Orthogonal Space-Time Block Coding without Channel State Information

Efficient Maximum-Likelihood Receiver Implementations, Identifiability, and Code Constructions

Wing-Kin (Ken) Ma

Department of Electronic Engineering
The Chinese University of Hong Kong
Hong Kong SAR, PRC

Technical Seminar in USTHK, Nov. 27, 2009.

Outline

1. Background
   - Orthogonal space-time block codes (OSTBCs): The basics
   - Detection without channel state information (CSI)

2. Blind Maximum-Likelihood (ML) OSTBC detection: Implementations
   - Flat fading channels
   - Extension: 16-QAM constellations
   - Extension: OSTBC-OFDM for frequency selective block fading channels

3. Blind Identifiability and Code Constructions
   - Probability 1 identifiability
   - Perfect identifiability and code constructions

Outline

1. Background
   - Orthogonal space-time block codes (OSTBCs): The basics
   - Detection without channel state information (CSI)
2. Blind Maximum-Likelihood (ML) OSTBC detection: Implementations
   - Flat fading channels
   - Extension: 16-QAM constellations
   - Extension: OSTBC-OFDM for frequency selective block fading channels
3. Blind Identifiability and Code Constructions
   - Probability 1 identifiability
   - Perfect identifiability and code constructions

Outline

1. Background
   - Orthogonal space-time block codes (OSTBCs): The basics
   - Detection without channel state information (CSI)
2. Blind Maximum-Likelihood (ML) OSTBC detection: Implementations
   - Flat fading channels
   - Extension: 16-QAM constellations
   - Extension: OSTBC-OFDM for frequency selective block fading channels
3. Blind Identifiability and Code Constructions
   - Probability 1 identifiability
   - Perfect identifiability and code constructions

How OSTBCs look like

Real constellations [1]: let $s_1, \ldots, s_K \in \mathbb{R}$ be a set of real symbols.

$$\begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix}, \quad \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \end{bmatrix}, \quad \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \\ s_4 & s_3 & -s_2 & s_1 \end{bmatrix}$$

Complex constellations [2]: let $u_1, \ldots, u_K \in \mathbb{C}$ be a set of complex syms.

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}, \quad \begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \end{bmatrix}, \quad \begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \\ 0 & -u_3^* & u_2^* & u_1 \end{bmatrix}$$

[1] V. Tarokh, *et al.*, "Space-time block codes from orthogonal designs," *IEEE Trans. IT*, 1999.

[2] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. IT*, 2003.

How OSTBCs look like

Real constellations [1]: let $s_1, \ldots, s_K \in \mathbb{R}$ be a set of real symbols.

$$\begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix}, \quad \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \end{bmatrix}, \quad \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \\ s_4 & s_3 & -s_2 & s_1 \end{bmatrix}$$

Complex constellations [2]: let $u_1, \ldots, u_K \in \mathbb{C}$ be a set of complex syms.

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}, \quad \begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \end{bmatrix}, \quad \begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \\ 0 & -u_3^* & u_2^* & u_1 \end{bmatrix}$$

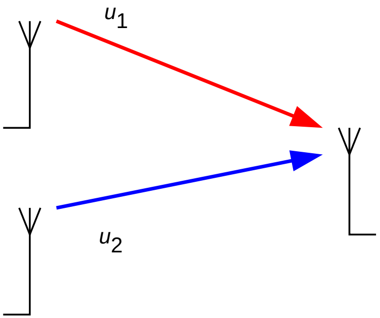
[1] V. Tarokh, *et al.*, "Space-time block codes from orthogonal designs," *IEEE Trans. IT*, 1999.
[2] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. IT*, 2003.

