\bar{\mathbf{X}}$ is feasible for problem (23) and has the same objective value $\mathrm{Tr}(\mathbf{G}\bar{\mathbf{X}})/\mathrm{Tr}(\mathbf{D}\bar{\mathbf{X}}) = \mathrm{Tr}(\mathbf{G}\mathbf{Z})$. On the other hand, it can be seen from (20), (23c) and (23d) that for any feasible $\mathbf{X}$ of problem (23), $\mathrm{Tr}(\mathbf{D}\mathbf{X}) = \sum_{k=1}^{n-1}[\mathbf{X}]_{k,k} + s_{1,1}^2 > 0$. Let $\bar{\mathbf{Z}} = \mathbf{X}/\mathrm{Tr}(\mathbf{D}\mathbf{X})$. Then it is also easy to show that $\bar{\mathbf{Z}}$ is feasible for problem (24) and has the same objective value $\mathrm{Tr}(\mathbf{G}\bar{\mathbf{Z}}) = \mathrm{Tr}(\mathbf{G}\mathbf{X})/\mathrm{Tr}(\mathbf{D}\mathbf{X})$. Hence we conclude that problems (23) and (24) are equivalent and $\mathbf{X}^\star = \mathbf{Z}^\star/[\mathbf{Z}^\star]_{n,n}$. ■

Proposition 1 implies that the optimum solution $\mathbf{X}^\star$ of (23) can simply be obtained by solving the SDP (24) in lieu of the bisection method. The SDP (24) can be solved in polynomial time using an interior-point algorithm (IPA) [26]. While (24) can be solved conveniently by calling popular, general-purpose SDP solvers such as SeDuMi [21], we can build a specialized IPA for (24) to further improve the computational efficiency.

Table I shows our custom-built IPA. This specialized IPA can be shown to have a worst-case complexity of $O(n^{3.5})$ (through counting the arithmetic operations and using known results in convergence of interior-point methods). The specialized IPA follows the primal-dual path following principle in [26] (see also [24], [27]), but carefully exploits structures of the inequality and equality constraints of (24) to trim down the computations. In particular, the search direction computations in Step 2 of Table I are specially designed for (24). Since the development involves tedious, laborous derivations, the complete details are omitted here.

To give some insights, let us briefly describe how the developed IPA works in principle. Essentially we consider solving (24) by solving its dual which can be shown to be

$$\min \ \nu \tag{26a}$$
$$\text{s.t. } \nu \in \mathbb{R}, \quad \mathbf{t} \in \mathbb{R}^{2(n-1)}, \quad \mathbf{Y} \in \mathbb{R}^{n \times n}, \tag{26b}$$
$$\mathbf{Y} \succeq \mathbf{0}, \quad \mathbf{t} \succeq \mathbf{0}, \tag{26c}$$
$$\mathbf{Y} = \mathrm{Diag}\left(\begin{bmatrix} \nu \mathbf{1}_{n-1} - \mathbf{t}_1 + \mathbf{t}_2 \\ s_{1,1}^2 \nu + \mathbf{1}_{n-1}^T(\mathbf{t}_1 - (2^q-1)^2 \mathbf{t}_2) \end{bmatrix}\right) - \mathbf{G}, \tag{26d}$$

where $(\mathbf{Y}, \mathbf{t}, \nu)$ are the dual variables of (24), $\mathbf{1}_{n-1}$ is an all-one vector with dimension $n-1$, $\mathrm{Diag}(\mathbf{x})$ denotes a diagonal matrix with the diagonal elements given by the elements of $\mathbf{x}$, and $\mathbf{t}_1, \mathbf{t}_2 \in \mathbb{R}^{n-1}$ respectively represent the upper and lower part of $\mathbf{t}$; i.e., $\mathbf{t} = [\ \mathbf{t}_1^T \ \mathbf{t}_2^T\ ]^T$. The idea is to apply a logarithmetic barrier approximation to (26) to implicitly handle the constraints $\mathbf{Y} \succeq \mathbf{0}$ and $\mathbf{t} \succeq \mathbf{0}$:

$$\min \ \nu - \mu \left(\log\det(\mathbf{Y}) + \sum_{i=1}^{2(n-1)} \log t_i \right) \tag{27a}$$
$$\text{s.t. } \nu \in \mathbb{R}, \quad \mathbf{t} \in \mathbb{R}^{2(n-1)}, \quad \mathbf{Y} \in \mathbb{R}^{n \times n}, \tag{27b}$$
$$\mathbf{Y} = \mathrm{Diag}\left(\begin{bmatrix} \nu \mathbf{1}_{n-1} - \mathbf{t}_1 + \mathbf{t}_2 \\ s_{1,1}^2 \nu + \mathbf{1}_{n-1}^T(\mathbf{t}_1 - (2^q-1)^2 \mathbf{t}_2) \end{bmatrix}\right) - \mathbf{G}, \tag{27c}$$

where $\mu > 0$ is called the barrier parameter. It is known that (27) approaches (26) as $\mu \to 0$. At each iteration of the proposed IPA, we reduce μ in a data adaptive fashion (Step 2 in Table I), and then compute a primal-dual search direction that approximates the Karush-Kuhn-Tucker conditions of (27) with respect to the updated μ (Steps 2-3 in Table I). The IPA terminates when μ is sufficiently small. One essential implementation aspect about the specialized IPA is that we need a primal-dual strictly feasible point $(\mathbf{Z}, \mathbf{Y}, \mathbf{t}, \nu)$ as an initialization. Here we provide a simple closed-form initial point: Let $\mathbf{q} \in \mathbb{R}^n$ with the ith element given by

$$q_i = \alpha \sum_{j=1}^{n} |[\mathbf{G}]_{i,j}| \tag{28}$$

for some $\alpha > 1$. The following point can be shown to be strictly primal-dual feasible (and thus can serve as an initialization):

