MIMO Signaling:
Knowing the Classics Can Make a Difference

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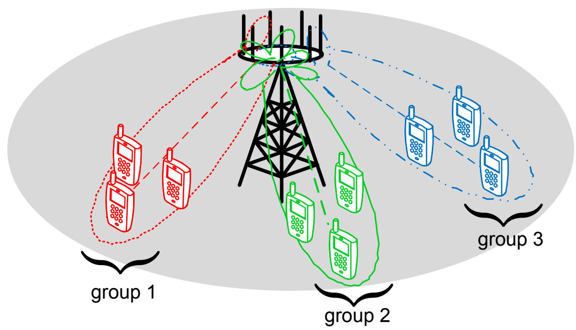
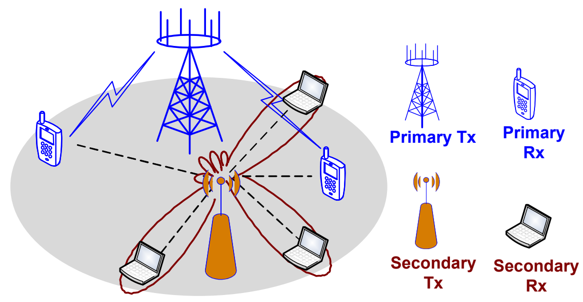
Acknowledgement: Xiaoxiao Wu, Anthony Man-Cho So, Mingjie Shao, Qiang Li, Lee Swindlehurst

Beamforming and Optimization

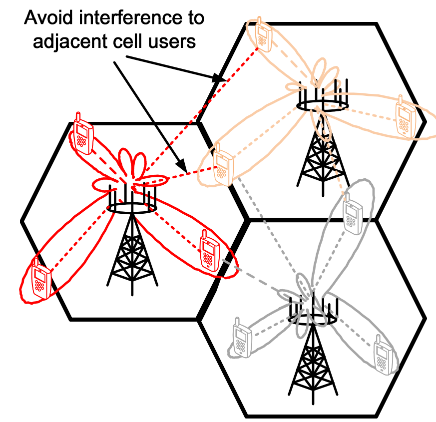
- beamforming, powered by optimization, is almost everywhere!



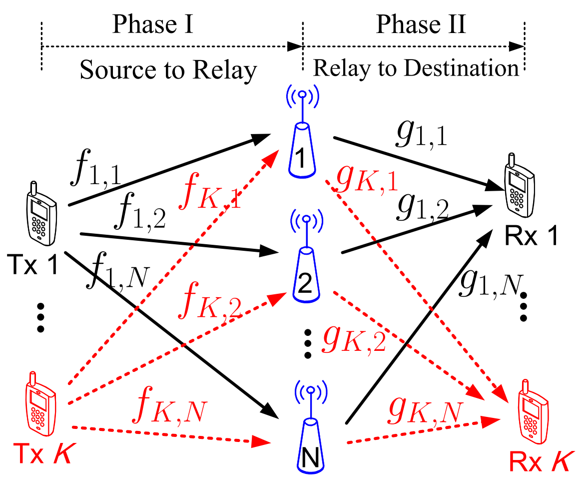
(a) multigroup multicasting



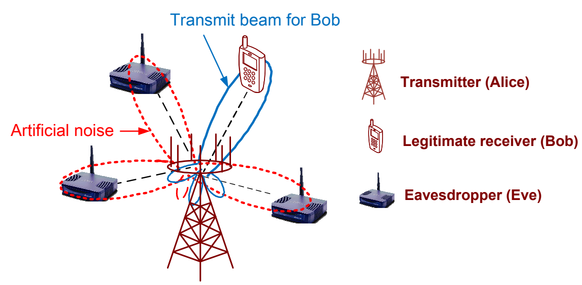
(b) cognitive radio



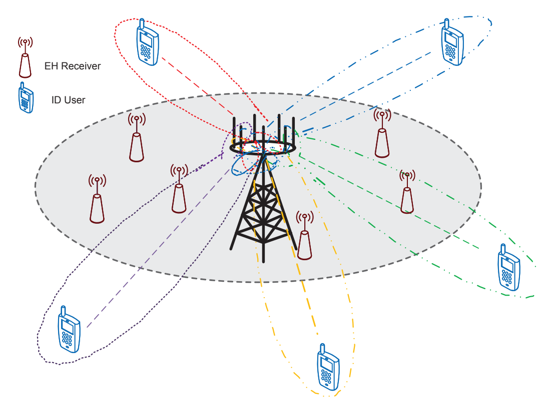
(c) multicell



(d) relay beamforming

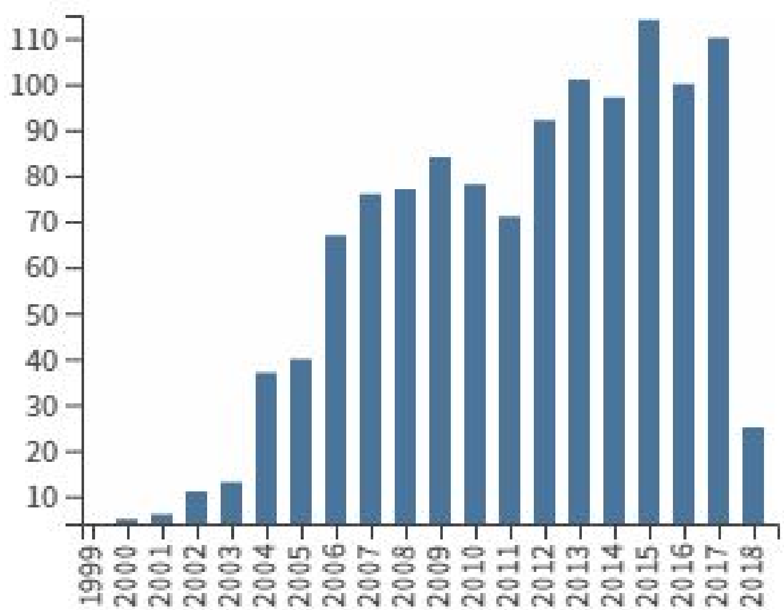


(e) physical-layer security

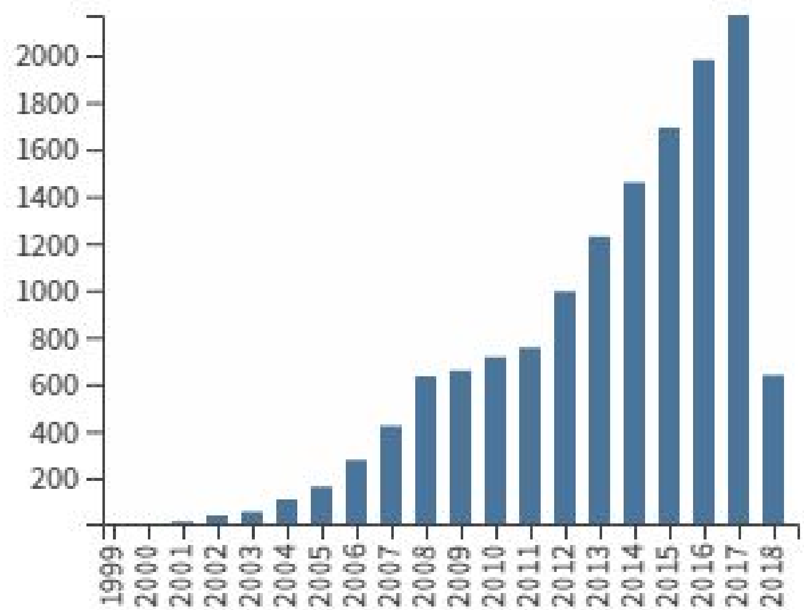


(f) energy harvesting

Transmit Beamforming



(a) Total Publications by Year



(b) Sum of Times Cited by Year

The number of published papers having the keyword "transmit beamforming" and the corresponding citations. Data obtained from SCI-Expanded database.

This Talk

- I am not going to talk about optimization today
- I would like to go back to the basics, and share with you how classical wisdom helps
- we will look into two different topics
  - topic 1: multicast beamforming
  - topic 2: one-bit massive MIMO precoding

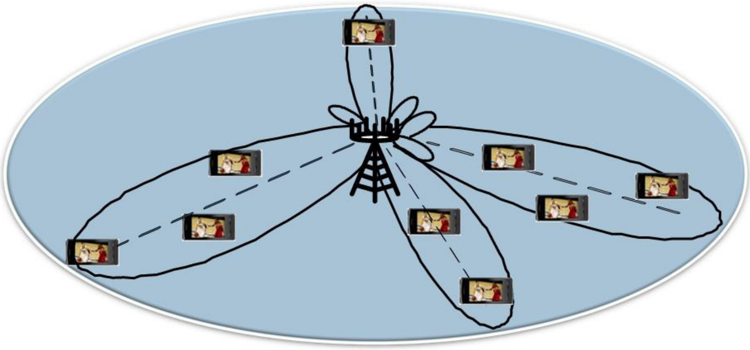
Topic 1: Multicast Beamforming

Scenario

- scenario: K-user MISO downlink, common info. broadcast, perfect channel state information at the transmitter (CSIT)

- received signal at user i:

$$y_i(t) = \boldsymbol{h}_i^H \boldsymbol{x}(t) + v_i(t),$$



where

$\boldsymbol{h}_i \in \mathbb{C}^N$ is the user i channel;
$\boldsymbol{x}(t) \in \mathbb{C}^N$ is the transmit signal;
$v_i(t)$ is noise.

Scenario

- scenario: K-user MISO downlink, common info. broadcast, perfect CSIT

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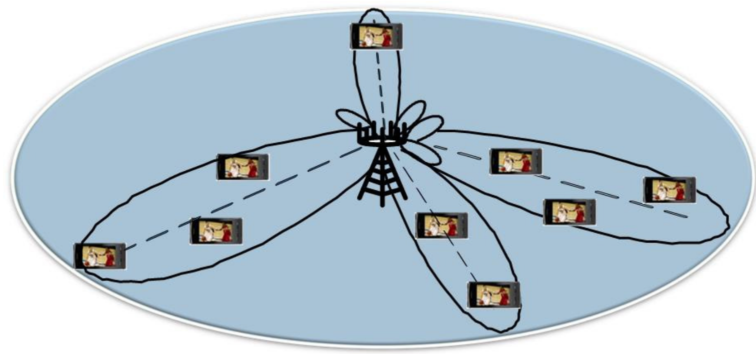

$$y_i(t) = \boldsymbol{h}_i^H \boldsymbol{x}(t) + v_i(t).$$

- transmit scheme: Beamforming (BF)

$$\boldsymbol{x}(t) = \boldsymbol{w} s(t),$$

where $s(t) \in \mathbb{C}$ is a data stream; $\boldsymbol{w} \in \mathbb{C}^N$ is the beamformer

Scenario

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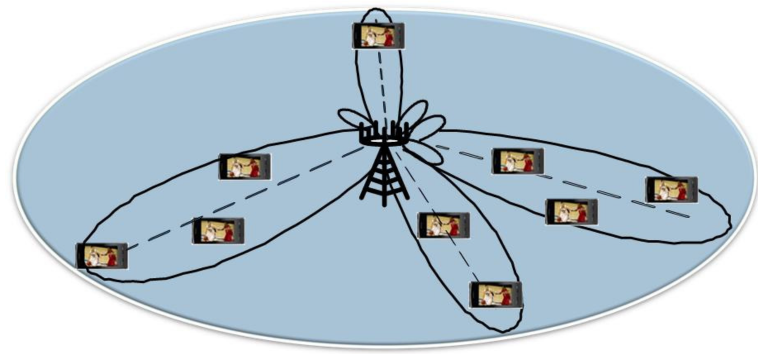

- transmit scheme: Beamforming (BF) $\boldsymbol{x}(t) = \boldsymbol{w}s(t)$.

- **Problem:** minimize the average transmit power, subject to SNR constraints

$$\min_{\boldsymbol{w} \in \mathbb{C}^N} \mathbb{E}[\|\boldsymbol{x}(t)\|^2] = \|\boldsymbol{w}\|^2$$

$$\text{s.t. } \mathsf{SNR}_i = \frac{|\boldsymbol{w}^H \boldsymbol{h}_i|^2}{\sigma_i^2} \geq \gamma, \qquad i = 1, \ldots, K,$$

where $\sigma_i^2 = \mathbb{E}[|v_i(t)|^2]$; γ is the SNR requirement; we assume $\mathbb{E}[|s_i(t)|^2] = 1$

Multicast Beamforming

- a classic problem, popularized by **[Sidiropoulos-Davidson-Luo'06]**
- a topic dominated by optimization
  - semidefinite relaxation (SDR) is the most famous
  - numerous non-convex algorithms were also proposed
- a keystone that triggered SDR research for many, many, many BF problems
  - note: another keystone is with unicast BF **[Bengtsson-Ottersten'01]**
- it's an old problem, so it's still practically meaningful?

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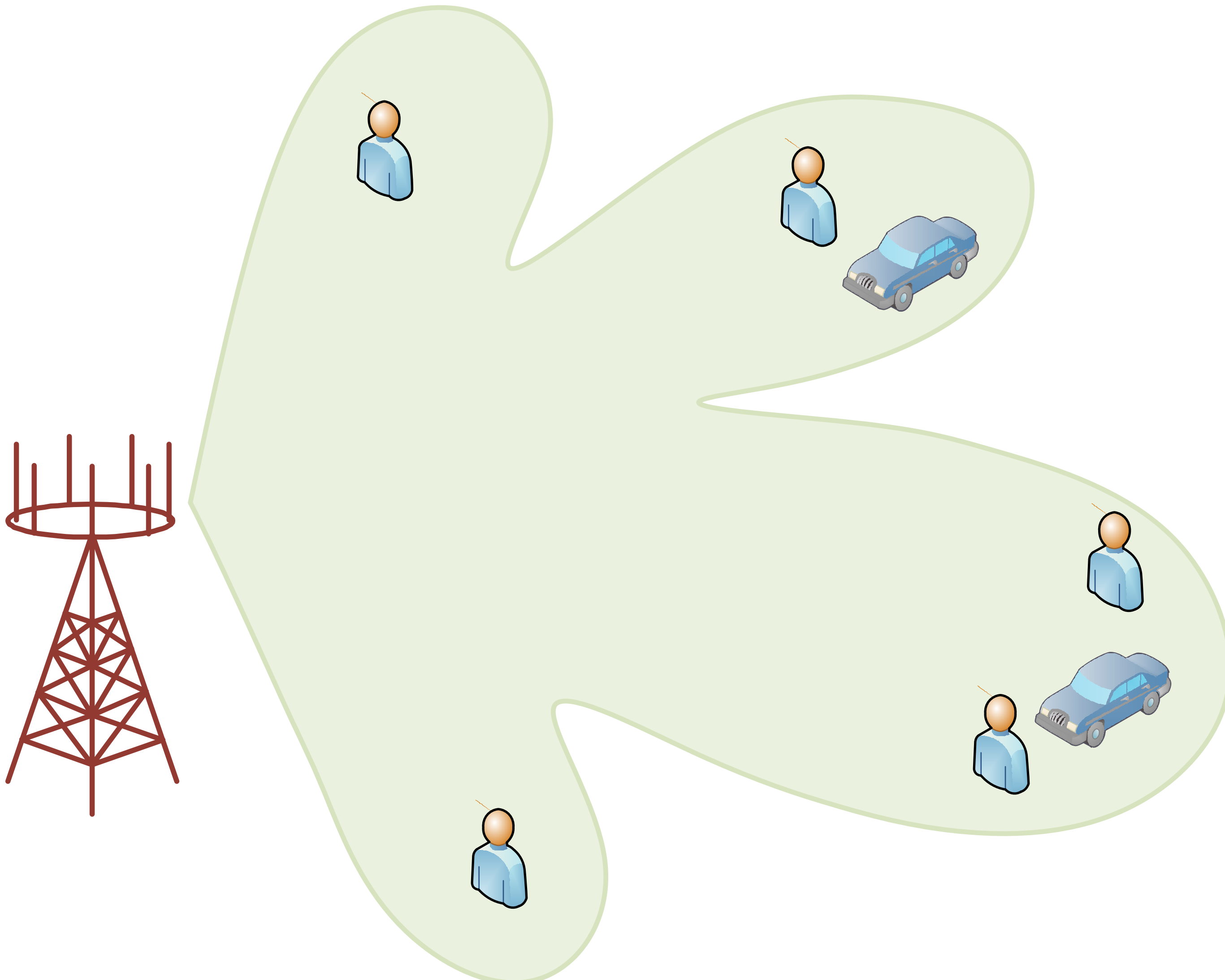
- it's an old problem, so it's still practically meaningful?
  - it depends
  - live streaming, V2V info broadcast, etc., are all sound applications
  - I see fewer works on massive common info broadcast