Illustration of OSTBC Transmission

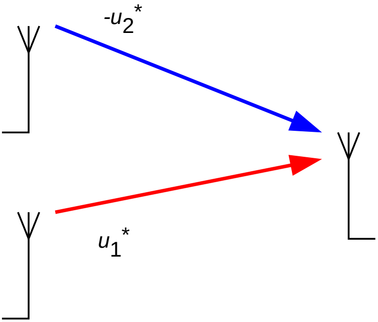
Take the complex Alamouti code as an example:

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}$$

Transmission at time instant 1:



Transmission at time instant 2:

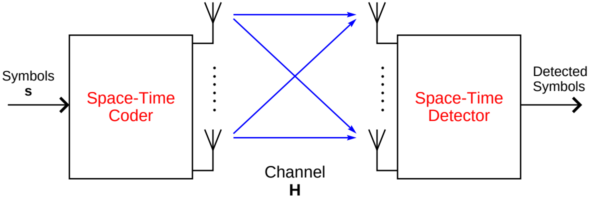


Signal Model

Received signal model for a generic space-time block code (STBC):

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s}) + \mathbf{V}$$

$\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$	MIMO channel (frequency flat);
(M_r, M_t)	number of receiver and transmitter antennas;
$\mathbf{s} \in \mathbb{R}^K$	symbol vector;
$\mathbf{C} : \mathbb{R}^K \to \mathbb{C}^{M_t \times T}$	STBC mapping function with length T;
$\mathbf{V} \in \mathbb{C}^{M_r \times T}$	additive white Gaussian noise (AWGN).



OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k \mathbf{X}_\ell^H = -\mathbf{X}_\ell \mathbf{X}_k^H$ for $k \neq \ell$.

OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k \mathbf{X}_\ell^H = -\mathbf{X}_\ell \mathbf{X}_k^H$ for $k \neq \ell$.

Example: BPSK Alamouti code

$$\begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix} = s_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + s_2 \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k\mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k\mathbf{X}_\ell^H = -\mathbf{X}_\ell\mathbf{X}_k^H$ for $k \neq \ell$.

Example: QPSK Alamouti code

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix} = \Re\{u_1\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \Im\{u_1\} \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} + \Re\{u_2\} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \Im\{u_2\} \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix}$$

OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k \mathbf{X}_\ell^H = -\mathbf{X}_\ell \mathbf{X}_k^H$ for $k \neq \ell$.

Well known results

- OSTBC codewords have orthogonal rows

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|_2^2 \mathbf{I} \tag{2}$$

- From (1)–(2) OSTBCs were shown to attain the **max. diversity order** and have **simple ML detector structures**, when $\mathbf{H}$ is known at the receiver.

OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k \mathbf{X}_\ell^H = -\mathbf{X}_\ell \mathbf{X}_k^H$ for $k \neq \ell$.

Remark: Generalized orthogonal designs (GODs)

- Most OSTBCs available today are from GODs.
- In GODs, each entry of $\mathbf{C}(.)$ has to be a symbol, its conjugate, or zero; e.g., the Alamouti code

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}$$

OSTBC Expression

- Assume BPSK or QPSK constellations.
- An OSTBC is a *real* linear combination of complex basis matrices

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k, \qquad \mathbf{s} \in \{\pm 1\}^K \tag{1}$$

where $\mathbf{X}_k$ satisfy i) $\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}$, & ii) $\mathbf{X}_k \mathbf{X}_\ell^H = -\mathbf{X}_\ell \mathbf{X}_k^H$ for $k \neq \ell$.

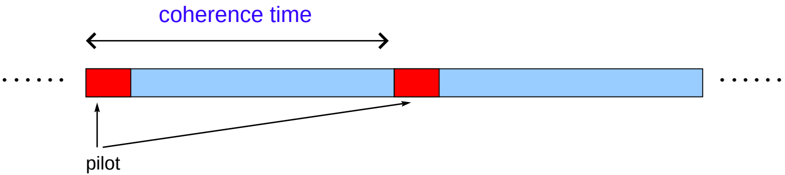
Remark: GODs (cont'd)

- Here is a non-GOD (rarely used)

$$\begin{bmatrix} u_1 & -u_2^* & \frac{1}{\sqrt{2}} u_3^* & \frac{1}{\sqrt{2}} u_3^* \\ u_2 & u_1^* & \frac{1}{\sqrt{2}} u_3^* & -\frac{1}{\sqrt{2}} u_3^* \\ \frac{1}{\sqrt{2}} u_3 & \frac{1}{\sqrt{2}} u_3 & \frac{1}{2}(-u_1 - u_1^* + u_2 - u_2^*) & \frac{1}{2}(u_2 + u_2^* + u_1 - u_1^*) \end{bmatrix}$$