$$\mathbf{Z} = \frac{1}{5(n-1) + s_{1,1}^2} \begin{bmatrix} 5\mathbf{I}_{n-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \succ \mathbf{0}, \tag{29a}$$
$$\mathbf{Y} = \mathrm{Diag}\left(\begin{bmatrix} \nu \mathbf{1}_{n-1} + \mathbf{q}_{1:n-1} \\ q_n \end{bmatrix}\right) - \mathbf{G} \succ \mathbf{0}, \tag{29b}$$
$$\mathbf{t} = [\mathbf{t}_1^T, \ \mathbf{t}_2^T]^T, \ \mathbf{t}_1 = \mathbf{q}_{1:n-1}, \ \mathbf{t}_2 = 2\mathbf{q}_{1:n-1}, \tag{29c}$$
$$\nu = \left[\ q_n + \mathbf{1}_{n-1}^T\left((2^q-1)^2 \mathbf{t}_2 - \mathbf{t}_1\right)\right] / s_{1,1}^2, \tag{29d}$$

where $\mathbf{q}_{1:n-1} \in \mathbb{R}^{n-1}$ contains the first $n-1$ elements of $\mathbf{q}$.

We provide the MATLAB source codes of the specialized IPA in http://www.ee.cuhk.edu.hk/~wkma/SDR/download/blind_lfsdr.rar, for readers who are interested in implementing our method.

C. Solution Approximation Procedures

The development above has enabled an efficient way to compute the optimum solution $\mathbf{X}^\star$ of the LFSDR problem (23). We now turn our attention to the last step of the proposed LFSDR approach: Using $\mathbf{X}^\star$ to find a feasible, rank-1 approximate solution of the original problem (21). One straightforward method to do this is to compute the principal eigenvector of $\mathbf{X}^\star$ (thereby performing rank-1 approximation), and then quantize the principal eigenvector into one belonging to the set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}^{n-1} \times \{\pm1\}$. Another method practically proven to be effective is the Gaussian randomization [19], [28]. In this method, we first generate L random vectors $\boldsymbol{\xi}^{(\ell)} \in \mathbb{R}^n$, $\ell = 1, \ldots, L$, following the Gaussian distribution $\mathcal{N}(\mathbf{0}, \mathbf{X}^\star)$ (i.e., zero mean and covariance matrix equal to $\mathbf{X}^\star$), and then quantize $\boldsymbol{\xi}^{(\ell)}$ into one belonging to the set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}^{n-1} \times \{\pm1\}$. Denote by $\hat{\boldsymbol{x}}^{(\ell)} \in \{\pm1, \pm3, \ldots, \pm(2^q-1)\}^{n-1} \times \{\pm1\}$ the quantized vector of $\boldsymbol{\xi}^{(\ell)}$, that is,

$$\hat{\boldsymbol{x}}^{(\ell)} = [\ \sigma_{\mathrm{PAM}}(\xi_1^{(\ell)}), \ldots, \sigma_{\mathrm{PAM}}(\xi_{n-1}^{(\ell)}), \ \mathrm{sgn}(\xi_n^{(\ell)})\ ]^T,$$

where $\mathrm{sgn} : \mathbb{R} \to \{\pm1\}$ is the sign function, and $\sigma_{\mathrm{PAM}} : \mathbb{R} \to \{\pm1, \pm3, \ldots, \pm(2^q-1)\}$ is a function in which $\sigma_{\mathrm{PAM}}(x)$ is obtained by rounding x to an integer in the set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}$. We pick the quantized vector that yields the largest objective value, i.e.,

$$\ell^\star = \arg \max_{\ell=1,\ldots,L} \frac{(\hat{\boldsymbol{x}}^{(\ell)})^T \mathbf{G} \hat{\boldsymbol{x}}^{(\ell)}}{(\hat{\boldsymbol{x}}^{(\ell)})^T \mathbf{D} \hat{\boldsymbol{x}}^{(\ell)}},$$

and choose $\hat{\boldsymbol{x}}^{(\ell^\star)}$ as the approximate solution of problem (21). By our experience, $L = 50 \sim 100$ is typically sufficient to obtain a good approximation performance.

D. Relationships with Other Relaxation Methods

In the subsection, we present some other relaxation methods for the higher-order QAM blind ML OSTBC detection problem and study their connections to the proposed LFSDR.

One simple approach to approximating the higher-order QAM blind ML OSTBC detection problem in (17) is actually to relax the discrete set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}$ to the real space $\mathbb{R}$:

$$f_{\mathrm{NR}} \triangleq \max_{\mathbf{s} \in \mathbb{R}^{2PK}} \frac{\mathbf{s}^T \mathbf{F} \mathbf{s}}{\mathbf{s}^T \mathbf{s}}. \tag{30}$$

which we call the *norm relaxed* blind ML problem. It can be seen that the principal eigenvector of $\mathbf{F}$ is the associated optimum solution. A feasible, approximate solution to (17) can then be obtained by quantizing the principal eigenvector into the set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}^{2PK}$. More specifically, let $\boldsymbol{v}^\star \in \mathbb{R}^{2PK}$ denote the principal eigenvector of $\mathbf{F}$, and assume that $s_{1,1}$ is known to the receiver. Then an approximate solution of (17) by norm relaxation is given by

$$\hat{\mathbf{s}}_{\mathrm{NR}} = \sigma_{\mathrm{PAM}}\left(\frac{s_{1,1}}{v_1^\star} \boldsymbol{v}^\star\right), \tag{31}$$

where $\sigma_{\mathrm{PAM}} : \mathbb{R}^{2PK} \to \{\pm1, \pm3, \ldots, \pm(2^q-1)\}^{2PK}$ is a function in which the ith element of $\sigma_{\mathrm{PAM}}(\boldsymbol{x})$ is obtained by rounding x_i to an integer in the set $\{\pm1, \pm3, \ldots, \pm(2^q-1)\}$.

TABLE I. Pseudo code of the specialized interior-point algorithm for solving (24).

Given a primal-dual strictly feasible initial point $(\mathbf{Z}, \mathbf{Y}, \nu, \mathbf{t})$ [see (29)] and a solution accuracy $\epsilon > 0$.

Step 1. Set $\mu := 0.5 \times [\nu - \mathrm{tr}(\mathbf{GZ})]/(3n-2)$.