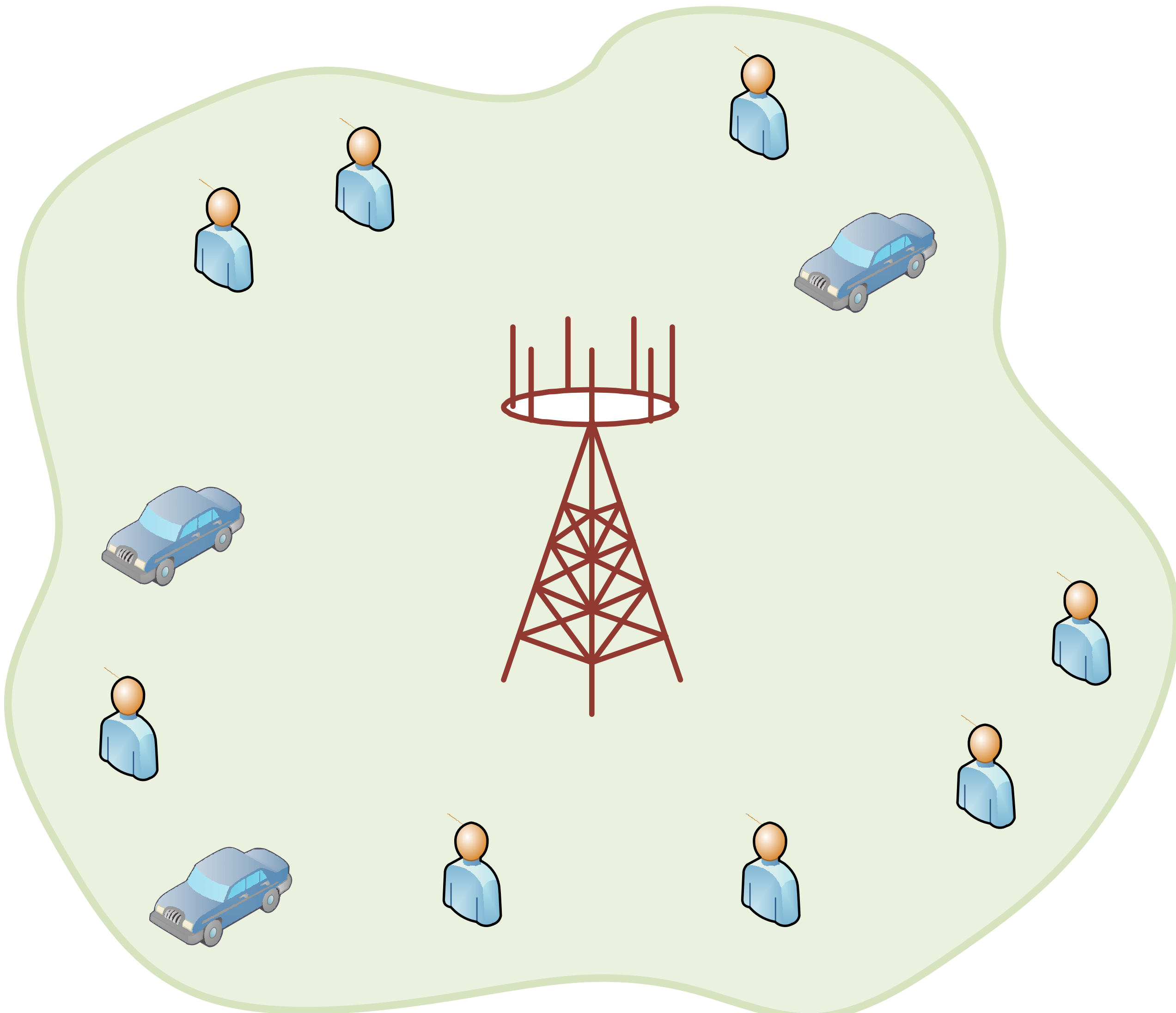
A Rethinking

- BF is spatial selective



A Rethinking

- **Question:** is BF always good? Or, does it always make sense to stick with BF?



From an Information Theory Viewpoint

- consider the multicast capacity (MC). Let $\boldsymbol{W} = \mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$.

$$C_{\mathsf{MC}}(P) = \max_{\boldsymbol{W}} \min_{i=1,\ldots,K} \log(1 + \boldsymbol{h}_i^H \boldsymbol{W} \boldsymbol{h}_i / \sigma_i^2))$$
$$\text{s.t. } \mathrm{Tr}(\boldsymbol{W}) \leq P, \; \boldsymbol{W} \succeq \mathbf{0}$$

Let $\boldsymbol{W}^\star$ be the MC-optimal solution

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  Let $\boldsymbol{W}^\star$ be the MC-optimal solution

- fact: if $\boldsymbol{W}^\star$ has rank one, or $\boldsymbol{W}^\star = \boldsymbol{w}^\star(\boldsymbol{w}^\star)^H$, BF is the optimal tx scheme

- fact: the SDR of the multicast BF rate max. problem

$$\max_{\|\boldsymbol{w}\|^2 \leq P} \min_{i=1,\ldots,K} \log(1 + |\boldsymbol{h}_i^H \boldsymbol{w}|^2 / \sigma_i^2)$$

  is identical to the MC capacity

  - the difference is that when $\boldsymbol{W}^\star$ has higher rank, we apply rank-one approx. with $\boldsymbol{W}^\star$ to get our beamformer $\boldsymbol{w}$

From an Information Theory Viewpoint

- consider the multicast capacity (MC). Let $\boldsymbol{W} = \mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$.

$$C_{\mathsf{MC}}(P) = \max_{\boldsymbol{W}} \min_{i=1,\ldots,K} \log(1 + \boldsymbol{h}_i^H \boldsymbol{W} \boldsymbol{h}_i / \sigma_i^2))$$
$$\text{s.t. } \mathrm{Tr}(\boldsymbol{W}) \leq P, \; \boldsymbol{W} \succeq \mathbf{0}$$

  Let $\boldsymbol{W}^\star$ be the MC-optimal solution

- **Question:** can we design a transceiver scheme that can do "higher-rank BF"?

Stochastic Beamforming: System Model

- consider an approach called **stochastic beamforming (SBF)** **[Wu-Ma-So'13]**[1]

- transmission scheme: a single-stream time-varying BF scheme

$$\boldsymbol{x}(t) = \boldsymbol{w}(t)s(t), \quad t = 1, 2, \ldots,$$

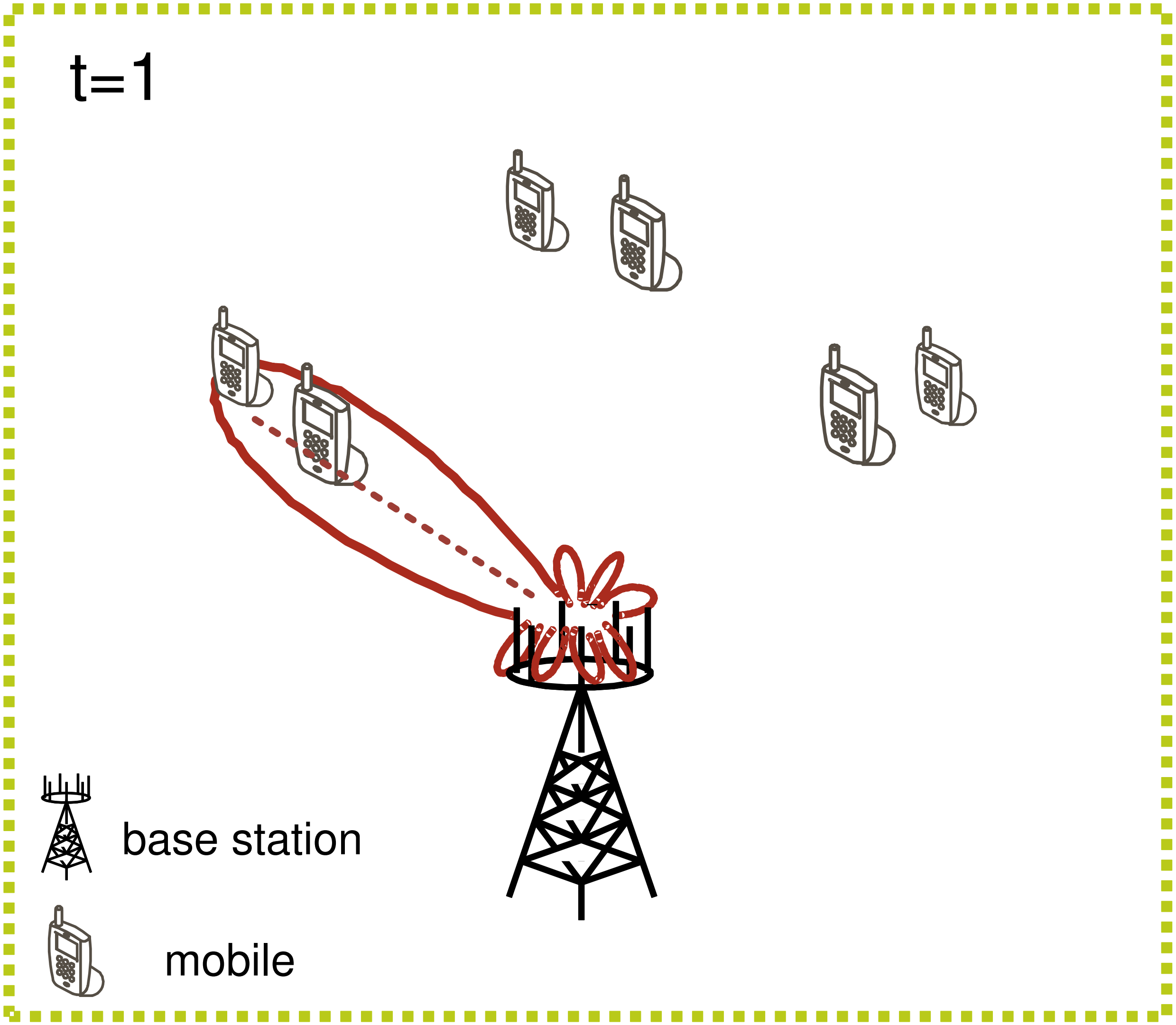
  where $\boldsymbol{w}(t)$ is a random-in-time beamformer

- idea: generate $\boldsymbol{w}(t)$ such that $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}(t)^H] = \boldsymbol{W}^\star$, the MC-optimal covariance

- caveat: $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}(t)^H] = \boldsymbol{W}^\star$ does not necessarily imply that SBF is MC-optimal

[1] X. Wu, W.-K. Ma, A. M.-C. So, "Physical-layer multicasting by stochastic transmit beamforming and Alamouti space-time coding," *IEEE TSP*, 2013.

Idea: Swing the Beamformer



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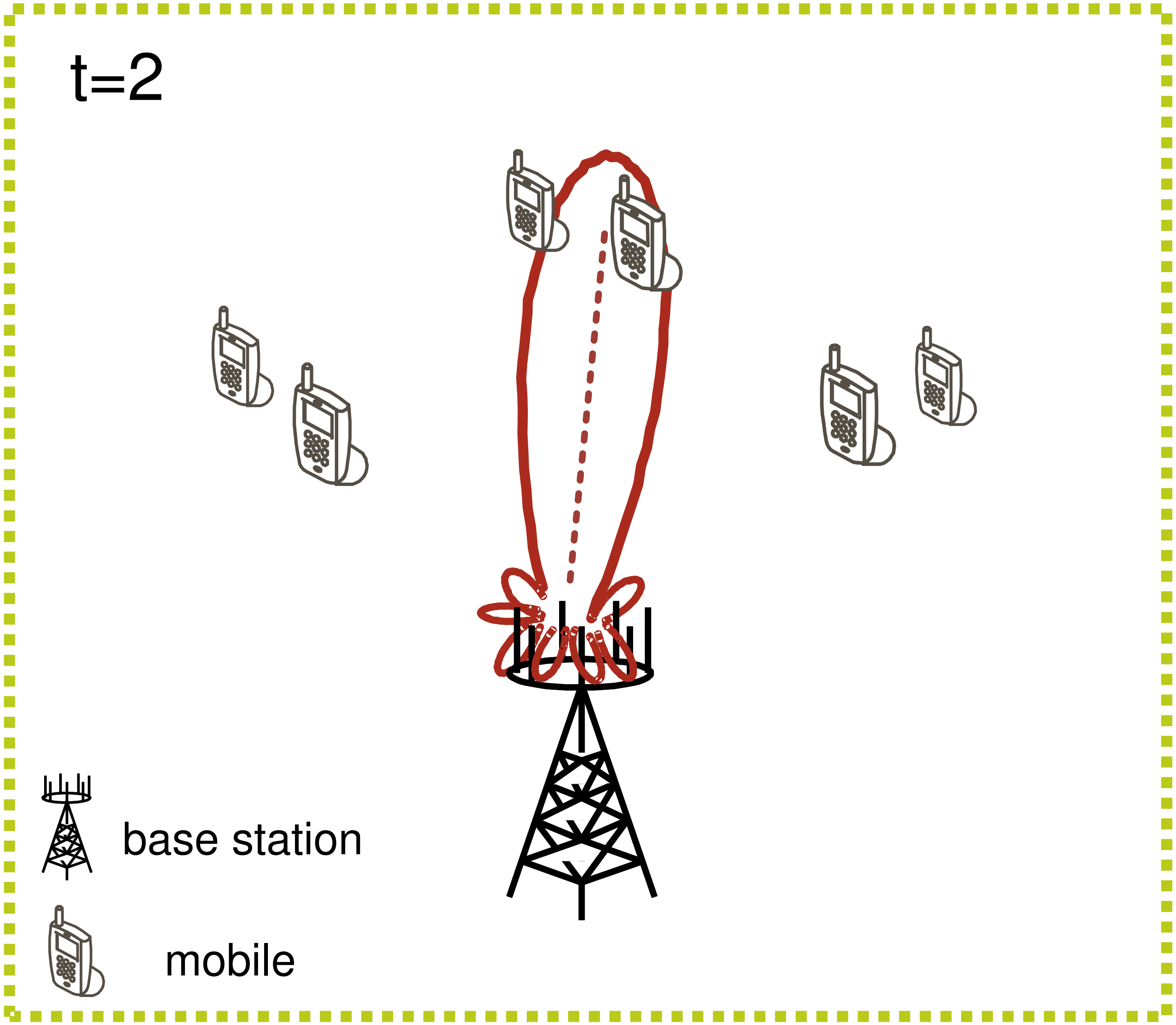

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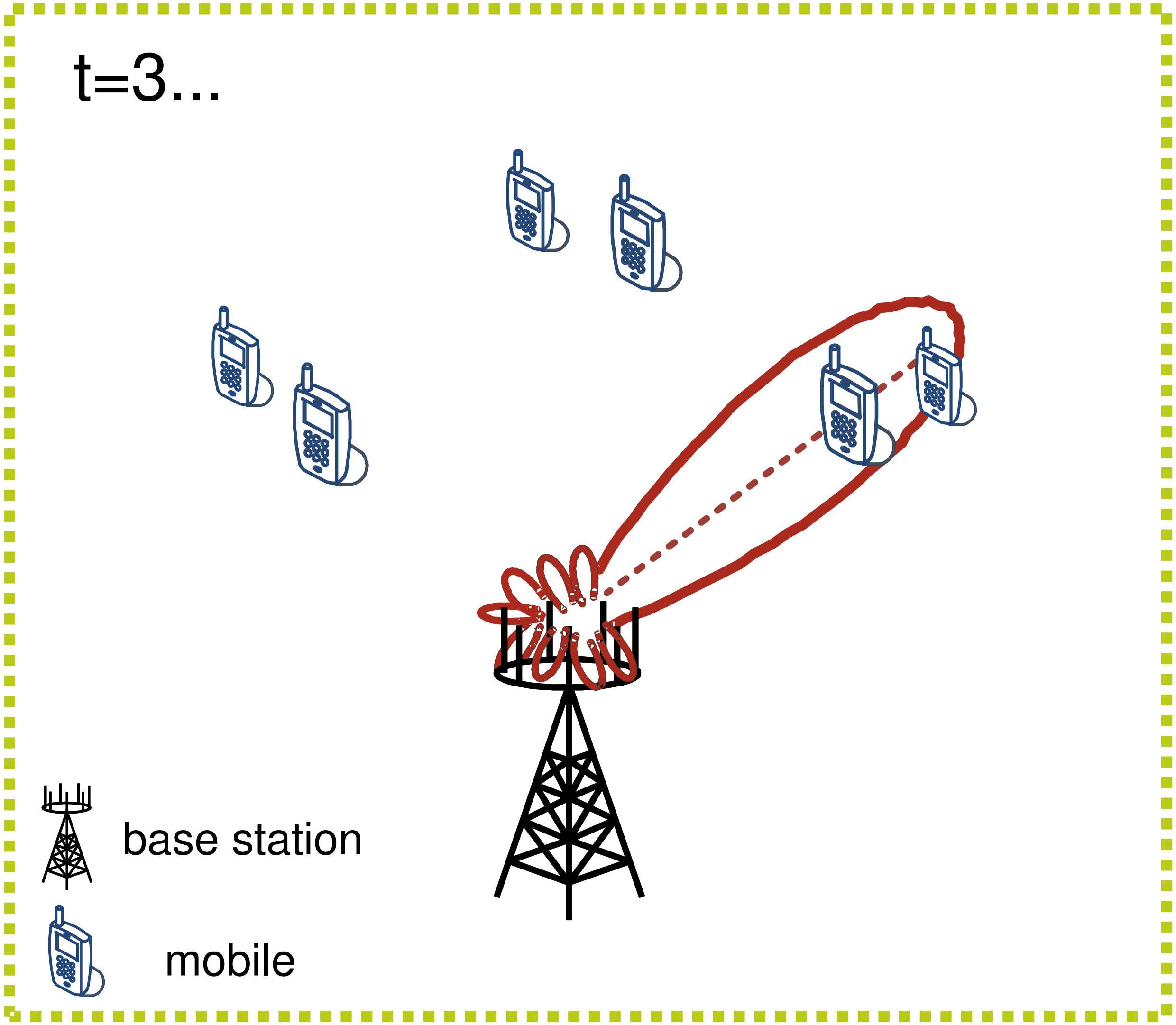