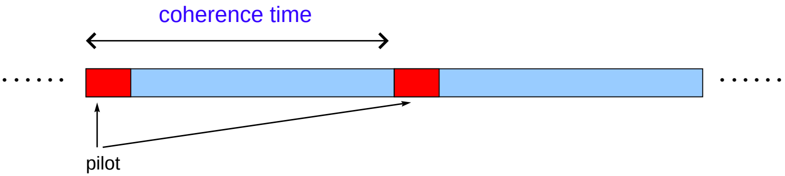
Detection without CSI

- This talk focuses on ML detection when $\mathbf{H}$ is unknown at the receiver.
- Channel state information (CSI) is usually obtained through training.
- But training is arguably inefficient when
  - the channel coherence time is small, and
  - the power & bandwidth overheads for training are unaffordable (e.g., uplink).



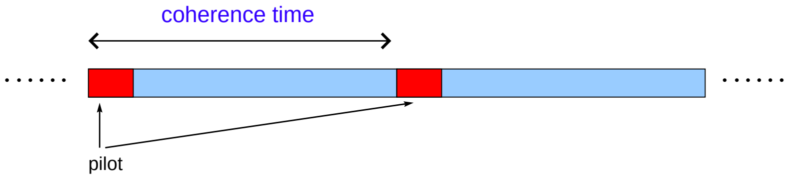
Detection without CSI

- This talk focuses on ML detection when $\mathbf{H}$ is unknown at the receiver.
- Channel state information (CSI) is usually obtained through training.
- But training is arguably inefficient when
  - the channel coherence time is small, and
  - the power & bandwidth overheads for training are unaffordable (e.g., uplink).



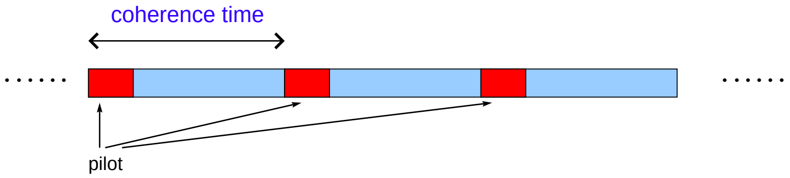
Detection without CSI

- This talk focuses on ML detection when $\mathbf{H}$ is unknown at the receiver.
- Channel state information (CSI) is usually obtained through training.
- But training is arguably inefficient when
  - the channel coherence time is small, and
  - the power & bandwidth overheads for training are unaffordable (e.g., uplink).



Detection without CSI

- This talk focuses on ML detection when $\mathbf{H}$ is unknown at the receiver.
- Channel state information (CSI) is usually obtained through training.
- But training is arguably inefficient when
  - the channel coherence time is small, and
  - the power & bandwidth overheads for training are unaffordable (e.g., uplink).

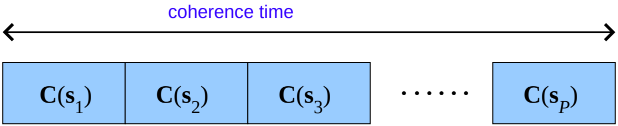


Outline

1. Background
   - Orthogonal space-time block codes (OSTBCs): The basics
   - Detection without channel state information (CSI)
2. Blind Maximum-Likelihood (ML) OSTBC detection: Implementations
   - Flat fading channels
   - Extension: 16-QAM constellations
   - Extension: OSTBC-OFDM for frequency selective block fading channels
3. Blind Identifiability and Code Constructions
   - Probability 1 identifiability
   - Perfect identifiability and code constructions

Signal Model

- Assume that $\mathbf{H}$ remains static over P consecutive code blocks.