Step 2. Compute the search directions $(\Delta\nu, \Delta\mathbf{t})$ by solving the linear system of equations

$$\mathbf{F}\begin{bmatrix}\Delta\nu\\ \Delta\mathbf{t}\end{bmatrix} = \mathbf{g},$$

where $\mathbf{F}$ and $\mathbf{g}$ are constructed by the following formulae

$$\mathbf{Y}^{-1}\odot\mathbf{Z} := \begin{bmatrix}\mathbf{W}_{11} & \mathbf{w}_{12}^T\\ \mathbf{w}_{12} & w_{22}\end{bmatrix},\ \mathbf{F} := \begin{bmatrix} f_{11} & \mathbf{f}_{21}^T & \mathbf{f}_{31}^T\\ \mathbf{f}_{21} & \mathbf{F}_{22} & \mathbf{F}_{32}^T\\ \mathbf{f}_{31} & \mathbf{F}_{32} & \mathbf{F}_{33}\end{bmatrix}$$

(in which $\odot$ denotes the Hadamard (componentwise) product of matrices),

$$f_{11} := \mathbf{1}_{n-1}^T\mathbf{W}_{11}\mathbf{1}_{n-1} + 2s_{1,1}^2\mathbf{1}_{n-1}^T\mathbf{w}_{12} + s_{1,1}^4 w_{22},$$

$$\mathbf{f}_{21} := -\mathbf{W}_{11}\mathbf{1}_{n-1} - s_{1,1}^2\mathbf{w}_{12} + (\mathbf{1}_{n-1}^T\mathbf{w}_{12} + s_{1,1}^2 w_{22})\mathbf{1}_{n-1},$$

$$\mathbf{f}_{31} := \mathbf{W}_{11}\mathbf{1}_{n-1} + s_{1,1}^2\mathbf{w}_{12} - (2^q-1)^2(\mathbf{1}_{n-1}^T\mathbf{w}_{12} + s_{1,1}^2 w_{22})\mathbf{1}_{n-1}$$

$$\mathbf{F}_{22} := \mathbf{W}_{11} - \mathbf{w}_{12}\mathbf{1}_{n-1}^T - \mathbf{1}_{n-1}\mathbf{w}_{12}^T + w_{22}\mathbf{1}_{n-1}\mathbf{1}_{n-1}^T + \mathbf{D}_1$$

$$\mathbf{F}_{32} := -\mathbf{W}_{11} + \mathbf{w}_{12}\mathbf{1}_{n-1}^T + (2^q-1)^2\mathbf{1}_{n-1}\mathbf{w}_{12}^T - (2^q-1)^2 w_{22}\mathbf{1}_{n-1}\mathbf{1}_{n-1}^T$$

$$\mathbf{F}_{33} := \mathbf{W}_{11} - (2^q-1)^2\mathbf{w}_{12}\mathbf{1}_{n-1}^T - (2^q-1)^2\mathbf{1}_{n-1}\mathbf{w}_{12}^T + (2^q-1)^4 w_{22}\mathbf{1}_{n-1}\mathbf{1}_{n-1}^T + \mathbf{D}_2$$

$$\mathbf{D}_1 := \mathrm{Diag}\left(\begin{bmatrix} t_1^{-1}([\mathbf{Z}]_{1,1} - [\mathbf{Z}]_{n,n})\\ \vdots\\ t_{n-1}^{-1}([\mathbf{Z}]_{n-1,n-1} - [\mathbf{Z}]_{n,n})\end{bmatrix}\right),$$

$$\mathbf{D}_2 := \mathrm{Diag}\left(\begin{bmatrix} t_n^{-1}((2^q-1)^2[\mathbf{Z}]_{n,n} - [\mathbf{Z}]_{1,1})\\ \vdots\\ t_{2(n-1)}^{-1}((2^q-1)^2[\mathbf{Z}]_{n,n} - [\mathbf{Z}]_{n-1,n-1})\end{bmatrix}\right)$$

$$\mathbf{g} := \mu\begin{bmatrix}\sum_{i=1}^{n-1}[\mathbf{Y}^{-1}]_{i,i} + s_{1,1}^2[\mathbf{Y}^{-1}]_{n,n}\\ \boldsymbol{\kappa}_1\\ \boldsymbol{\kappa}_2\end{bmatrix} + \begin{bmatrix}-1\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$$

$$\boldsymbol{\kappa}_1 := \begin{bmatrix} -[\mathbf{Y}^{-1}]_{1,1} + [\mathbf{Y}^{-1}]_{n,n} + t_1^{-1}\\ \vdots\\ -[\mathbf{Y}^{-1}]_{n-1,n-1} + [\mathbf{Y}^{-1}]_{n,n} + t_{n-1}^{-1}\end{bmatrix},$$

$$\boldsymbol{\kappa}_2 := \begin{bmatrix} [\mathbf{Y}^{-1}]_{1,1} - (2^q-1)^2[\mathbf{Y}^{-1}]_{n,n} + t_n^{-1}\\ \vdots\\ [\mathbf{Y}^{-1}]_{n-1,n-1} - (2^q-1)^2[\mathbf{Y}^{-1}]_{n,n} + t_{2(n-1)}^{-1}\end{bmatrix}$$

Step 3. Compute the search directions

$$\Delta\mathbf{Y} := \mathrm{Diag}\left(\begin{bmatrix}\Delta\nu\mathbf{1}_{n-1} - \Delta\mathbf{t}_1 + \Delta\mathbf{t}_2\\ s_{1,1}^2\Delta\nu + \mathbf{1}_{n-1}^T(\Delta\mathbf{t}_1 - (2^q-1)^2\Delta\mathbf{t}_2)\end{bmatrix}\right),$$

$$\Delta\mathbf{Z} := \mu\mathbf{Y}^{-1} - \mathbf{Z} - \mathbf{Y}^{-1}(\Delta\mathbf{Y})\mathbf{Z}$$

and symmetrize $\Delta\mathbf{Z}$ by $\Delta\mathbf{Z} := (\Delta\mathbf{Z} + (\Delta\mathbf{Z})^T)/2$.