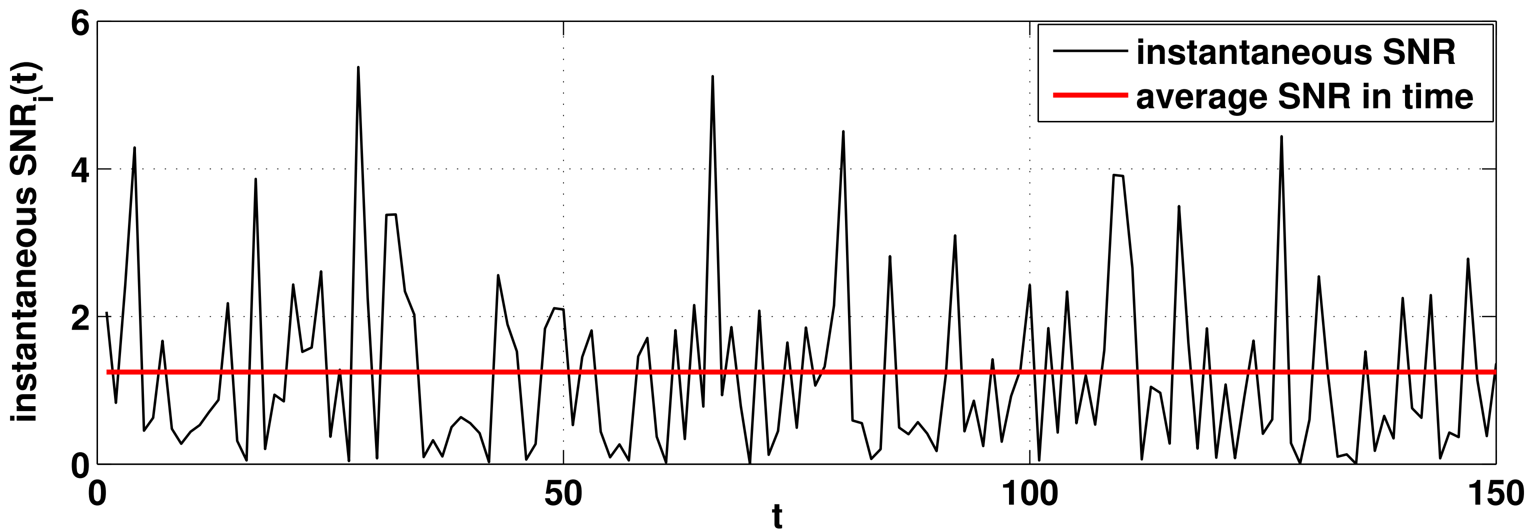
Stochastic Beamforming: System Model

- received signals:

$$y_i(t) = \boldsymbol{h}_i^H \boldsymbol{w}(t) s(t) + v_i(t), \qquad t = 1, 2, \ldots$$

The SNRs fluctuates in time, with

$$\text{SNR}_i(t) = |\boldsymbol{h}_i^H \boldsymbol{w}(t)|^2 / \sigma_i^2$$

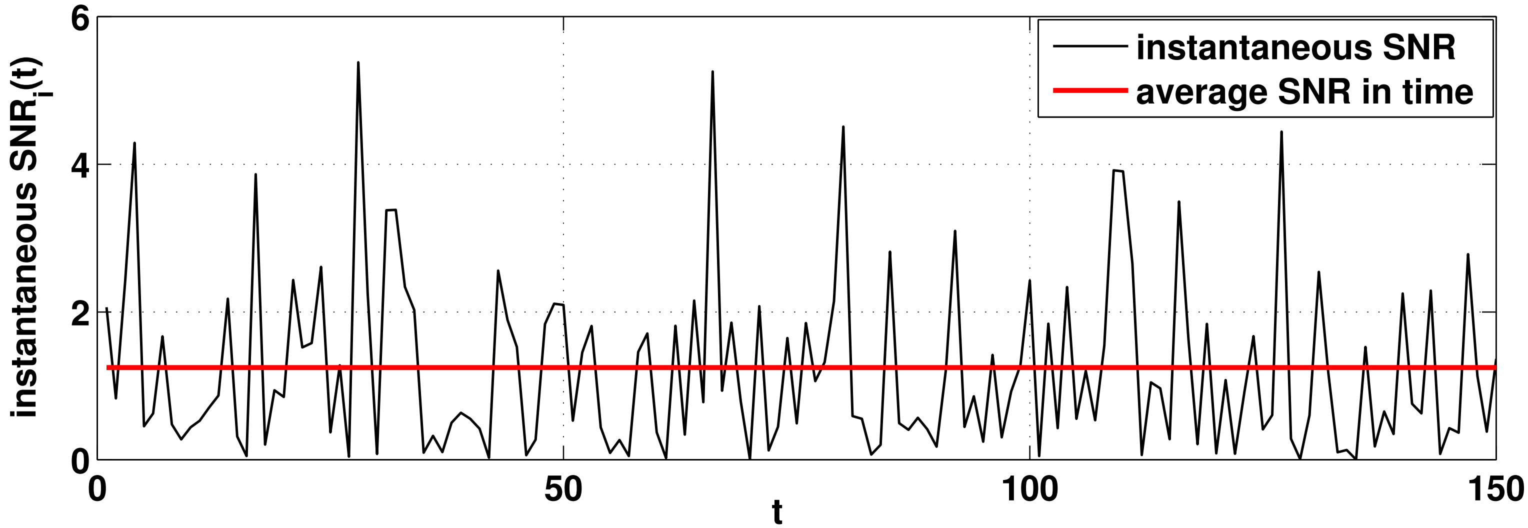


Traditional Wisdom: How We Fight Fast Fading

- consider a SISO model under fast fading

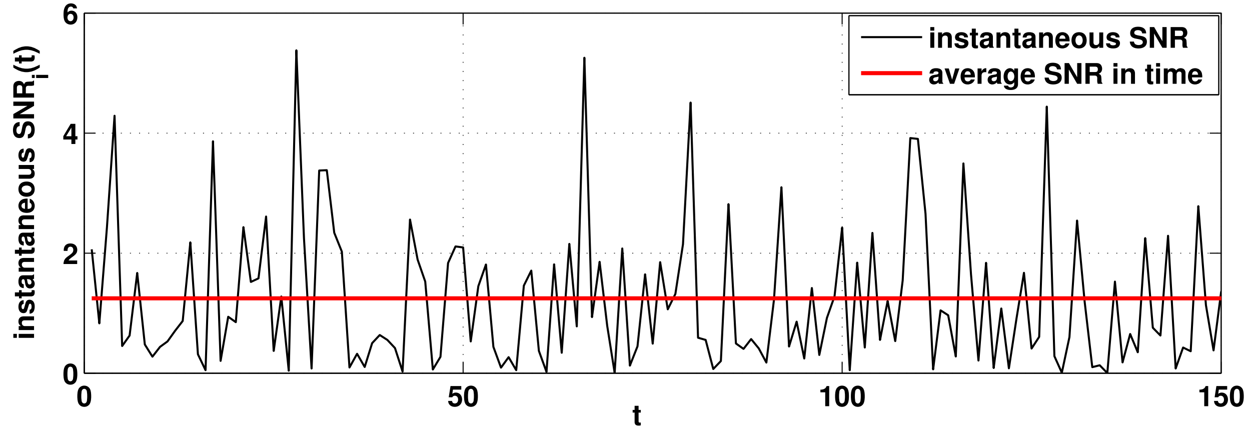
$$y(t) = \alpha(t)s(t) + v(t), \quad t = 1, 2, \ldots$$

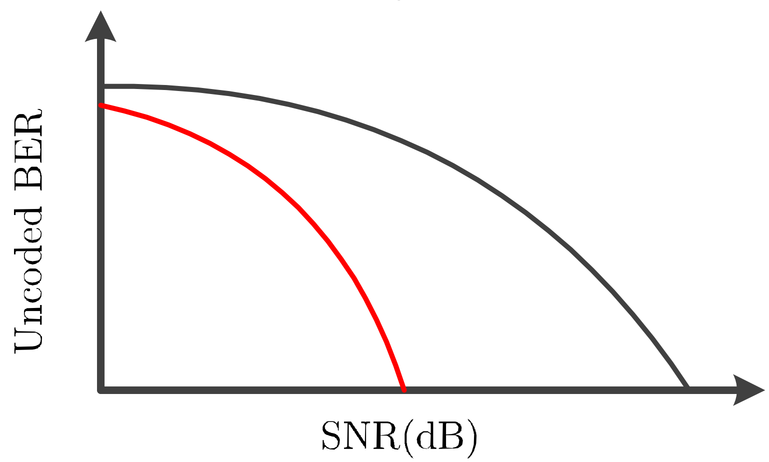
where $\alpha(t)$ is the fading coefficient; $s(t)$ is a symbol; $v(t)$ is noise



Traditional Wisdom: How We Fight Fast Fading

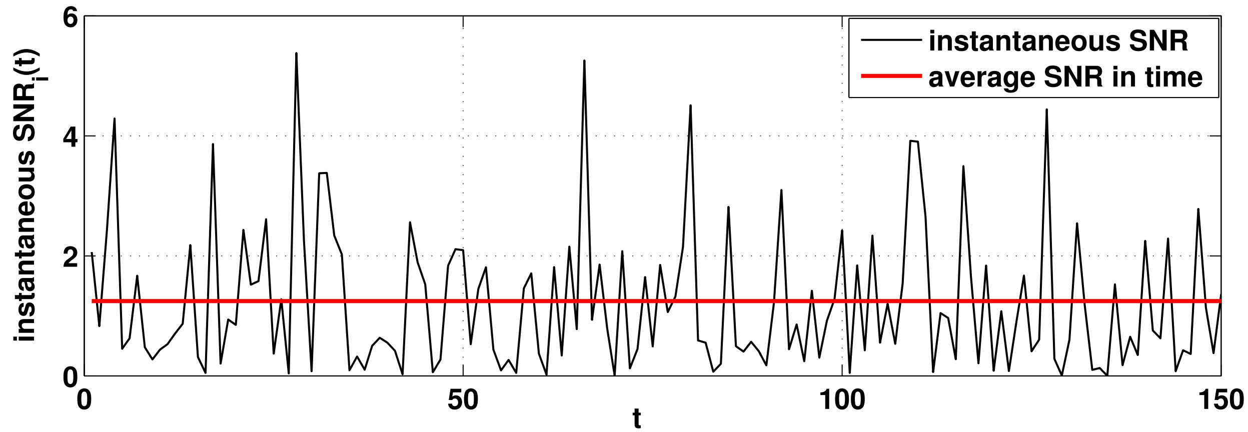
- model: $y(t) = \alpha(t)s(t) + v(t), \quad t = 1, 2, \ldots$
- issue: uncoded BERs are dominated by deep fade instances

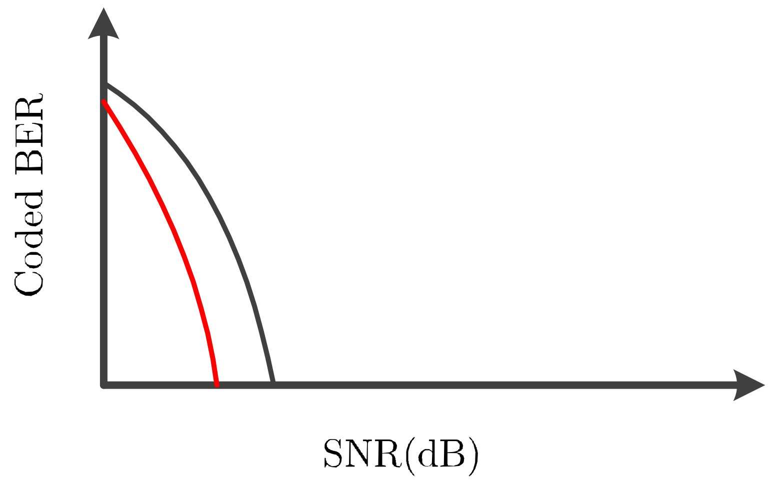




Traditional Wisdom: How We Fight Fast Fading

- model: $y(t) = \alpha(t)s(t) + v(t), \quad t = 1, 2, \ldots$
- solution: use channel coding to "average out" deep fades (used in 2G!)





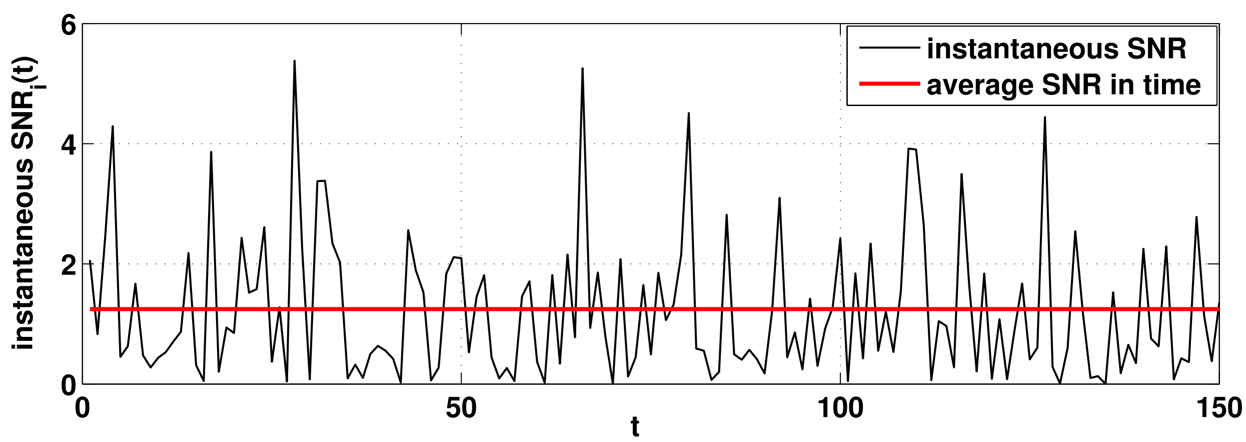
Traditional Wisdom: How We Fight Fast Fading

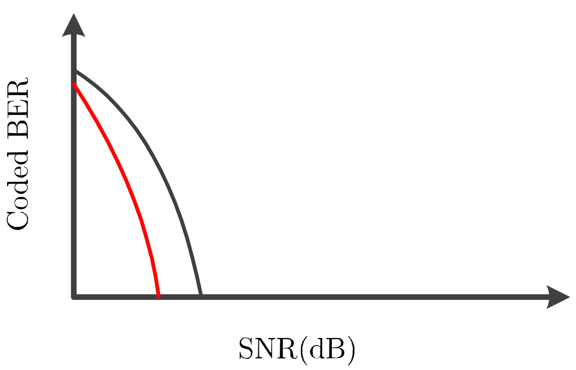
- model: $y(t) = \alpha(t)s(t) + v(t), \quad t = 1, 2, \ldots$
- solution: use channel coding to "average out" deep fades
- from an information theory viewpoint, the capacity is

$$C(P) = \mathbb{E}_{\alpha \sim \mathcal{D}}[\log(1 + \alpha P/\sigma^2)]$$

  where $\mathcal{D}$ is the distribution of α.

- in practice, the above capacity may be approached if we apply a near-ideal scalar channel code such as Turbo code and LDPC





Stochastic Beamforming: How to Randomize the BF?

- so we will apply channel coding
- but how should we randomize the SBF vector $\boldsymbol{w}(t)$?

Stochastic Beamforming: How to Randomize the BF?