- The received signal model:

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{s}_p) + \mathbf{V}_p, \qquad p = 1, \ldots, P$$

- By letting $\boldsymbol{\mathcal{Y}} = [\ \mathbf{Y}_1, \ldots, \mathbf{Y}_P\ ]$, $\boldsymbol{\mathcal{C}}(\mathbf{s}) = [\ \mathbf{C}(\mathbf{s}_1), \ldots, \mathbf{C}(\mathbf{s}_P)\ ]$, & $\boldsymbol{s} = [\ \mathbf{s}_1^T, \ldots, \mathbf{s}_P^T\ ]^T$, the model can be conveniently expressed as

$$\boldsymbol{\mathcal{Y}} = \mathbf{H}\boldsymbol{\mathcal{C}}(\boldsymbol{s}) + \boldsymbol{\mathcal{V}}$$

Blind ML Detection Formulation

- We consider the deterministic blind ML detector [or the generalized likelihood ratio test (GLRT)]:

$$\hat{s} = \arg \min_{\mathbf{s} \in \{\pm 1\}^{KP}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{M_r \times M_t}} \|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2 \right\} \tag{3}$$

In this formulation, $\mathbf{H}$ is treated as a deterministic unknown.

- Given a generic space-time coding function $\mathbf{C}(.)$, Eq. (3) is a challenging optimization problem.
- A common approx. method for (3) is to use cyclic minimization[3]: minimize $\|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2$ w.r.t. $\mathbf{H}$ only at one time, minimize $\|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2$ w.r.t. $\mathbf{s}$ only at another time.

[3] E. G. Larsson, P. Stoica, & J. Li, "Orthogonal space-time block codes: ML detection for unknown channles and unstructured interferences," *IEEE Trans. SP*, 2003.

Blind ML Detection Formulation

- We consider the deterministic blind ML detector [or the generalized likelihood ratio test (GLRT)]:

$$\hat{\boldsymbol{s}} = \arg \min_{\mathbf{s} \in \{\pm 1\}^{KP}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{M_r \times M_t}} \|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2 \right\} \qquad (3)$$

  In this formulation, $\mathbf{H}$ is treated as a deterministic unknown.

- Given a generic space-time coding function $\mathbf{C}(.)$, Eq. (3) is a challenging optimization problem.

- A common approx. method for (3) is to use cyclic minimization[3]: minimize $\|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2$ w.r.t. $\mathbf{H}$ only at one time, minimize $\|\boldsymbol{\mathcal{Y}} - \mathbf{H}\boldsymbol{\mathcal{C}}(\mathbf{s})\|_F^2$ w.r.t. $\mathbf{s}$ only at another time.

[3] E. G. Larsson, P. Stoica, & J. Li, "Orthogonal space-time block codes: ML detection for unknown channles and unstructured interferences," *IEEE Trans. SP*, 2003.

Difficulty with Blind ML Detection

- The inner min. problem is a least-squares problem

$$\min_{\mathbf{H}\in\mathbb{C}^{M_r\times M_t}} \|\boldsymbol{\mathcal{Y}}-\mathbf{H}\boldsymbol{\mathcal{C}}(\boldsymbol{s})\|_F^2 = \|\boldsymbol{\mathcal{Y}}-\mathbf{H}\boldsymbol{\mathcal{C}}(\boldsymbol{s})\|_F^2\Big|_{\mathbf{H}=\boldsymbol{\mathcal{Y}}\boldsymbol{\mathcal{C}}^H(\mathbf{s})[\boldsymbol{\mathcal{C}}(\boldsymbol{s})\boldsymbol{\mathcal{C}}^H(\boldsymbol{s})]^{-1}} = \|\boldsymbol{\mathcal{Y}}(\mathbf{I}-\boldsymbol{\Pi}(\boldsymbol{s}))\|_F^2 \quad (4)$$

where $\boldsymbol{\Pi}(\boldsymbol{s}) = \boldsymbol{\mathcal{C}}^H(\boldsymbol{s})[\boldsymbol{\mathcal{C}}(\boldsymbol{s})\boldsymbol{\mathcal{C}}^H(\boldsymbol{s})]^{-1}\boldsymbol{\mathcal{C}}(\boldsymbol{s})$.