Step 4. Use line search to find a primal step-size $\alpha_p \in (0,1]$ such that $\mathbf{Z} + \alpha_p(\Delta\mathbf{Z}) \succ \mathbf{0}$ and $[\mathbf{Z}]_{n,n} + [\Delta\mathbf{Z}]_{n,n} \le [\mathbf{Z}]_{i,i} + [\Delta\mathbf{Z}]_{i,i} \le (2^q-1)^2([\mathbf{Z}]_{n,n} + [\Delta\mathbf{Z}]_{n,n})$ for $i = 1,\ldots,n-1$.

Step 5. Use line search to find a dual step size $\alpha_d \in (0,1]$ such that $\mathbf{Y} + \alpha_d(\Delta\mathbf{Y}) \succ \mathbf{0}$ and $\mathbf{t} + \alpha_d(\Delta\mathbf{t}) \succ \mathbf{0}$.

Step 6. Update $\mathbf{Z} := \mathbf{Z} + \alpha_p(\Delta\mathbf{Z})$, $\mathbf{Y} := \mathbf{Y} + \alpha_d(\Delta\mathbf{Y})$, $\nu := \nu + \alpha_d(\Delta\nu)$, and $\mathbf{t} := \mathbf{t} + \alpha_d(\Delta\mathbf{t})$.

Step 7. If $\nu - \mathrm{tr}(\mathbf{GZ}) \le \epsilon$ (i.e., duality gap is less than ϵ), then terminate and output $(\mathbf{Z}, \mathbf{Y}, \nu, \mathbf{t})$; otherwise go to Step 1.

It can be proved that the proposed LFSDR approach has an approximation accuracy at least no worse than this simple norm relaxation method, as stated in the following proposition.

Proposition 2 *Let* $f_{\rm LFSDR} \triangleq \mathrm{Tr}(\mathbf{GZ}^\star)$ *be the optimum objective value of the SDP problem* (24)*, and recall that* $f_{\rm ML}$ *and* $f_{\rm NR}$ *are the optimum values of the original blind ML detection problem [in* (19)*] and the norm relaxed problem [in* (30)*], respectively. Then*

$$|f_{\rm ML} - f_{\rm LFSDR}| \le |f_{\rm ML} - f_{\rm NR}|.$$

Proof: The idea of this proof follows that of Theorem 1 in [6]. Since $f_{\rm LFSDR} \ge f_{\rm ML}$ and $f_{\rm NR} \ge f_{\rm ML}$ (a basic result in relaxation), it suffices to show that $f_{\rm LFSDR} \le f_{\rm NR}$. Suppose that

$$\mathbf{Z}^\star = \begin{bmatrix}\mathbf{P} & \mathbf{q}\\ \mathbf{q}^T & r\end{bmatrix} \succeq \mathbf{0},$$

where $\mathbf{P} \in \mathbb{R}^{(n-1)\times(n-1)}$, $\mathbf{q} \in \mathbb{R}^{n-1}$ and $r \in \mathbb{R}$. Let

$$\tilde{\mathbf{Z}} = \begin{bmatrix} s_{1,1}^2 r & s_{1,1}\mathbf{q}^T\\ s_{1,1}\mathbf{q} & \mathbf{P}\end{bmatrix} \succeq \mathbf{0}.$$

Then one can readily show from (18), (20) and (24) that

$$\mathrm{Tr}(\mathbf{GZ}^\star) = \mathrm{Tr}(\mathbf{F}\tilde{\mathbf{Z}}), \tag{32}$$

$$\mathrm{Tr}(\mathbf{DZ}^\star) = \mathrm{Tr}(\tilde{\mathbf{Z}}) = 1. \tag{33}$$

Consider the eigenvalue decomposition of $\tilde{\mathbf{Z}} = \sum_{k=1}^n \lambda_k \mathbf{g}_k\mathbf{g}_k^T$, where $\lambda_k \ge 0$ is the kth eigenvalue of $\tilde{\mathbf{Z}}$, and $\mathbf{g}_k \in \mathbb{R}^n$ is the associated unit-norm eigenvector. Then

$$\mathrm{Tr}(\mathbf{F}\tilde{\mathbf{Z}}) = \sum_{k=1}^n \lambda_k \mathbf{g}_k^T\mathbf{F}\mathbf{g}_k \le \left(\sum_{k=1}^n \lambda_k\right)\max_{\|\mathbf{g}\|_2=1}\mathbf{g}^T\mathbf{F}\mathbf{g} = \mathrm{Tr}(\tilde{\mathbf{Z}})f_{\rm NR}, \tag{34}$$

where the last equality is due to (30) and $\mathrm{Tr}(\tilde{\mathbf{Z}}) = \sum_{k=1}^n \lambda_k$. By (32), (33) and (34), we obtain $f_{\rm LFSDR} = \mathrm{Tr}(\mathbf{GZ}^\star) = \mathrm{Tr}(\mathbf{F}\tilde{\mathbf{Z}}) \le f_{\rm NR}$. ■

In fact, we will further show by simulations in Section IV that this simple norm relaxation method has symbol error performance far from what the proposed LFSDR approach offers.

In addition to the LFSDR proposed in the previous subsections, there are two other possible ways of relaxations that may also provide effective approximations to the higher-order QAM blind ML OSTBC detection problem. Specifically, by applying the virtually-antipodal SDR (VA-SDR) [29] and polynomial-inspired SDR (PI-SDR) [30] concepts respectively (which were developed for coherent higher-order QAM MIMO detection), we can propose two more relaxation methods to the blind ML problem. But, interestingly, our recent theoretical analysis in coherent MIMO detection has shown [27], [31] that the rationale adopted in the proposed LFSDR, and the relaxation methods based on VA-SDR, and PI-SDR are equivalent in attaining the same optimal values. More importantly, the SDR equivalence theorems in [31] are directly applicable to the blind ML OSTBC detection problem here [1]. Hence we conclude that

Proposition 3 *The proposed LFSDR, given in* (23)*, is equivalent to the two relaxation alternatives where the VA-SDR [29] and PI-SDR [30] are respectively applied to the higher-order QAM blind ML OSTBC detection problem [in* (21)*]. The equivalence lies in the identical optimum objective values for the three relaxation methods.*

The details of the VA-SDR and PI-SDR alternatives and their equivalence to the proposed LFSDR are given in a separate technical report [32] rather than in this paper due to space limit. That technical report also gives useful simulation results, namely, verification of the SDR equivalence in Proposition 3, and numerical complexity comparisons. There one can see that the proposed LFSDR costs less amount of computations than its VA-SDR and PI-SDR alternatives.