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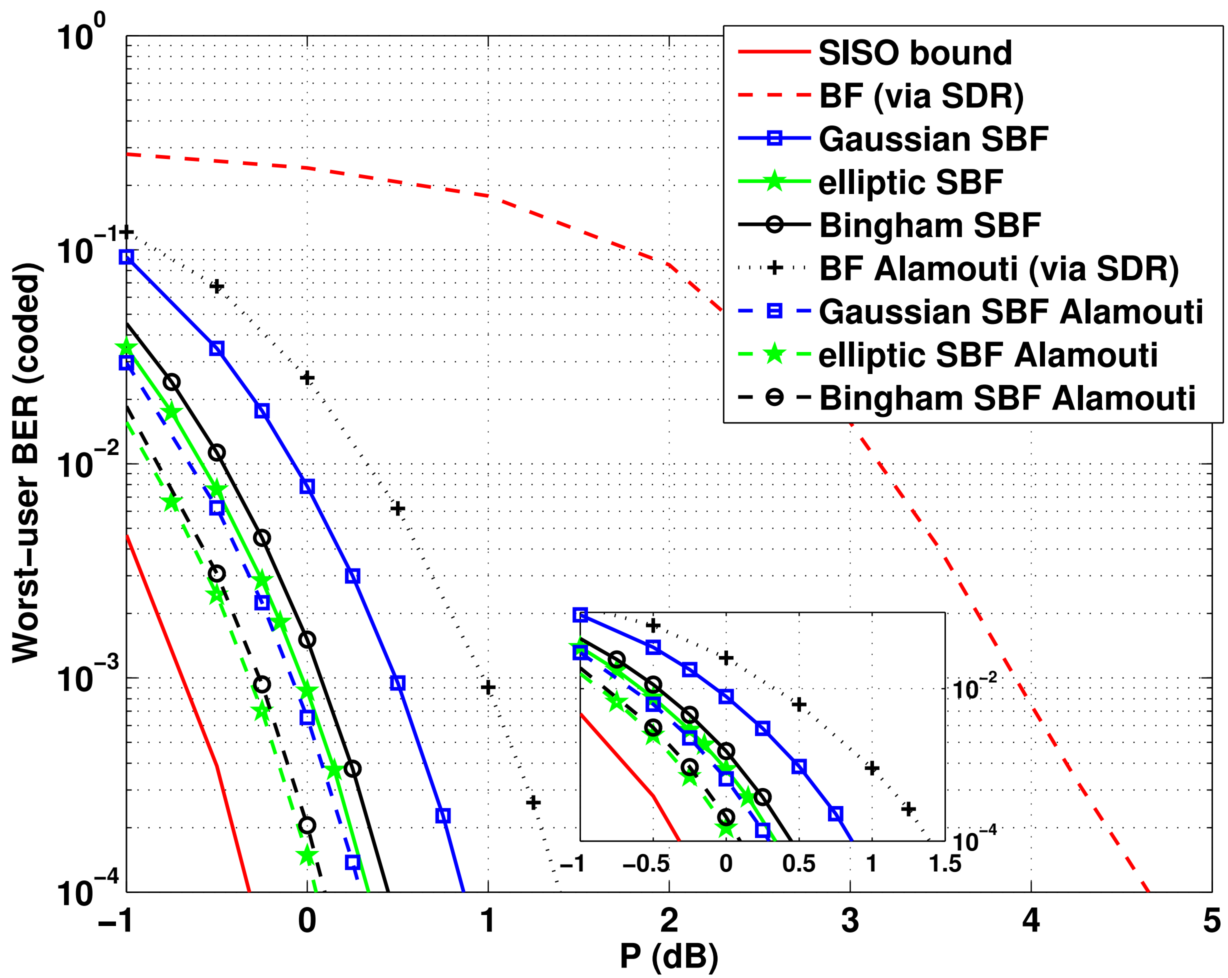
- but how should we randomize the SBF vector $\boldsymbol{w}(t)$?

- let's do this heuristics—generate the SBF vectors by

$$\boldsymbol{w}(t) \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{W}^\star)$$

  in the time-i.i.d. fashion

Simulation Results: SBF Does Work!



8 tx antennas, 32 users, QPSK, BICM, rate-$1/3$ Turbo code with a code length 2880. Compare BF (via SDR), **SBF with Gaussian randomizations** and SISO bound for the moment.

Stochastic Beamforming: Achievable Rate Analysis

- assuming ideal channel coding of $s(t)$, the SBF achievable multicast rate is

$$C_{\mathsf{SBF}}(P) = \min_{i=1,\ldots,K} \mathbb{E}_{\boldsymbol{w}\sim\mathcal{D}}[\log(1 + \boldsymbol{h}_i^H \boldsymbol{w}\boldsymbol{w}^H \boldsymbol{h}_i)],$$

where $\mathcal{D} = \mathcal{CN}(\mathbf{0}, \boldsymbol{W}^\star)$ is the SBF distribution; note that $\boldsymbol{W}^\star$ depends on P

- recall the multicast capacity

$$C_{\mathsf{MC}}(P) = \max_{\boldsymbol{W}\succeq\mathbf{0}, \mathrm{Tr}(\boldsymbol{W})\leq P} \ \min_{i=1,\ldots,K} \log(1 + \boldsymbol{h}_i^H \boldsymbol{W} \boldsymbol{h}_i)$$

The achievable rate gap of the Gaussian SBF satisfies, for all $P \geq 0$,

$$C_{\mathsf{MC}}(P) - C_{\mathsf{SBF}}(P) \leq 0.8314 \text{ bits/s/Hz},$$

- this result does not depend on the number of users K, while SDR performance tends to deteriorate as K increases

Stochastic Beamforming: Further Endeavor

- Gaussian SBF is no good in terms of peak-to-average power spread
- **Elliptic SBF:** let $r = \text{rank}(\boldsymbol{W}^\star)$; factorize $\boldsymbol{W}^\star = \boldsymbol{L}^H \boldsymbol{L}$, $\boldsymbol{L} \in \mathbb{C}^{r \times N}$

$$\boldsymbol{w} = \frac{\boldsymbol{L}^H \boldsymbol{\alpha}}{\|\boldsymbol{\alpha}\|/\sqrt{r}}, \quad \boldsymbol{\alpha} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_r)$$

  – $\|\boldsymbol{w}\|^2 \in [r\lambda_{\min}^{+}(\boldsymbol{W}^\star), r\lambda_{\max}(\boldsymbol{W}^\star)]$ with prob. 1; $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}^H(t)] = \boldsymbol{W}^\star$

- **Bingham SBF:**

$$\boldsymbol{w} = \frac{\boldsymbol{L}^H \boldsymbol{\alpha}}{\|\boldsymbol{L}^H \boldsymbol{\alpha}\|}, \quad \boldsymbol{\alpha} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_r).$$

  – $\|\boldsymbol{w}\|^2 = 1$ (zero power spread!); $\mathbb{E}[\boldsymbol{w}(t)\boldsymbol{w}^H(t)] \neq \boldsymbol{W}^\star$

For all $P \geq 0$, both the elliptic and Bingham SBFs have

$$C_{\mathsf{MC}}(P) - C_{\mathsf{SBF}}(P) \leq 0.8314 \text{ bits/s/Hz}$$

Stochastic Beamforming: Further Endeavor

- a more powerful way of using SBF is to combine SBF with the (rank-2) Alamouti space-time code.

> For all $P \geq 0$, the combinations of Alamouti space-time coding and the aforementioned SBF schemes lead to a rate gap no worst than 0.39 bits/s/Hz.

- SBF can be applied to almost all other BF problems where SDR is applicable, eliminating the need to do rank-one approx. in SDR
  - this was shown to be working for multigroup multicasting **[Wu-Li-So-Ma'16]**

Take-Home Point

- in SDR, when we get higher rank SDR solutions we generally see this as a weakness
- SBF tells us to embrace the higher rank solution—and turn the weakness into benefits—by rethinking the transceiver design
- it is the traditional wisdom of combating fast fading channels that presents us with this opportunity

Topic 2: One-Bit Massive MIMO Precoding

One-Bit Massive MIMO

- massive MIMO: promise many nice things
- issue:
  - massive no. of antennas = massive no. of RF front-ends and ADCs/DACs
  - high-resolution ADCs/DACs are expensive
  - RF power amplifiers that provides a wide linear dynamic range operate in high backoff mode, this wastes a lot of energy
- one-bit MIMO:
  - replace the high-resolution ADCs/DACs with the cheap one-bit ADCs/DACs
  - lead to constant envelope transmission, low-backoff RF power amplifiers can be used, energy saved

One-Bit Massive MIMO

- challenge: many one-bit MIMO precoding designs require solving binary optimization problems with a massive scale **[Jacobsson-Durisi-Coldrey-Goldstein-Studer'17]**, **[Sohrabi-Liu-Yu'18]**, **[Shao-Li-Ma-So'19]** (and more)

- we can also consider conventional linear precoding first, and then one-bit quantize that precoder

- but performance of such quantized linear precoding can be bad as quantization error can be severe

- **Question:** is there a way we can do the precode-then-quantize route in a more reliable way?

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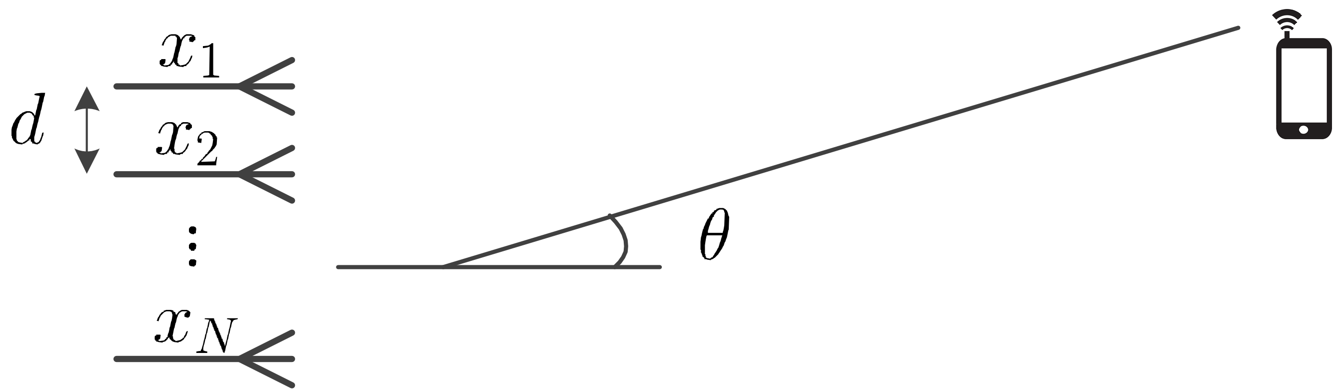
- **Question:** is there a way we can do the precode-then-quantize route in a more reliable way?
  - we try to answer that question **[Shao-Ma-Li-Swindlehurst'19]**[2]

[2] M. Shao, W.-K. Ma, Q. Li, and L. Swindlehurst, "One-bit massive MIMO precoding," *ArXiv*, 2019.

Model



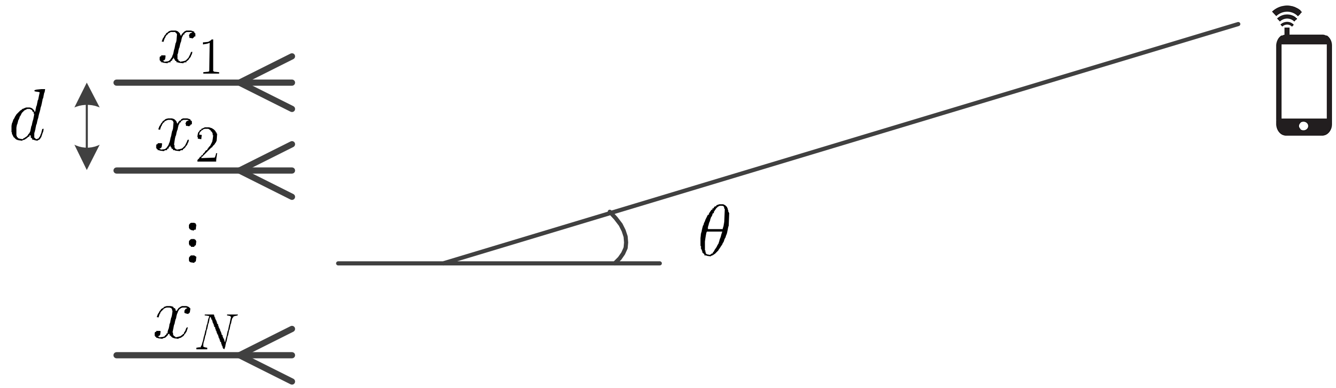
- model:

$$y = \sqrt{\frac{P}{2N}}\boldsymbol{h}^T\boldsymbol{x} + v$$

where $y \in \mathbb{C}$ is the received signal; $\boldsymbol{x} \in \mathbb{C}^N$ is the transmitted signal; $\boldsymbol{h} \in \mathbb{C}^N$ is the channel; v is noise; P is the transmit power; N is the no. of antennas

- constraint: $\boldsymbol{x} \in \{\pm 1 \pm j\}^N$

Model

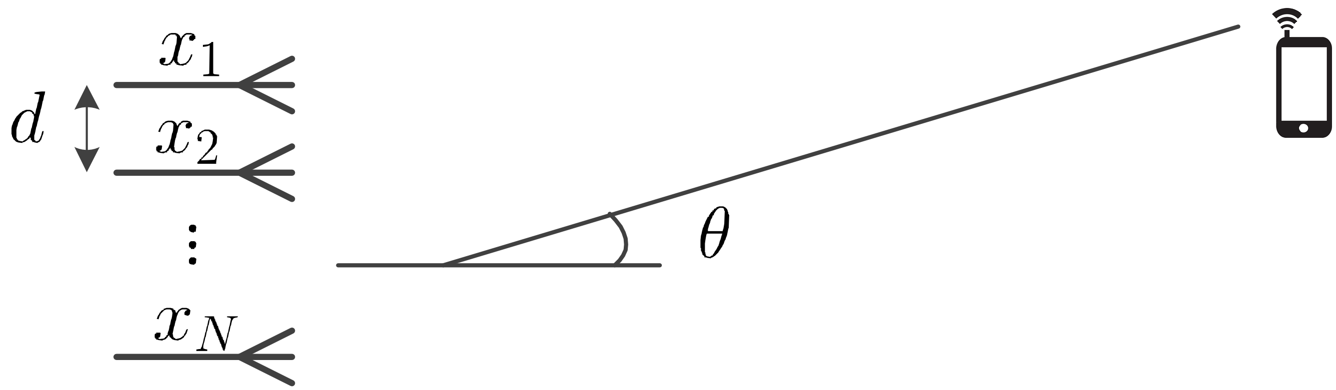


- model: $y = \sqrt{\frac{P}{2N}} \boldsymbol{h}^T \boldsymbol{x} + v$
- constraint: $\boldsymbol{x} \in \{\pm 1 \pm j\}^N$
- channel model: uniform linear array, one-path propagation

$$\boldsymbol{h} = \alpha \boldsymbol{a}(\theta), \quad \boldsymbol{a}(\theta) = (1, e^{-j\phi}, \cdots, e^{-j\phi(N-1)}), \quad \phi = \frac{2\pi d}{\lambda} \sin(\theta)$$

where $\alpha \in \mathbb{C}$ is the complex channel gain; θ is the angle of departure; λ is the carrier wavelength; $d \leq \lambda/2$ is the inter-antenna spacing

Model



- model: $y = \sqrt{\frac{P}{2N}} \boldsymbol{h}^T \boldsymbol{x} + v$
- constraint: $\boldsymbol{x} \in \{\pm 1 \pm j\}^N$
- channel model: $\boldsymbol{h} = \alpha \boldsymbol{a}(\theta), \boldsymbol{a}(\theta) = (1, e^{-j\phi}, \cdots, e^{-j\phi(N-1)}), \phi = \frac{2\pi d}{\lambda} \sin(\theta)$
- aim: design $\boldsymbol{x} \in \{\pm 1 \pm j\}^N$ such that

$$\boldsymbol{h}^T \boldsymbol{x} \approx c \cdot s$$

where $c > 0$ is scalar; s is a data symbol

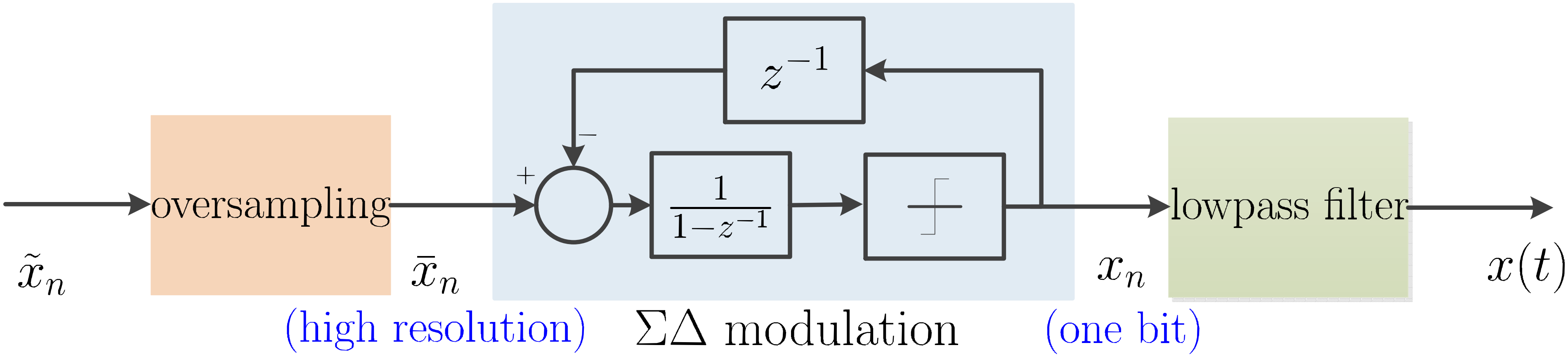
Temporal $\Sigma\Delta$ Modulation

- when you listen to music with your smart phone, DAC is being used.
- suppose that DAC is a 16-bit DAC, say, according to the technical specification
- do you think it's a real 16-bit DAC?