- Substituting (4) into the blind ML problem, we arrive at

$$\min_{\boldsymbol{s}\in\{\pm 1\}^{KP}} \|\boldsymbol{\mathcal{Y}}(\mathbf{I}-\boldsymbol{\Pi}(\boldsymbol{s}))\|_F^2 \quad (5)$$

where the objective function is highly nonlinear & nonconvex.

- A complete search for (5) would cost us a complexity of $\mathcal{O}(2^{KP})$.
- But, for BPSK or QPSK OSTBCs, we can prove that[4]

[4] W.-K. Ma, B.-N. Vo, T. N. Davidson, & P. C. Ching, "Blind ML detection of OSTBCs: Efficient high-performance implementations," *IEEE Trans. SP*, Feb. 2006.

BQP Reformulation

Proposition

The blind ML detection problem for BPSK or QPSK OSTBCs

$$\min_{\boldsymbol{s}\in\{\pm 1\}^{KP}} \|\boldsymbol{\mathcal{Y}}(\mathbf{I}-\mathbf{\Pi}(\boldsymbol{s}))\|_F^2$$

can be reformulated as a Boolean quadratic program (BQP)

$$\max_{\boldsymbol{s}\in\{\pm 1\}^{KP}} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$

where

$$\boldsymbol{\mathcal{R}} = \begin{bmatrix} \mathbf{R}_{11} & \ldots & \mathbf{R}_{1,P} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{P,1} & \ldots & \mathbf{R}_{P,P} \end{bmatrix},$$

and $\mathbf{R}_{pq} \in \mathbb{R}^{K\times K}$ with $[\mathbf{R}_{pq}]_{k\ell} = \Re\{\mathrm{tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_\ell \mathbf{Y}_q^H\}\}$.

Idea behind the BQP Reformulation

- For BPSK or QPSK OSTBCs,

$$\mathcal{C}(\mathbf{s})\mathcal{C}^H(\mathbf{s}) = \|\mathbf{s}\|_2^2\mathbf{I} = KP\mathbf{I}.$$

- Hence, the projector $\mathbf{\Pi}(\mathbf{s})$ can be reduced to

$$\mathbf{\Pi}(\mathbf{s}) = \mathcal{C}^H(s)[\mathcal{C}(s)\mathcal{C}^H(s)]^{-1}\mathcal{C}(s) = \frac{1}{KP}\mathcal{C}^H(s)\mathcal{C}(s)$$

- Substituting this into the blind ML problem, the proposition will be obtained.

On Solving the BQP

- The reformulated blind ML problem

$$\max_{\boldsymbol{s}\in\{\pm1\}^{KP}} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$

is much simplified compared to its original counterpart.

- Still, it is a hard discrete optimization problem (NP-hard).
- We propose two alternatives to handling the opt., namely
  - semidefinite relaxation (SDR)[5], an efficient high-performance approximation method; &
  - sphere decoding[6], an exact solver with good efficiency for small to moderate problem sizes.

[5] W.-K. Ma, T. N. Davidson, K. M. Wong, Z.-Q. Luo, & P. C. Ching, "Quasi-ML multiuser detection using SDR with application to sync. CDMA," *IEEE Trans. SP*, Apr. 2002.

[6] O. Damen, H. El Gamal, & G. Caire, "On ML detection and the search for the closest lattice point," *IEEE Trans. IT*, 2003.

On Solving the BQP

- The reformulated blind ML problem

$$\max_{\boldsymbol{s} \in \{\pm 1\}^{KP}} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$

is much simplified compared to its original counterpart.

- Still, it is a hard discrete optimization problem (NP-hard).
- We propose two alternatives to handling the opt., namely
  - semidefinite relaxation (SDR)[5], an efficient high-performance approximation method; &
  - sphere decoding[6], an exact solver with good efficiency for small to moderate problem sizes.

[5] W.-K. Ma, T. N. Davidson, K. M. Wong, Z.-Q. Luo, & P. C. Ching, "Quasi-ML multiuser detection using SDR with application to sync. CDMA," *IEEE Trans. SP*, Apr. 2002.