IV. Simulation Results

Extensive simulation results are given in this section to demonstrate the effectiveness of the proposed LFSDR-based higher-order QAM blind ML OSTBC detector. The channel coefficients in $\mathbf{H}$ were independent and identically distributed (i.i.d.) circular complex Gaussian random variables with zero mean and unit variance. The signal-to-noise ratio (SNR) was defined as

$$\text{SNR} = \frac{\text{E}\{\|\mathbf{HC}(\mathbf{s}_p)\|_F^2\}}{\text{E}\{\|\mathbf{W}_p\|_F^2\}} = \frac{\gamma N_t K}{T\sigma_w^2},$$

where $\gamma = 10$ for 16-QAM and $\gamma = 42$ for 64-QAM. If not mentioned specifically, the complex 3×4 OSTBC ($N_t = 3$, $T = 4$, $K = 3$) [33]

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 + js_2 & -s_3 + js_4 & -s_5 + js_6 & 0 \\ s_3 + js_4 & s_1 - js_2 & 0 & -s_5 + js_6 \\ s_5 + js_6 & 0 & s_1 - js_2 & s_3 - js_4 \end{bmatrix} \quad (35)$$

was used in the simulation, and the LFSDR problem (24) was solved by the specialized IPA in Table I. An approximate solution of problem (21) was obtained either by quantizing the principal eigenvector of $\mathbf{X}^\star$ or by the Gaussian randomization procedure in Section III-C with 100 random vectors ($L = 100$) generated. The detector performance was evaluated using average symbol error rate (SER), and at least $10,000$ trials were performed for each simulation result.

A. Performance Comparison with Some Existing Methods

Here we present the performance comparison results of the proposed LFSDR blind ML detector, the norm relaxed blind ML detector (i.e., Eqn. (31)), the blind subspace channel estimator by Shahbazpanahi *et al.* [2], the cyclic ML method [7] (initialized by the norm relaxed blind ML detector), and the coherent ML detector (which assumes perfect channel state information (CSI)). Note that for the white Gaussian noise case, the Shahbazpanahi's blind subspace channel estimator is equivalent to the norm relaxed blind ML detector [2], [6] from a theoretical viewpoint. However, the former employs a different method of using the pilot to fix the channel ambiguity (please see [2] for the details). As a result, the two methods will be seen to exhibit different simulation performances. Figure 1(a) and Figure 1(b) show the performance results (SER vs. SNR) for the case of 16-QAM OSTBC, and Fig. 1(c) and Fig. 1(d) display the results for the 64-QAM OSTBC.

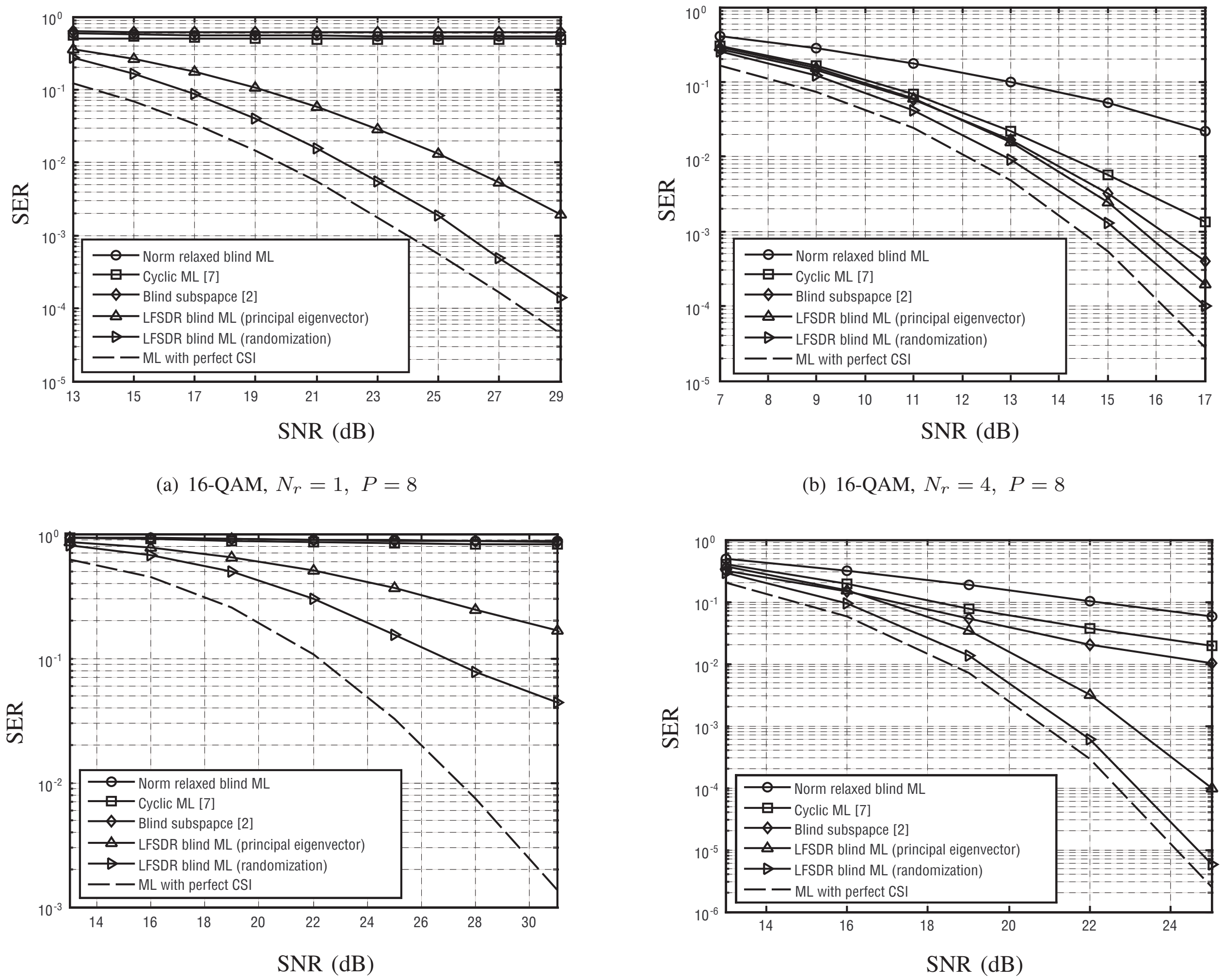
One can see from Fig. 1(a) and Fig. 1(c) that Shahbazpanahi's subspace method, the norm relaxed blind ML detector, and the cyclic ML method cannot properly decode the transmitted OSTBCs when $N_r = 1$. In comparison with these two methods, the proposed LFSDR-based blind ML detector exhibits consistent SER performance. For the multiple-receive-antenna case ($N_r = 4$) as presented in Fig. 1(b) and Fig. 1(d), Shahbazpanahi's subspace method can properly identify the transmitted symbols (some theoretical reasoning for the significant performance difference of the subspace method in the one-receive-antenna and multiple-receive-antenna cases has been provided in [4]). Nevertheless, one can see from these figures that the LFSDR-based blind ML detector outperforms the subspace method as well as the norm relaxed blind ML detector, thereby providing a numerical support to Proposition 2. By comparing Fig. 1(a) and Fig. 1(c), it can be seen that for $N_r = 1$, the performance difference between the proposed LFSDR and the coherent ML detector for 64-QAM OSTBC is larger than that for 16-QAM OSTBC. However, as observed from Fig. 1(b) and Fig. 1(d) where $N_r = 4$, the performance differences between the LFSDR-based blind ML detector (randomization) and the coherent ML detector at SER=10^{-4} can be less than 1 dB and 0.5 dB, respectively. These results illustrate that in the case of $N_r = 4$ and either for 16-QAM or 64-QAM OSTBCs, the proposed LFSDR approach is accurate in the approximation of the true blind ML solution. The simulation results in Fig. 1 also indicate that the Gaussian randomization procedure is a better approximation method than the principal eigenvector procedure.