Temporal $\Sigma\Delta$ Modulation

- when you listen to music with your smart phone, DAC is being used.
- suppose that DAC is a 16-bit DAC, say, according to the technical specification
- do you think it's a real 16-bit DAC?
- unlikely. It is too expensive to build a real 16-bit DAC (which requires outputting 65536 voltage levels)
- the DAC used is likely to be a (much) improved version of the one-bit $\Sigma\Delta$ modulator

Temporal ΣΔ Modulation

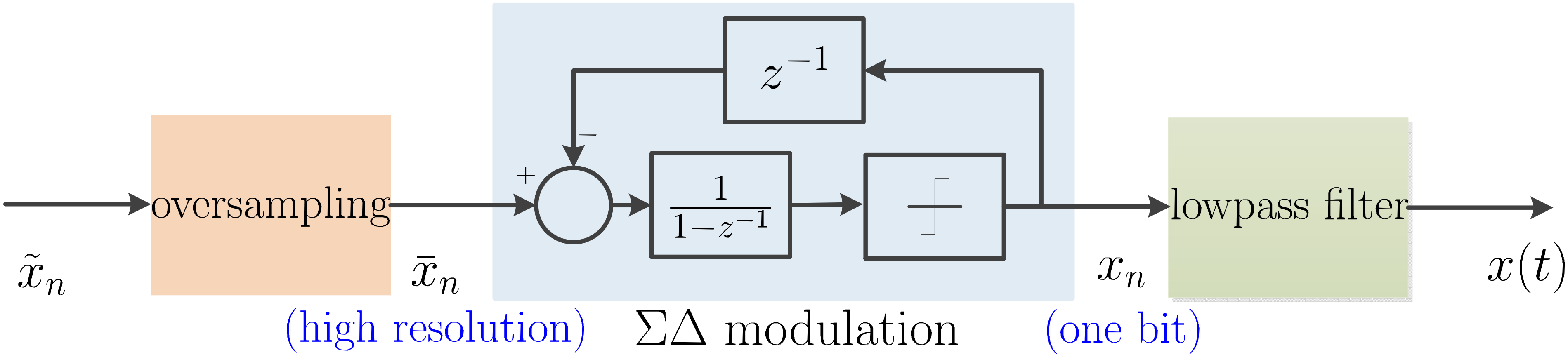


- operation: $x_n = \text{sgn}(b_n), b_n = b_{n-1} + (\bar{x}_n - x_{n-1})$
- we have

$$x_n = \bar{x}_n + q_n - q_{n-1}$$

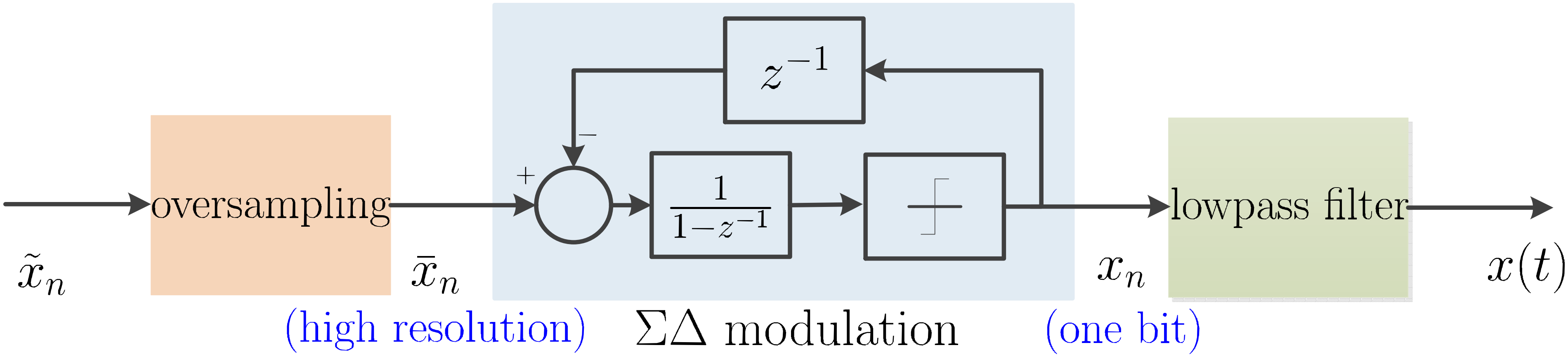
where $q_n = \text{sgn}(b_n) - b_n$ is the quantization error

Temporal ΣΔ Modulation



- $x_n = \bar{x}_n + q_n - q_{n-1}$, $\quad q_n = \mathrm{sgn}(b_n) - b_n$ is quant. error
- **Fact (no overloading):** if $|\bar{x}_n| \leq 1$ for all n, then $|q_n| \leq 1$ for all n
- could have $|q_n| \to \infty$ if $|\bar{x}_n| \leq 1$ for all n does not hold

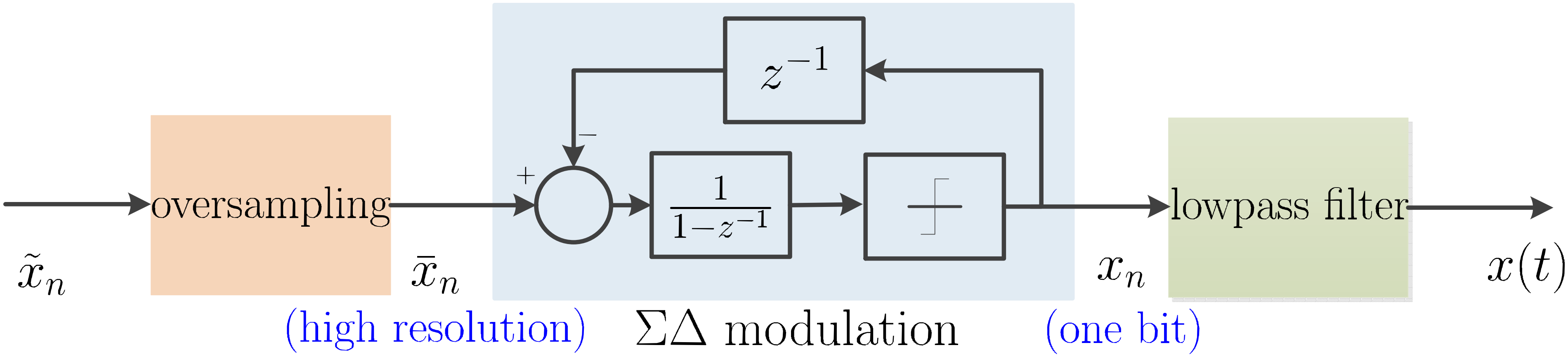
Temporal $\Sigma\Delta$ Modulation



- $x_n = \bar{x}_n + q_n - q_{n-1}, \quad q_n = \mathrm{sgn}(b_n) - b_n$ is q. error
- assumption (no overloading): $|\bar{x}_n| \leq 1$ for all n, so $|q_n| \leq 1$ for all n
- assumption (debatable, though used almost everywhere):

  $\{q_n\}$ is white, uniformly distributed on $[-1, 1]$, and independent of $\{\bar{x}_n\}$

Temporal ΣΔ Modulation



- model:

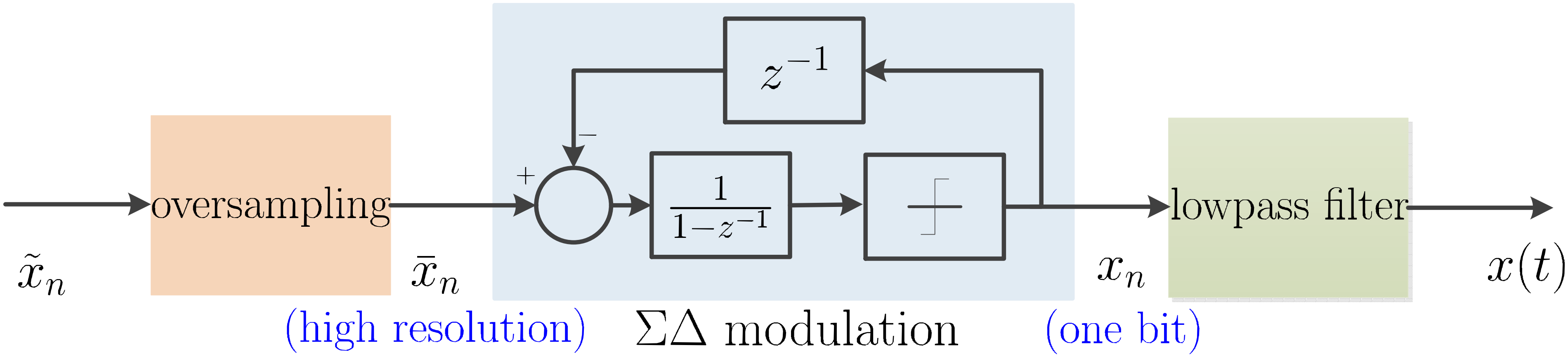
$$x_n = \bar{x}_n + q_n - q_{n-1}$$

where $|\bar{x}_n| \leq 1$ for all n; $\{q_n\}$ is white, uniformly distributed on $[-1, 1]$, and independent of $\{\bar{x}_n\}$

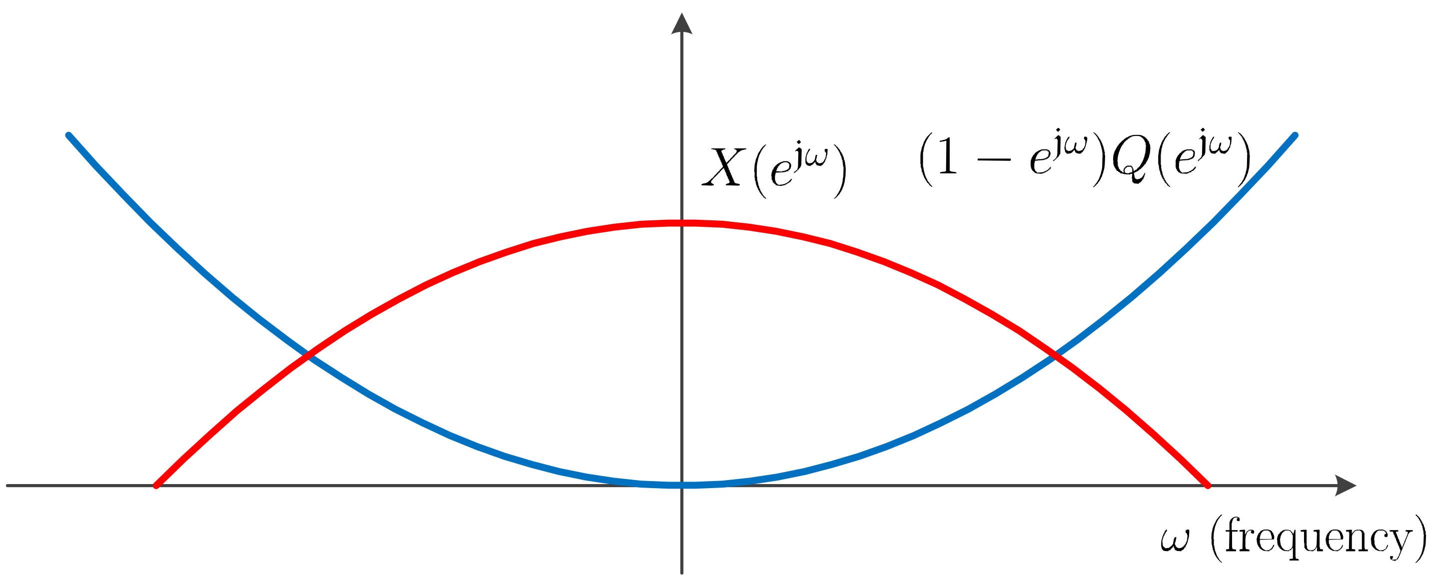
- observation:

$$X(z) = \bar{X}(z) + (1 - z^{-1})Q(z)$$

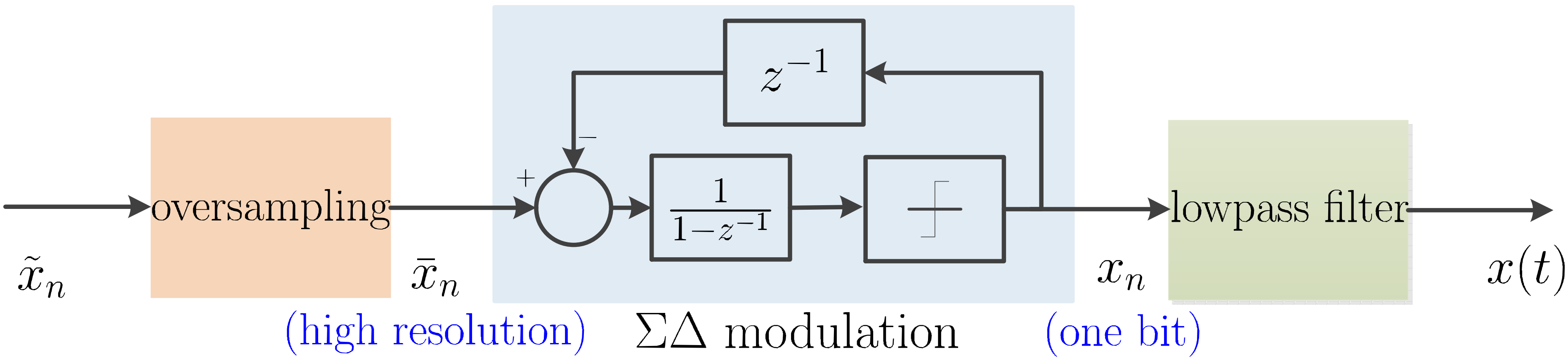
Temporal ΣΔ Modulation



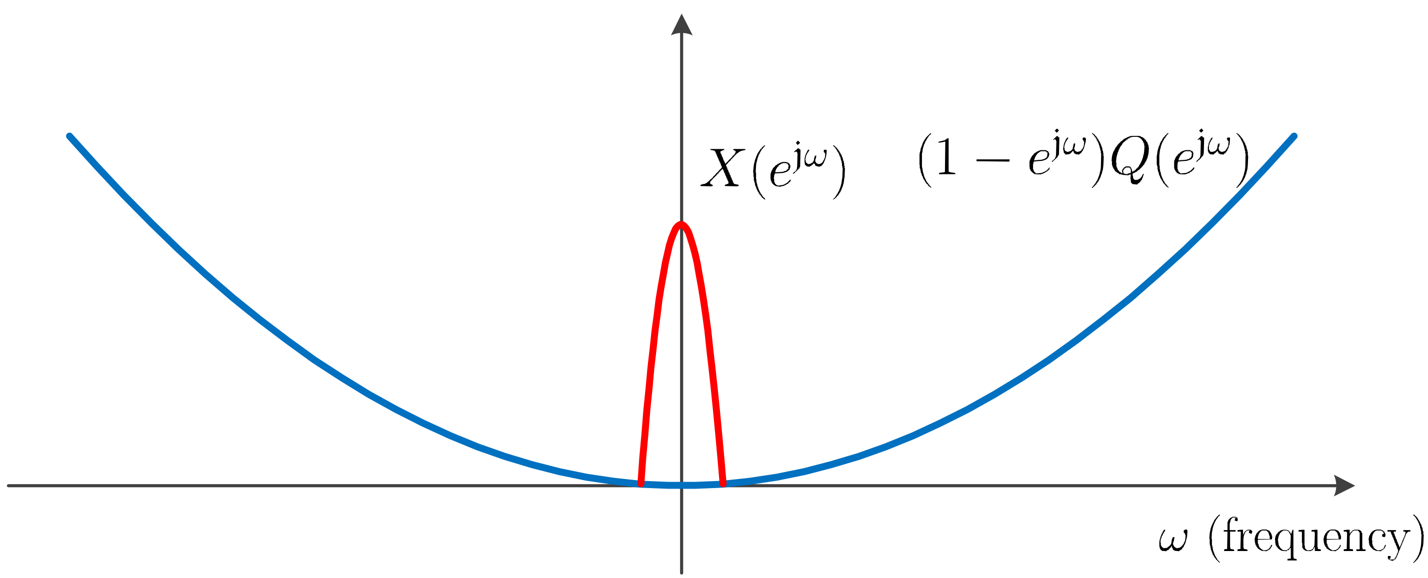
- observation: $X(z) = \bar{X}(z) + (1 - z^{-1})Q(z)$; q. noise is shaped as highpass