[6] O. Damen, H. El Gamal, & G. Caire, "On ML detection and the search for the closest lattice point," *IEEE Trans. IT*, 2003.

On Solving the BQP

- The reformulated blind ML problem

$$\max_{\boldsymbol{s} \in \{\pm 1\}^{KP}} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$

  is much simplified compared to its original counterpart.
- Still, it is a hard discrete optimization problem (NP-hard).
- We propose two alternatives to handling the opt., namely
  - semidefinite relaxation (SDR)[5], an efficient high-performance approximation method; &
  - sphere decoding[6], an exact solver with good efficiency for small to moderate problem sizes.

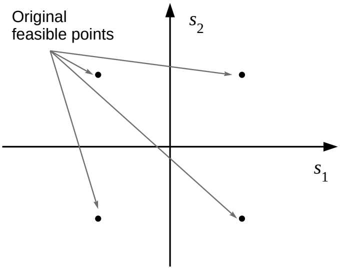
[5] W.-K. Ma, T. N. Davidson, K. M. Wong, Z.-Q. Luo, & P. C. Ching, "Quasi-ML multiuser detection using SDR with application to sync. CDMA," *IEEE Trans. SP*, Apr. 2002.

[6] O. Damen, H. El Gamal, & G. Caire, "On ML detection and the search for the closest lattice point," *IEEE Trans. IT*, 2003.

A Simple Suboptimal BQP Solution

The original BQP

$$\max \ \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$
$$\text{s.t. } \boldsymbol{s} \in \{\pm 1\}^{KP}$$



- The relaxed problem is the principal eigenvector problem, which has a closed form.
- This method is equivalent to the SVD method[7] & the subspace method[8].

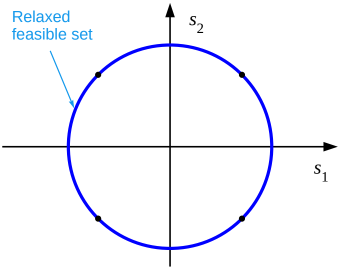
[7] P. Stoica & G. Ganesan, "Space-time block codes: trained, blind, and semi-blind detection," *Digital Signal Process.*, 2003.

[8] S. Shahbazpanahi, A. Gershman, & J. Manton, "Closed-form blind MIMO channel estimation for OSTBCs," *IEEE Trans. SP*, Dec. 2005.

A Simple Suboptimal BQP Solution

Norm relaxation

$$\max \; \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$
$$\text{s.t.} \; \|\boldsymbol{s}\|_2^2 = KP$$



- The relaxed problem is the principal eigenvector problem, which has a closed form.
- This method is equivalent to the SVD method[7] & the subspace method[8].

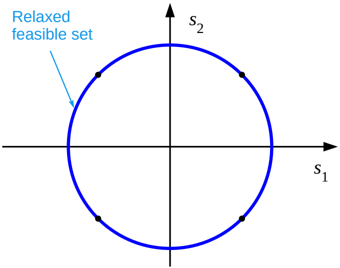
[7] P. Stoica & G. Ganesan, "Space-time block codes: trained, blind, and semi-blind detection," *Digital Signal Process.*, 2003.

[8] S. Shahbazpanahi, A. Gershman, & J. Manton, "Closed-form blind MIMO channel estimation for OSTBCs," *IEEE Trans. SP*, Dec. 2005.

A Simple Suboptimal BQP Solution

Norm relaxation

$$\max \ \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$
$$\text{s.t.} \ \|\boldsymbol{s}\|_2^2 = KP$$



- The relaxed problem is the principal eigenvector problem, which has a closed form.
- This method is equivalent to the SVD method[7] & the subspace method[8].

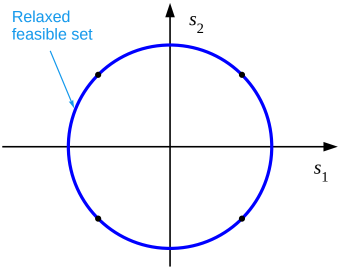
[7] P. Stoica & G. Ganesan, "Space-time block codes: trained, blind, and semi-blind detection," *Digital Signal Process.*, 2003.