In Fig. 2, we further present some performance comparison results for various numbers of block size P. One can see from both Fig. 2(a) and 2(b) that all the methods under test can have improved symbol error performance when P increases, but the proposed LFSDR based blind ML detector outperforms all the other methods for all P. The performance advantage is more significant for 64-QAM OSTBCs as shown in Fig. 2(b). These simulation results demonstrate that the proposed LFSDR blind ML approach is more effective than other methods when the channel is static for small to moderate number of OSTBC blocks, which is consistent with the results for BPSK/QPSK OSTBCs in [6], [8].

B. Performance Comparison with Higher-Order QAM Differential OSTBC Scheme

Traditionally the differential OSTBC scheme can only be applied to the constant modulus case, but a recent work in [34] has revealed that the differential OSTBC scheme can

[1] While the focus of [31] is on proving the equivalence of SDRs under the coherent MIMO detection context, the analysis there does not place an assumption on the objective function structures. For this reason, the SDR equivalence theorems in [31] can be applied to the blind ML problem.



(a) 16-QAM, $N_r = 1,\ P = 8$

(b) 16-QAM, $N_r = 4,\ P = 8$

(c) 64-QAM, $N_r = 1,\ P = 8$

(d) 64-QAM, $N_r = 4,\ P = 8$

Fig. 1. Performance (SER v.s. SNR) comparison results of the proposed LFSDR blind ML detector with some existing methods for the complex 3×4 OSTBC under various settings.

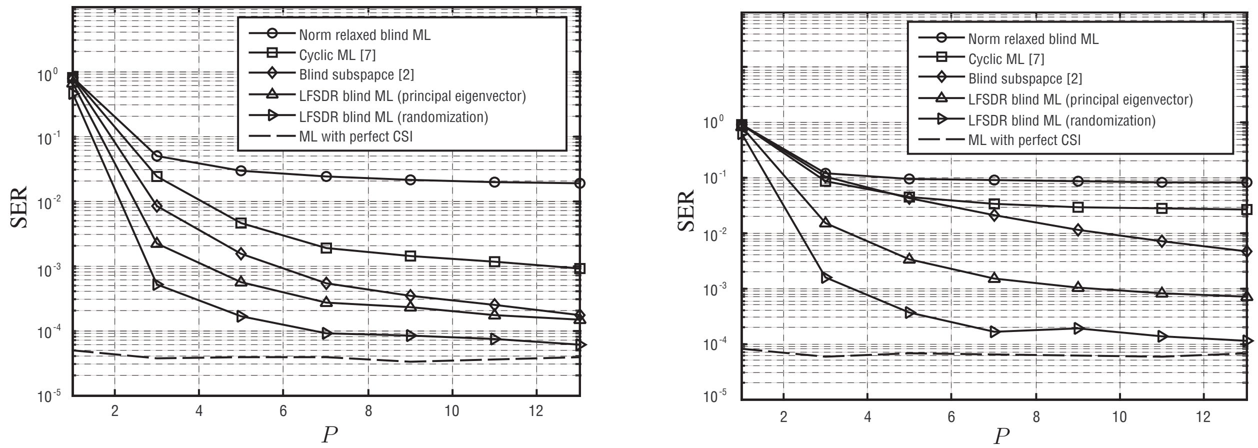
be extended to the nonconstant modulus case through some decision feedback procedure (see Eqn. (27) and Eqn. (30) in [34] for the details). This example aims to compare the differential OSTBC scheme and the proposed LFSDR-based blind ML detector. The following 4×4 OSTBC ($N_t = 4$, $T = 4$, $K = 3$) [22] was used in this simulation example

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 + js_2 & -s_3 + js_4 & -s_5 + js_6 & 0 \\ s_3 + js_4 & s_1 - js_2 & 0 & -s_5 + js_6 \\ s_5 + js_6 & 0 & s_1 - js_2 & s_3 - js_4 \\ 0 & -s_5 + js_6 & s_3 + js_4 & -s_1 + js_2 \end{bmatrix}. \tag{36}$$

Figure 3 shows the performance comparison results for $P = 8$, $N_r = 4$. One can see from this figure that, either for 16-QAM or 64-QAM QSTBC, the proposed LFSDR-based blind ML detector outperforms the differential scheme.