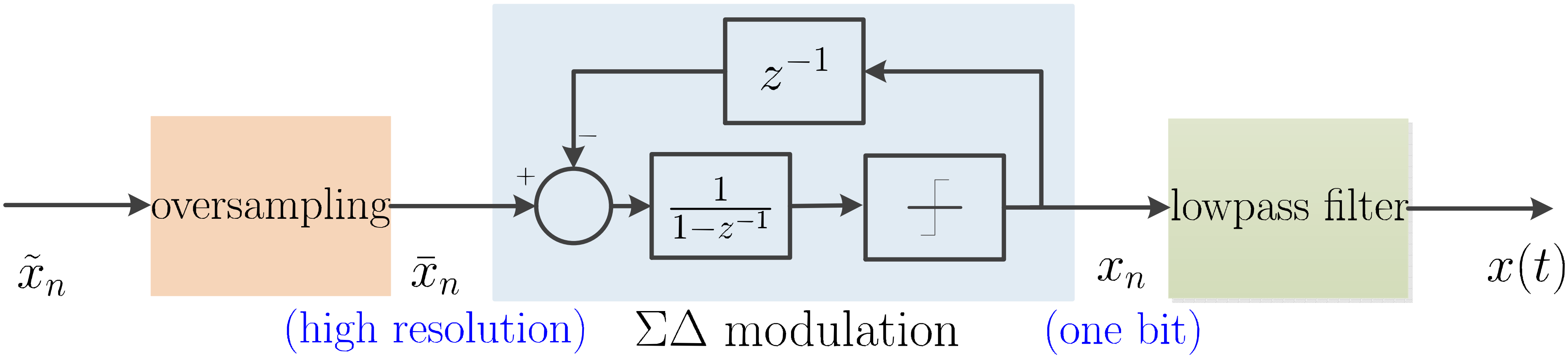
Temporal ΣΔ Modulation



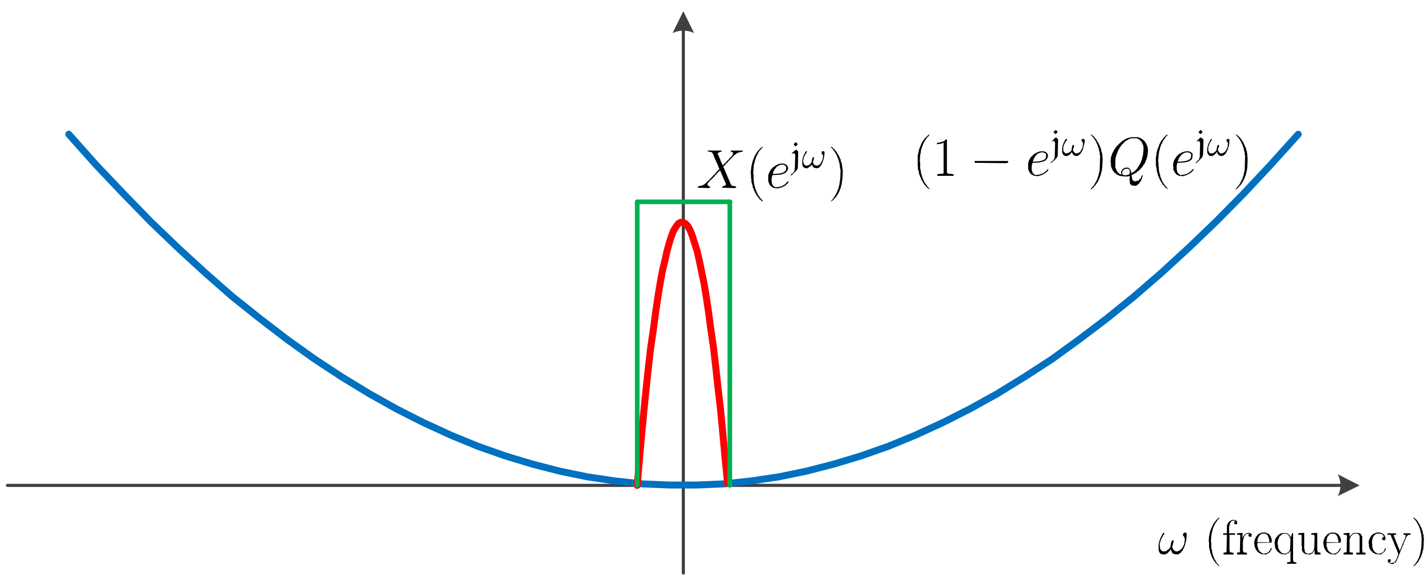
- we almost always oversample to avoid highpass region!



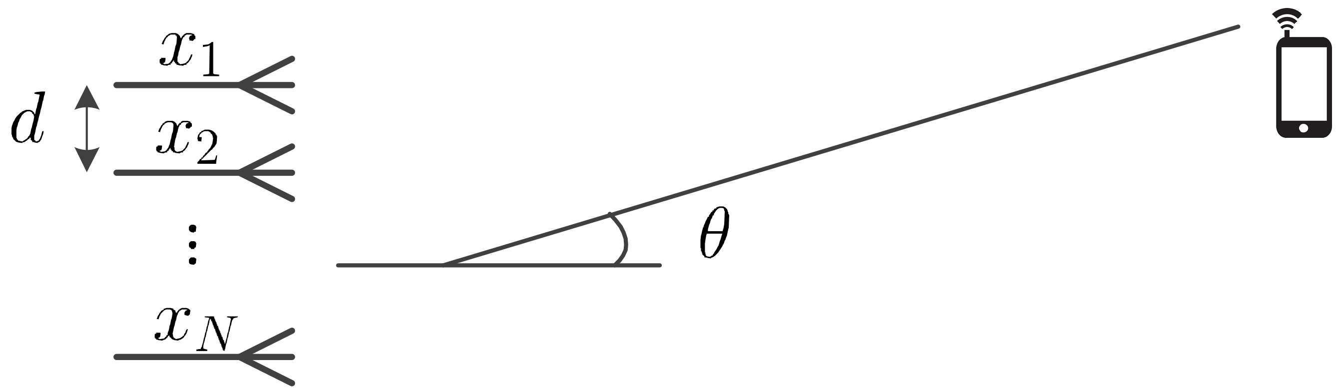
Temporal ΣΔ Modulation



- applying a lowpass filter finally does the task

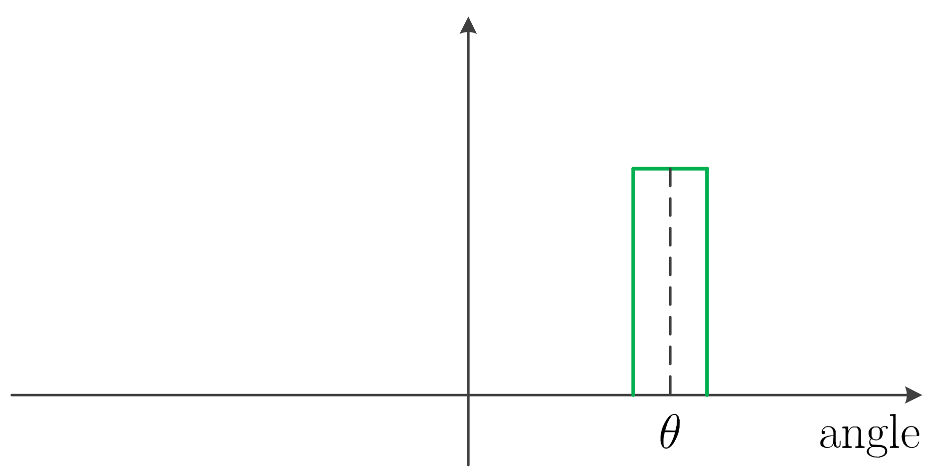


Back to MIMO

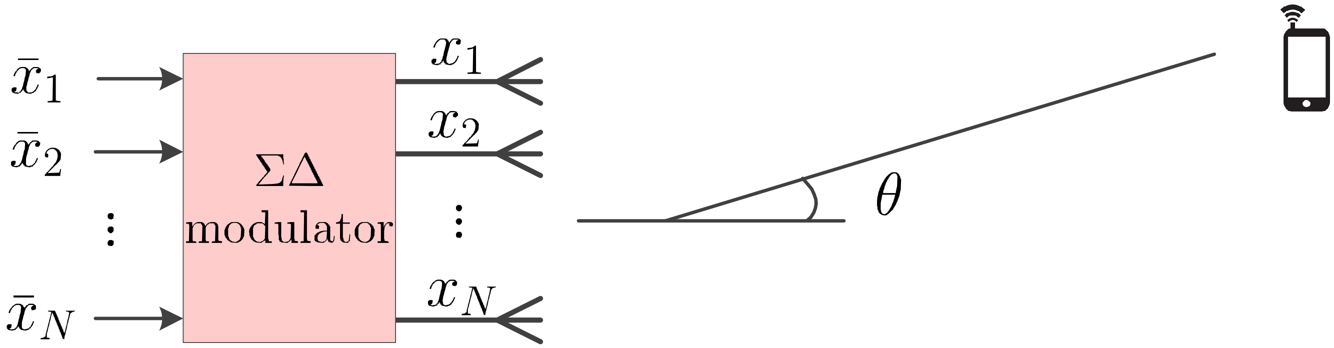


- recall $y = \sqrt{\frac{P}{2N}}\boldsymbol{h}^T\boldsymbol{x} + v$, $\boldsymbol{h} = \alpha\boldsymbol{a}(\theta), \boldsymbol{a}(\theta) = (1, e^{-j\phi}, \cdots, e^{-j\phi(N-1)}), \phi = \frac{2\pi d}{\lambda}\sin(\theta)$

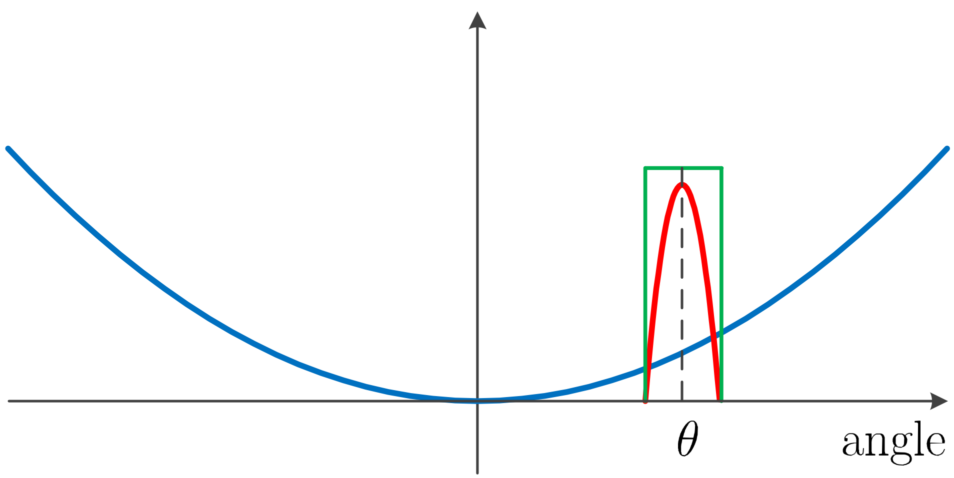
- the channel is a spatial bandpass filter



Spatial ΣΔ Modulation



- **how about applying ΣΔ modulation in space?**



$\Sigma\Delta$ Maximum Ratio Transmission (MRT)

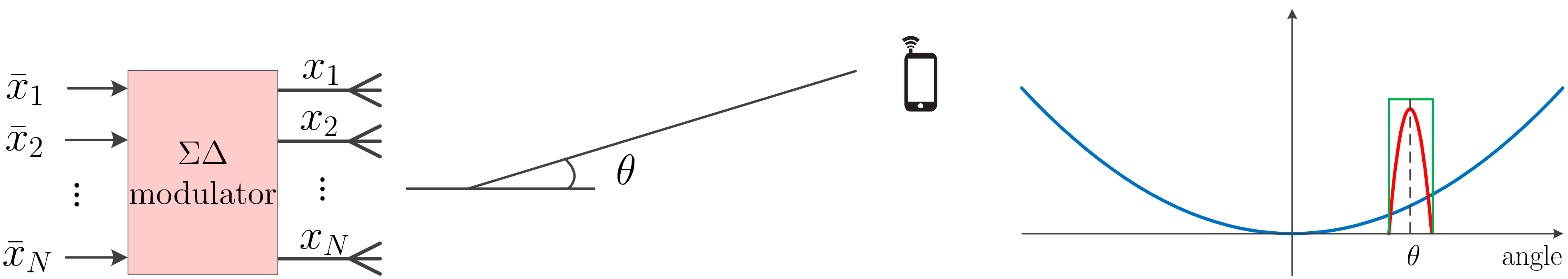
- $\Sigma\Delta$-MRT: 1) choose $\bar{\boldsymbol{x}} = (\bar{x}_1, \ldots, \bar{x}_n)$, the tx signal to be $\Sigma\Delta$-modulated, as

$$\bar{\boldsymbol{x}} = (e^{-j\angle\alpha} s) \cdot \boldsymbol{a}(\theta)^*$$

  where the symbol s is assumed to have $|s| \leq 1$;

  2) apply $\Sigma\Delta$ modulation on the real and imaginary part of $\bar{\boldsymbol{x}}$ to obtain $\boldsymbol{x}$

- note that $\bar{\boldsymbol{x}}$ satisfies $|\Re(\bar{x}_n)| \leq 1$, $|\Re(\bar{x}_n)| \leq 1$, so no overloading

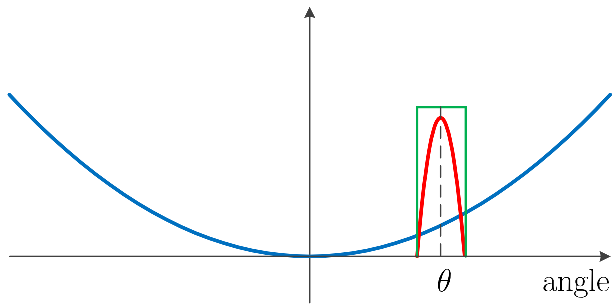


Scatter Plot of $\Sigma\Delta$-MRT; 8-ary PSK

Effective SNR of $\Sigma\Delta$-MRT

- the effective SNR:

$$\text{SNR}_{\text{eff}} = \frac{|\alpha|^2 PN}{\underbrace{\frac{8|\alpha|^2 P}{3}\left|\sin\left(\frac{\pi d}{\lambda}\sin(\theta)\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$



- implications:
  - increasing the power P does *not* reduce the q. noise power
  - increasing the no. of antennas N increases the effective SNR
    * what's favorable: massive antennas, even when each has very small power (P/N)

Effective SNR of $\Sigma\Delta$-MRT

- the effective SNR:

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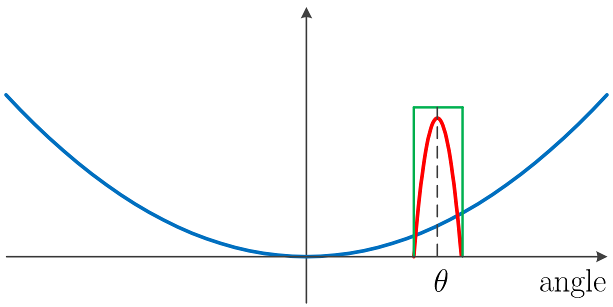
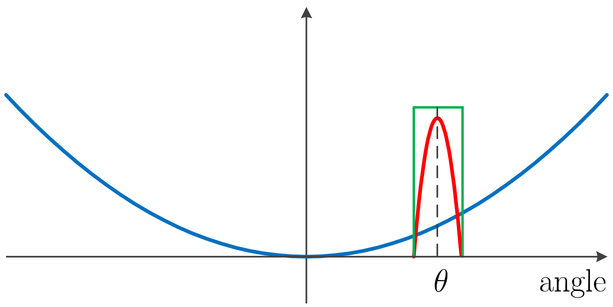