[8] S. Shahbazpanahi, A. Gershman, & J. Manton, "Closed-form blind MIMO channel estimation for OSTBCs," *IEEE Trans. SP*, Dec. 2005.

A Simple Suboptimal BQP Solution

Norm relaxation

$$\max \ \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$
$$\text{s.t.} \ \|\boldsymbol{s}\|_2^2 = KP$$



- The relaxed problem is the principal eigenvector problem, which has a closed form.
- This method is equivalent to the SVD method[7] & the subspace method[8].

[7] P. Stoica & G. Ganesan, "Space-time block codes: trained, blind, and semi-blind detection," *Digital Signal Process.*, 2003.

[8] S. Shahbazpanahi, A. Gershman, & J. Manton, "Closed-form blind MIMO channel estimation for OSTBCs," *IEEE Trans. SP*, Dec. 2005.

Semidefinite Relaxation

- The BQP can be reformulated as

$$\begin{aligned} \max\ & \mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\} && \Longleftrightarrow \boldsymbol{s}^T\boldsymbol{\mathcal{R}}\boldsymbol{s} \\ \text{s.t. } & S_{ii} = 1, \quad i = 1, \ldots, KP && \Longleftrightarrow s_i^2 = 1 \\ & \boldsymbol{S} = \boldsymbol{s}\boldsymbol{s}^T \end{aligned}$$

- The constraint $\boldsymbol{S} = \boldsymbol{s}\boldsymbol{s}^T$ implies that $\boldsymbol{S}$ is positive semidefinite (PSD), and has rank 1.
- The idea of SDR is to drop the rank-1 constraint:

$$\begin{aligned} \max\ & \mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\} \\ \text{s.t. } & S_{ii} = 1, \quad i = 1, \ldots, KP \\ & \boldsymbol{S} \succeq \boldsymbol{0} \end{aligned}$$

- The resultant problem is a convex optimization problem called semidefinite program, whose globally optimal solution can be numerically computed with a complexity of $\mathcal{O}((KP)^{3.5})$.

SDR Approximation Accuracy

Let

$$f_{\mathsf{ML}} = \max_{\boldsymbol{s}\in\{\pm 1\}^{KP}} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s},$$

$$f_{\mathsf{SDR}} = \max_{\boldsymbol{S}\succeq \boldsymbol{0}, \mathrm{diag}(\boldsymbol{S})=\boldsymbol{1}} \mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\},$$

$$f_{\mathsf{NR}} = \max_{\|\boldsymbol{s}\|_2^2 = KP} \boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}$$

denote the optimal values of the true blind ML, SDR, and norm relaxation, respectively.

Lemma

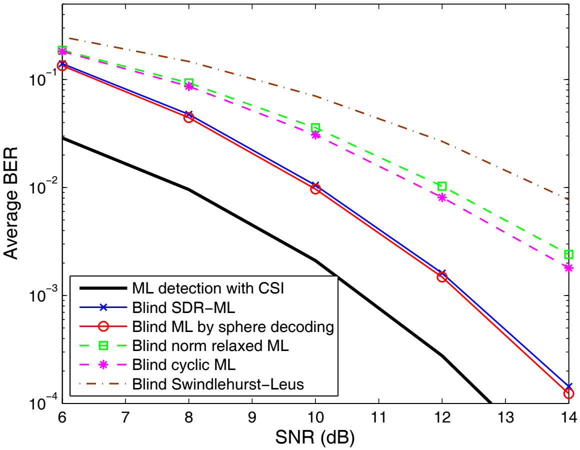
For any $\boldsymbol{\mathcal{R}}$,

$$|f_{\mathsf{ML}} - f_{\mathsf{SDR}}| \leq |f_{\mathsf{ML}} - f_{\mathsf{NR}}|$$

This implies that SDR should perform at least no worse than norm relaxation.