C. Performance Comparison with Cui-Tellambura Modified Sphere Decoder

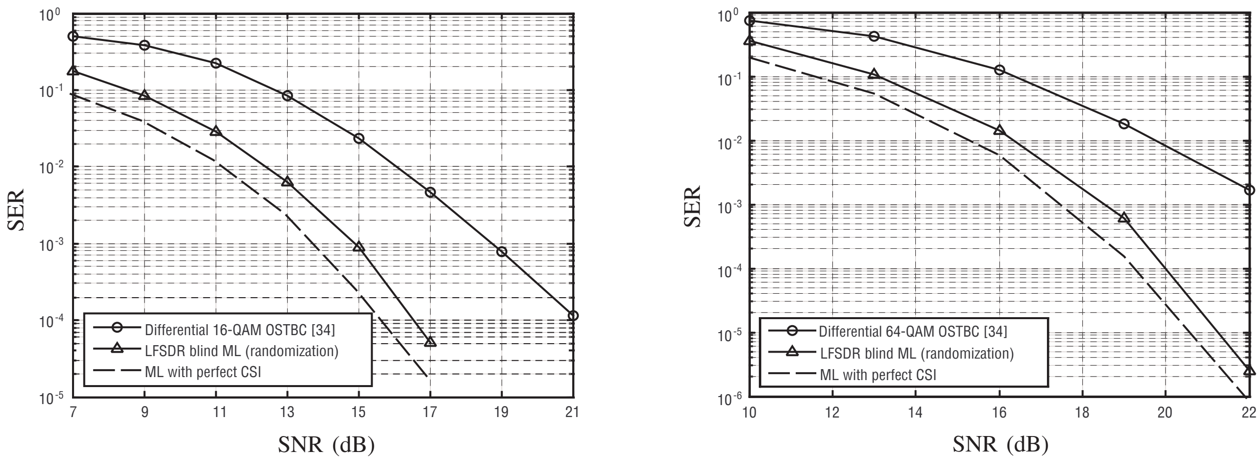
In this subsection, the proposed LFSDR is compared with an optimal blind ML detection method, namely the modified sphere decoder by Cui and Tellambura [9]. Let us first examine the computational complexity of the modified sphere decoder and the proposed LFSDR-based blind ML detector. Figures 4(a) and 4(b) present the average computer running time (in second) of the two methods with respective to the block size P and SNR, respectively. Our implementation for the modified sphere decoder was based on C language. To improve the search efficiency, we incorporated the Schnorr-Euchner enumeration [12] in the modified sphere decoder. For the proposed LFSDR approach, the specialized IPA in Table I was implemented in C language as well. To demonstrate the computational advantage of the specialized IPA, we also included the LFSDR implementation using the general-purpose SDP solver SeDuMi [21]. The simulation was conducted under MATLAB using a desktop computer with a 2.66GHz dual-



(a) 16-QAM, SNR= 17 dB

(b) 64-QAM, SNR= 23 dB

Fig. 2. Performance (SER v.s. P) comparison results of the proposed LFSDR blind ML detector with some existing methods for the complex 3×4 OSTBC.



(a) 16-QAM, $N_r = 4, \ P = 8$

(b) 64-QAM, $N_r = 4, \ P = 8$

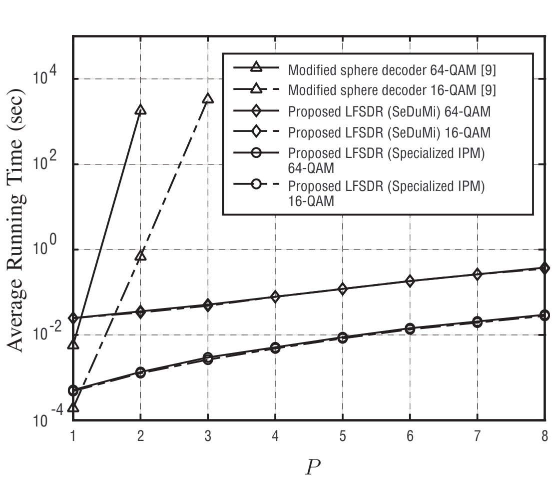
Fig. 3. Performance (SER v.s. SNR) comparison results of the LFSDR and the differential OSTBC scheme for the complex 4×4 OSTBC.

core CPU and 2GB RAM. The initial square search radius for the modified sphere decoder was obtained by using the norm relaxed blind ML solution (31). It can be seen from Fig. 4(a) that the average running time of the LFSDR-based blind ML detector either implemented by the specialized IPA or by SeDuMi increases with P at a much slower rate than the modified sphere decoder. Moreover, either for 16-QAM OSTBC or 64-QAM OSTBC, the proposed LFSDR approach has almost the same computational complexity. Besides, the modified sphere decoder quickly becomes impractical when $P > 2$ for 16-QAM and when $P > 1$ for 64-QAM. From Fig. 4(b), one can see that the average running time of the modified sphere decoder remains almost constant with respect to SNR, and is much higher than that of the LFSDR-based blind ML detector. This is in sharp contrast to its BPSK/QPSK counterpart in [6] where the computational time of the BPSK/QPSK sphere decoder decreases when SNR increases. From both figures, it can also be seen that the specialized IPA is around 10 times faster than SeDuMi, showing its advantages in practical implementations.