- implications:
  - decreasing the inter-antenna spacing d reduces the q. noise power
    - $*$ identical to over-sampling in temporal $\Sigma\Delta$ modulation
    - $*$ mutual coupling prohibits us from making d too small

Effective SNR of $\Sigma\Delta$-MRT

- the effective SNR:

$$\mathsf{SNR}_{\mathsf{eff}} = \frac{|\alpha|^2 PN}{\underbrace{\frac{8|\alpha|^2 P}{3}\left|\sin\left(\frac{\pi d}{\lambda}\sin(\theta)\right)\right|^2}_{\text{q. noise power}} + 2\sigma_v^2}$$



- implications:
  - increasing $|\theta|$ increases the q. noise power

Effective SNR of $\Sigma\Delta$-MRT

- the effective SNR:

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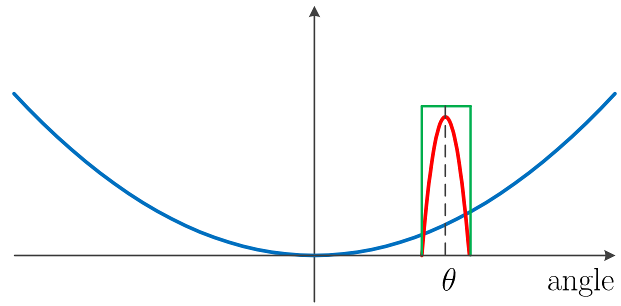
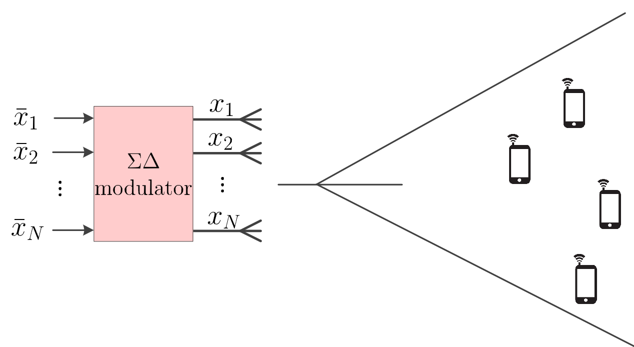
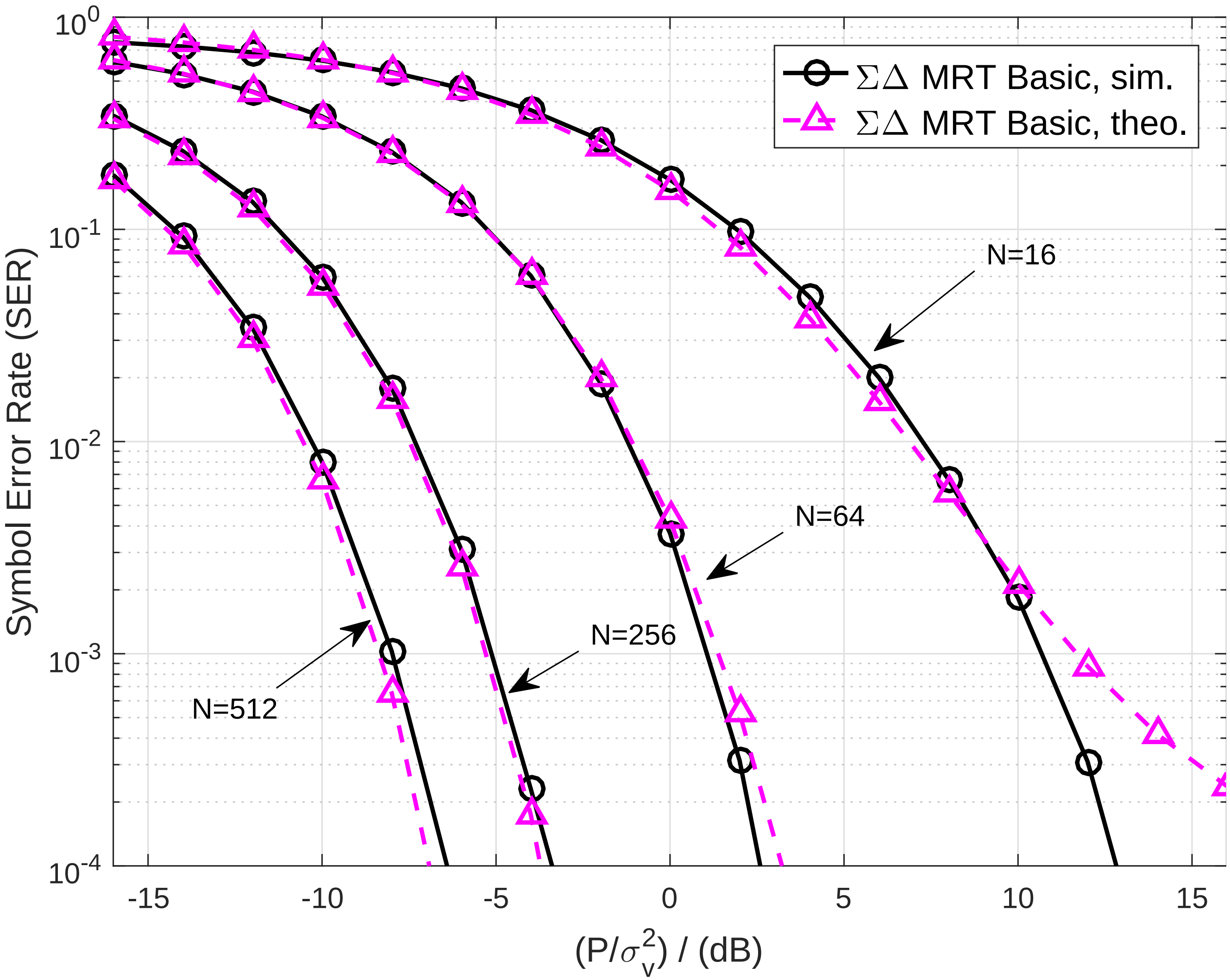


- implications:
  - increasing $|\theta|$ increases the q. noise power
    * in sectored antennas, where the angle range is limited, say, from -30° to 30°, the q. noise impact can be contained

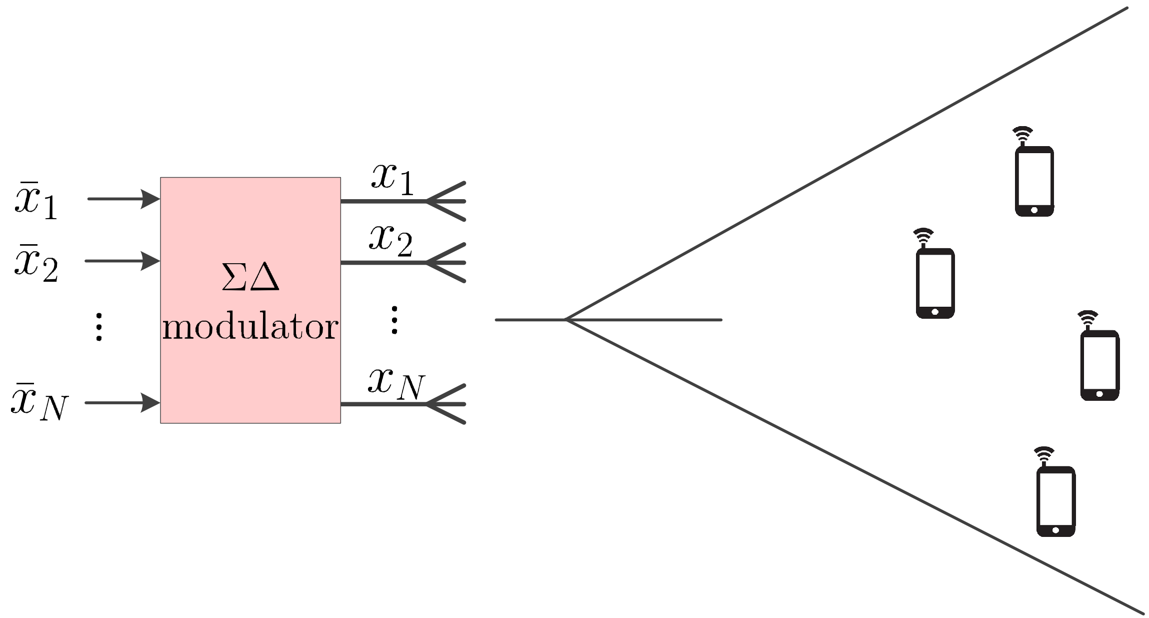


BER Performance of ΣΔ-MRT

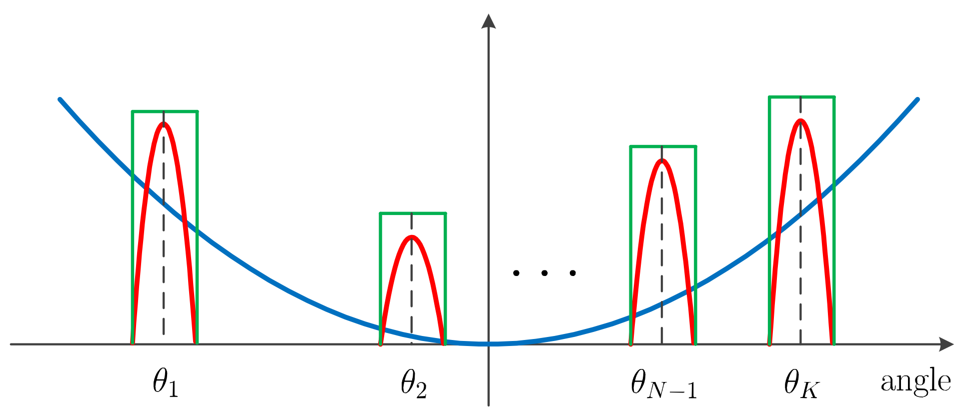
- inter-antenna spacing $d = \lambda/8$; angle $\theta = 60°$; 8-ary PSK



Spatial ΣΔ Modulation for the Multiuser Case

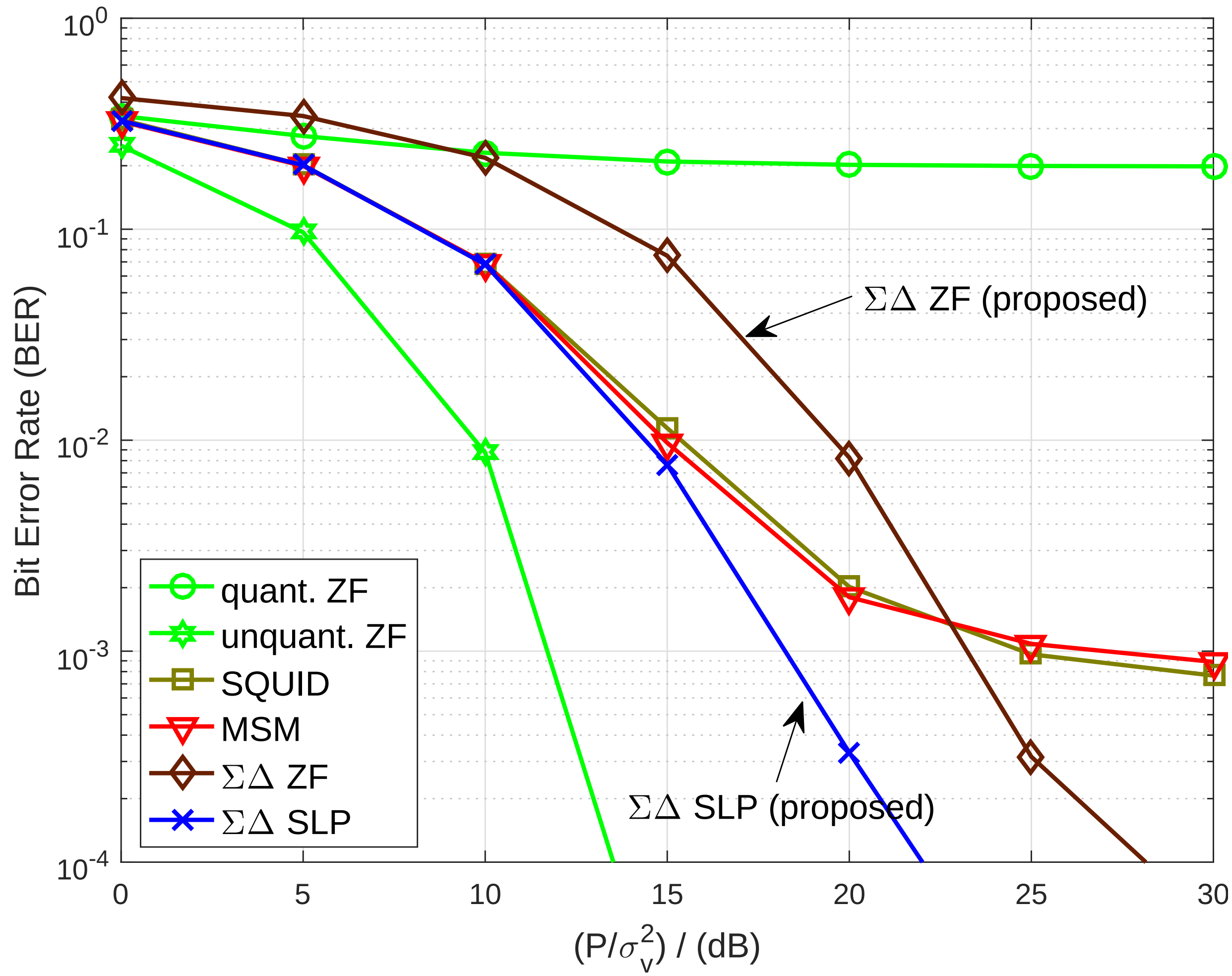


- the same idea applies; design precoding like zero-forcing (ZF) and symbol-level precoding (SLP) under the amplitude constraints $|\Re(\bar{x}_n)| \leq 1$, $|\Re(\bar{x}_n)| \leq 1$

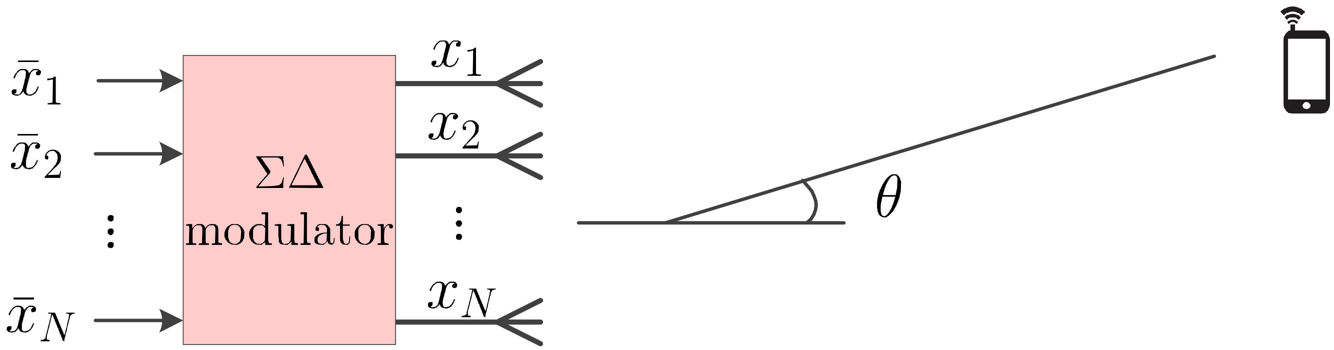


BER Performance of Multiuser $\Sigma\Delta$ Precoding Schemes

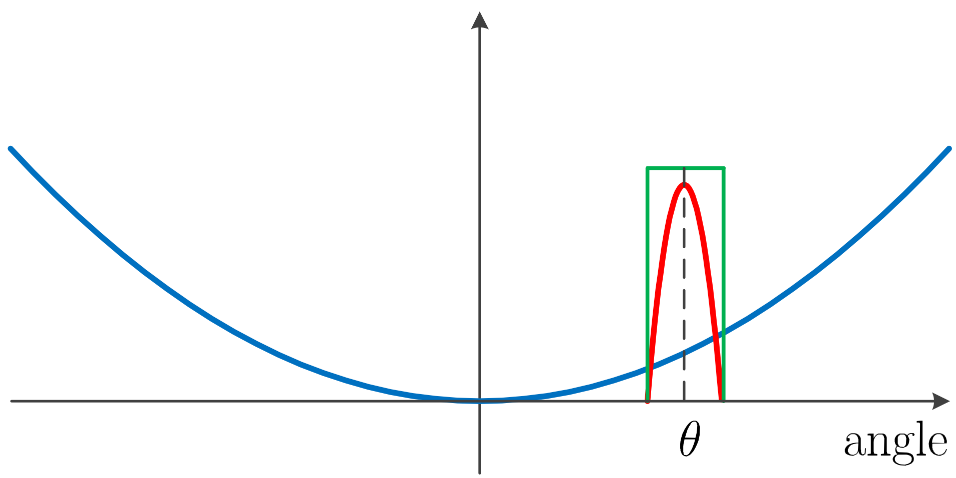
- number of antennas $N = 256$; number of users $K = 24$; inter-antenna spacing $d = \lambda/8$; angle range $[-22.5°, 22.5°]$; 8-ary PSK



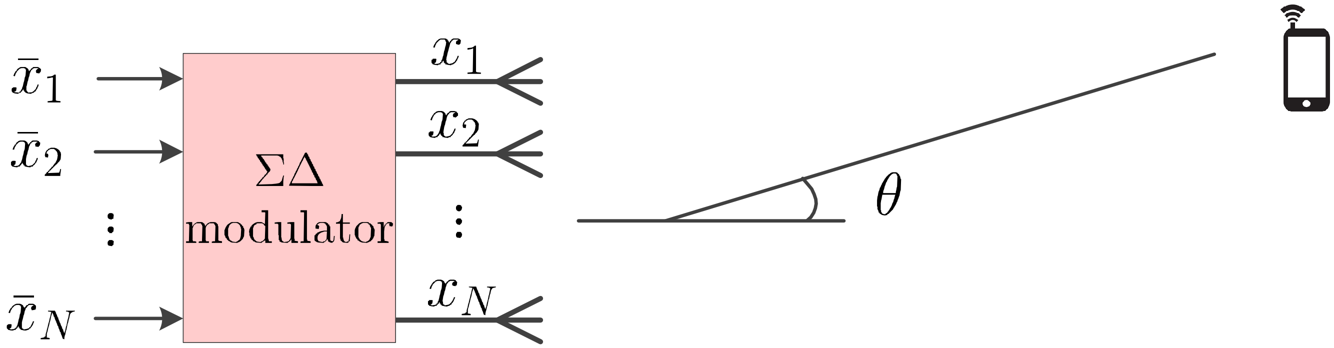
Steering the Angle for Spatial ΣΔ Modulation



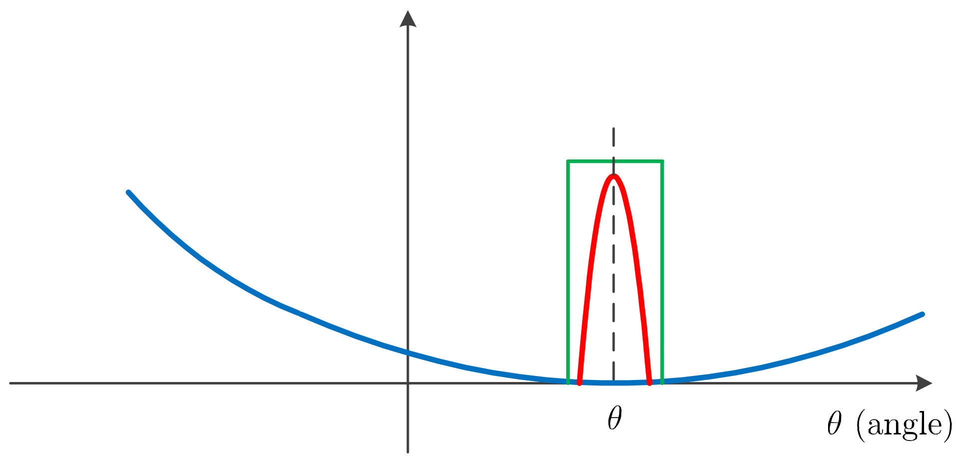
- q. noise increases as $|\theta|$ increases



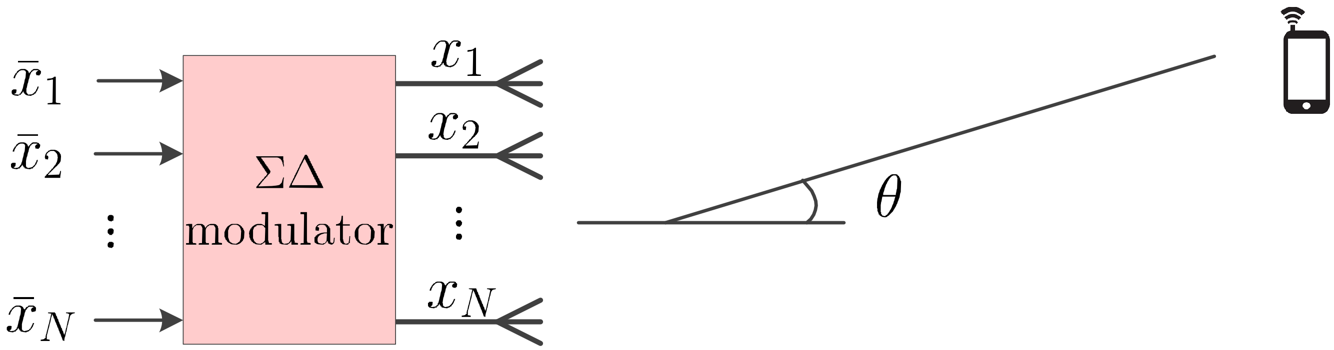
Steering the Angle for Spatial ΣΔ Modulation



- **Question:** can we angle-shift the noise spectrum, thereby allowing the user to experience (almost) zero q. noise?



Steering the Angle for Spatial ΣΔ Modulation



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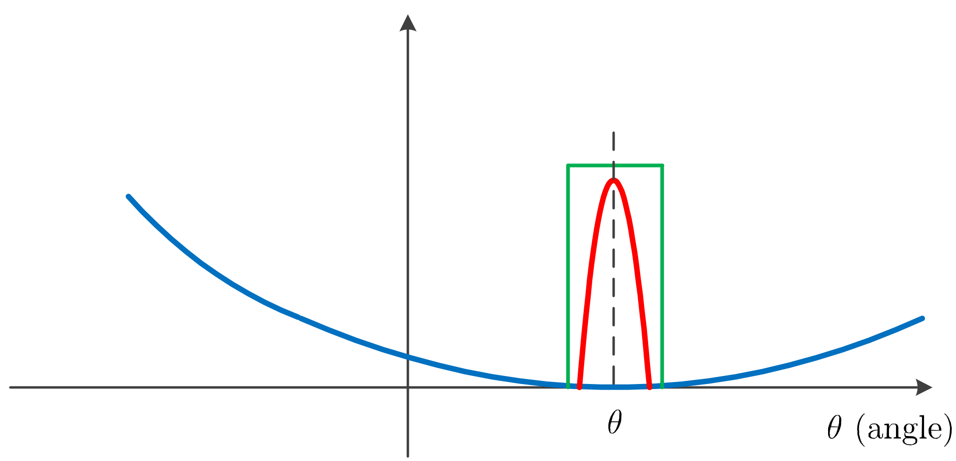
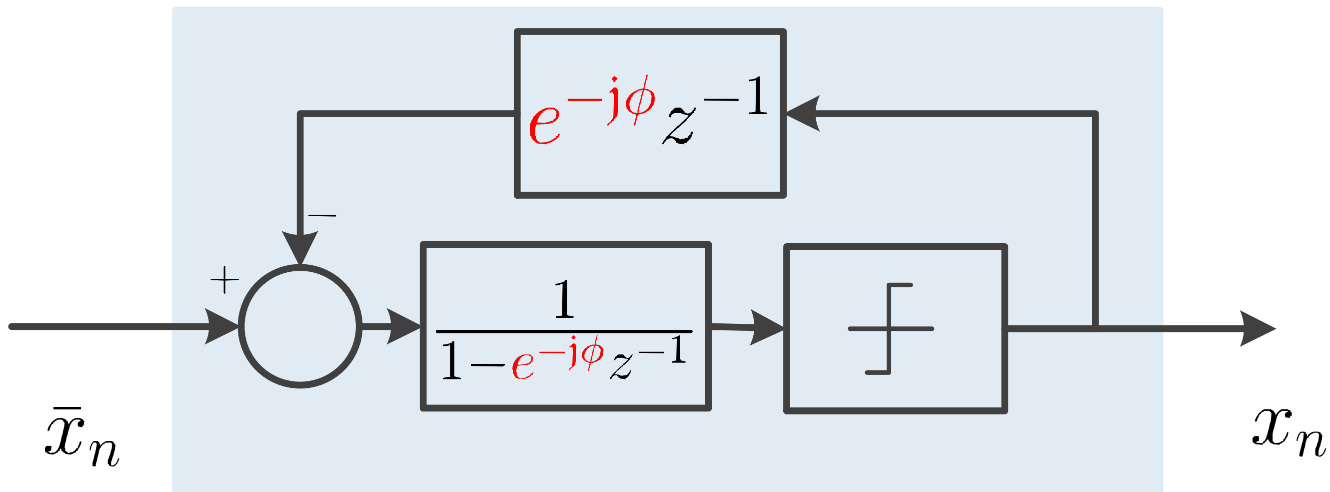


- **Answer:** yes, by borrowing insight from bandpass temporal ΣΔ modulation

Spatial ΣΔ Modulation with Angle Steering

- angle-steered ΣΔ modulator: given ϕ,



- it can be shown that

$$x_n = \bar{x}_n + q_n - e^{j\phi} q_{n-1}$$

and as such

$$X(z) = \bar{X}(z) + (1 - e^{j\phi} z^{-1}) Q(z)$$

Spatial ΣΔ Modulation with Angle Steering

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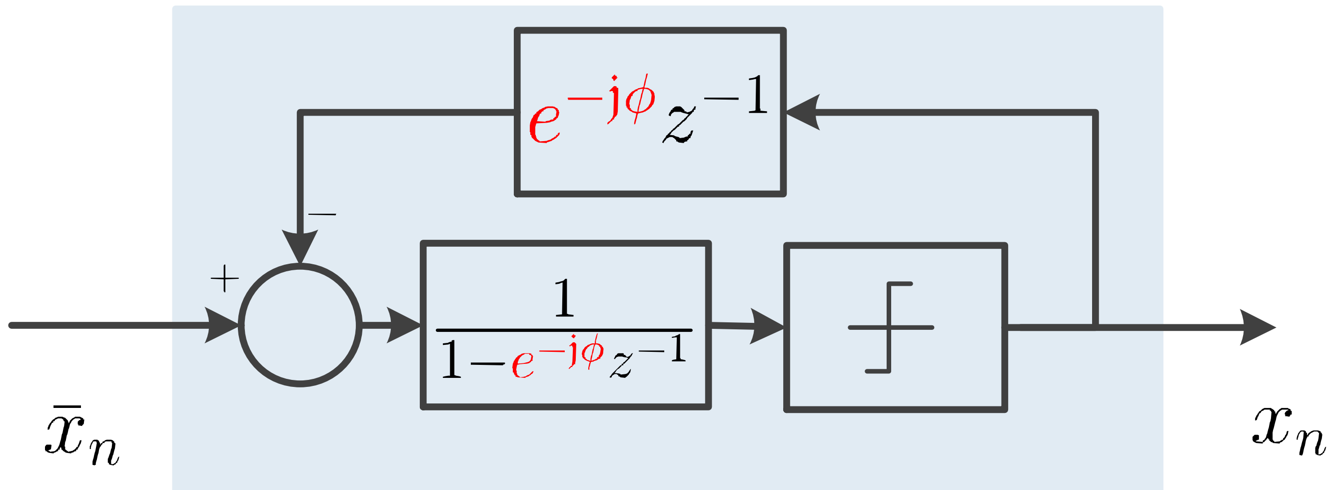


- **Fact (no overloading):** Let

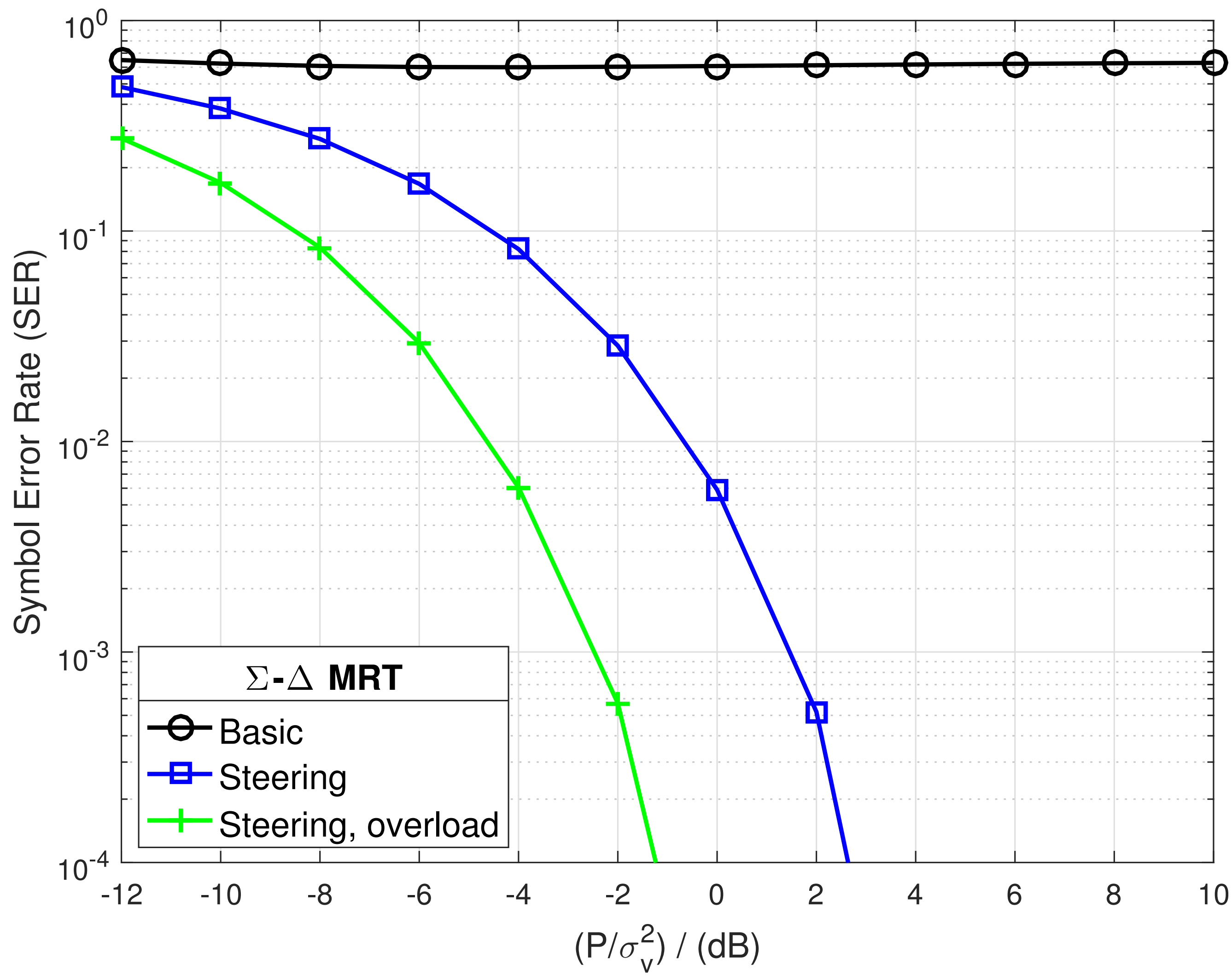
$$A = 2 - |\cos(\phi)| - |\sin(\phi)| \in [0.59, 1]$$

If $|\Re(x_n)| \leq A$, $|\Im(x_n)| \leq A$ for all n, then $|\Re(q_n)| \leq 1$, $|\Im(q_n)| \leq 1$ for all n

- implication: (almost) zero q. noise can be attained, but the signal amplitude can go down to 0.59 in the worst case

BER Performance of Angle-Steered $\Sigma\Delta$-MRT

- no. of antennas $N = 128$; inter-antenna spacing $d = \lambda/3$; angle $\theta = 90°$; 8-ary PSK



Take-Home Point

- spatial $\Sigma\Delta$ modulation allows us to design one-bit precoders in a continuous way
- assumption required: uniform linear array; one path propagation, extendable to more than one path
- in conventional precoding or BF, it is common to do

$$\begin{aligned} &\max \text{ QoS metric} \\ &\text{s.t. } \mathbb{E}[\|\boldsymbol{x}\|^2] \leq P \quad \text{(average power constraint)} \end{aligned}$$

- in $\Sigma\Delta$ precoding, we talk about

$$\begin{aligned} &\max \text{ QoS metric} \\ &\text{s.t. } \|\Re(\bar{\boldsymbol{x}})\|_\infty \leq A, \|\Im(\bar{\boldsymbol{x}})\|_\infty \leq A \quad \text{(amplitude constraint)} \end{aligned}$$

Take-Home Point

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- **that's it! Thank you very much!**