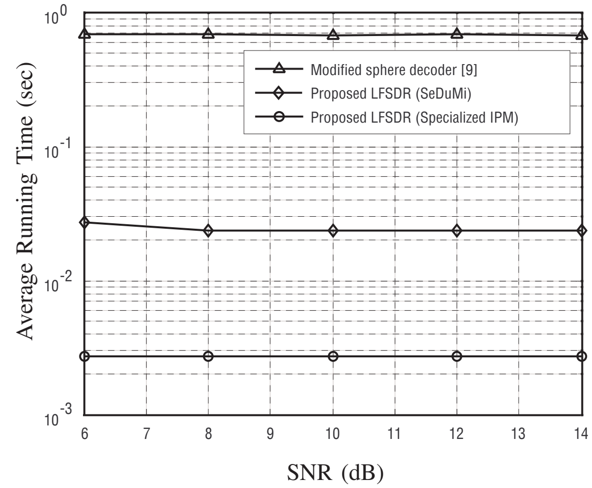
Figure 5 shows the performance comparison results for $P = 2$ and 16-QAM OSTBC (we cannot further increase P since we have seen that the complexity of the modified sphere decoder becomes overwhelming for $P > 2$). One can see from this figure that the performance gap between the LFSDR-based blind ML detector and the modified sphere decoder decreases when the number of receive antenna N_r increases. Together with the performance results in Fig. 1, we can see that the proposed LFSDR approach can yield promising approximation quality when $P \geq 8$ or when $N_r \geq 2$.

V. Conclusions

In the paper, we have presented a suboptimal LFSDR approach to blind ML detection of higher-order QAM OS-

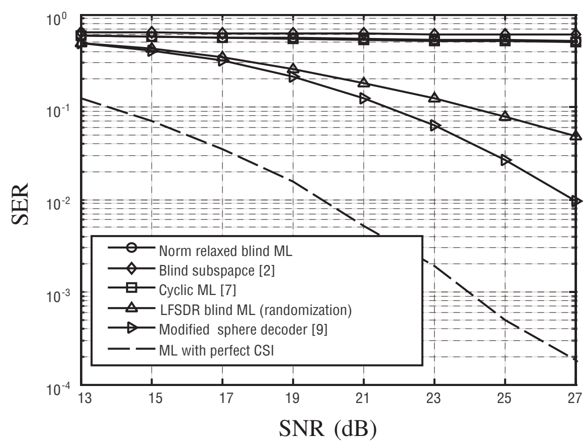


(a) SNR= 13 dB, $N_r = 4$

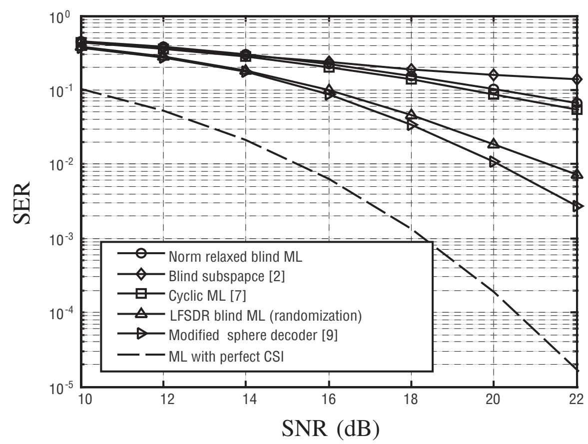


(b) 16-QAM, $N_r = 4$, $P = 2$

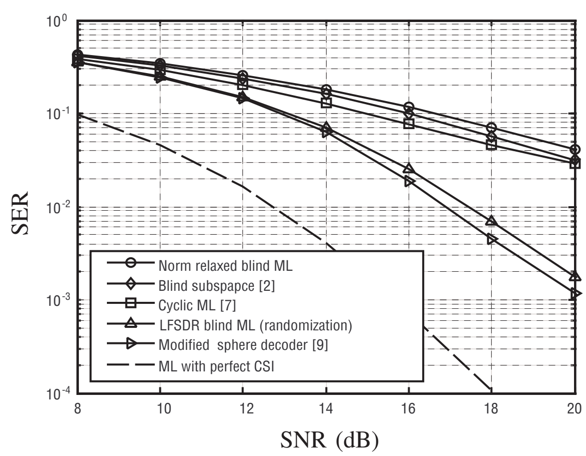
Fig. 4. Complexity comparison results of the proposed LFSDR (solved respectively by SeDuMi [21] and the specialized IPA in Table I) and Cui-Tellambura's modified sphere decoder [9].



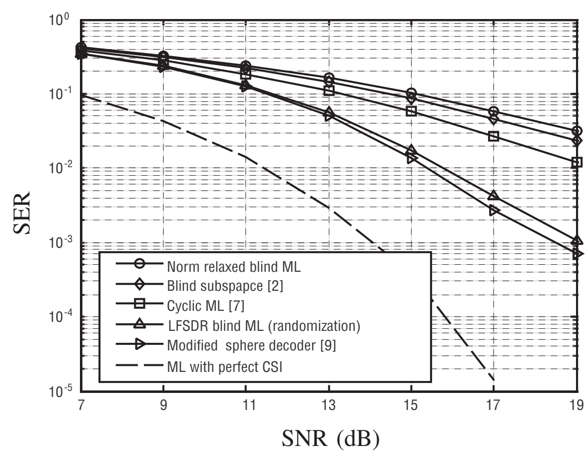
(a) $N_r = 1,\ P = 2$



(b) $N_r = 2,\ P = 2$



(c) $N_r = 3,\ P = 2$



(d) $N_r = 4,\ P = 2$

Fig. 5. Performance (SER) comparison results of the proposed LFSDR with some existing suboptimal methods and Cui-Tellambura's modified sphere decoder [9] under various settings.

TBCs. The proposed LFSDR approach is efficient, involving solving only one SDP followed by a simple rank-1 solution approximation procedure. Moreover, this approach has been shown to be at least no worse than the simple norm relaxation method. Extensive simulation results for both 16-QAM and 64-QAM OSTBCs have demonstrated that the proposed LFSDR approach outperforms some existing suboptimal methods. Moreover, we have seen that, for both 16-QAM and 64-QAM OSTBCs, the proposed LFSDR approach is effective in approximating the true blind ML detection solution, especially when there are multiple receive antennas.

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