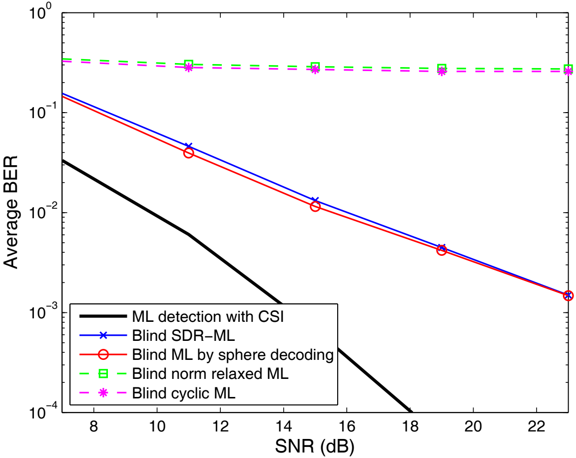
Simulation Results: Performance

3×4 BPSK OSTBC, $M_r = 4$, $P = 8$.



Simulation Results: Performance (Cont'd)

3×4 BPSK OSTBC, $M_r = 1$, $P = 8$.



Extension: 16-QAM Constellations

- For M-PSK constellations, the quadratic maximization reformulation remains valid. Again, SDR or sphere decoding can be used [9,10].
- But, for 16-QAM constellations where $s_k \in \{\pm 1, \pm 3\}$, the same reformulation will lead to a Rayleigh quotient maximization

$$\max_{\boldsymbol{s} \in \{\pm 1, \pm 3\}^{PK}} \frac{\boldsymbol{s}^T \boldsymbol{\mathcal{R}} \boldsymbol{s}}{\boldsymbol{s}^T \boldsymbol{s}}$$

where SDR and sphere decoding are no longer applicable (not directly, at least).

[9] T. Cui and C. Tellambura, "Efficient blind receiver design for OSTBCs," *IEEE Trans. WC*, May 2007.

[10] L. Zhou, J.-K. Zhang, and K.-M. Wong, "A novel signaling scheme for blind unique identification of Alamouti space-time block-coded channel," *IEEE Trans. SP*, June 2007.

Linear Fractional SDR for 16-QAM Constellations

Consider a bound-constrained SDR[11] for Rayleigh quotient max.:

$$\max \frac{\mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\}}{\mathrm{tr}\{\boldsymbol{S}\}}$$
$$\text{s.t. } 1 \leq S_{ii} \leq 9, \quad i = 1, \ldots, KP \quad \text{(in lieu of } S_{ii} \in \{1, 3^2\}\text{)}$$
$$\boldsymbol{S} \succeq \boldsymbol{0}$$

- The objective is linear fractional; quasi-convex but not convex.
- This linear fractional SDR can be solved by bisection, where a sequence of SDP feasibility problems are solved ($\Longrightarrow$ expensive).
- We can turn the linear fractional SDR to an SDP.[12]

[11] N. D. Sidiropoulos and Z.-Q. Luo, "A SDP approach to MIMO detection for higher-order constellations," *IEEE Signal Process. Lett.*, 2006.

[12] C.-W. Hsin, T.-H. Chang, W.-K. Ma, & C.-Y. Chi, A linear fractional SDR approach to ML detection of higher-order QAM OSTBC in unknown channels," to appear in *IEEE Trans. SP*, 2010.

Linear Fractional SDR for 16-QAM Constellations

Consider a bound-constrained SDR[11] for Rayleigh quotient max.:

$$\max \frac{\mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\}}{\mathrm{tr}\{\boldsymbol{S}\}}$$
$$\text{s.t. } 1 \leq S_{ii} \leq 9, \quad i = 1, \ldots, KP \quad \text{(in lieu of } S_{ii} \in \{1, 3^2\}\text{)}$$
$$\boldsymbol{S} \succeq \boldsymbol{0}$$

- The objective is linear fractional; quasi-convex but not convex.
- This linear fractional SDR can be solved by bisection, where a sequence of SDP feasibility problems are solved ($\Longrightarrow$ expensive).
- We can turn the linear fractional SDR to an SDP.[12]

[11] N. D. Sidiropoulos and Z.-Q. Luo, "A SDP approach to MIMO detection for higher-order constellations," *IEEE Signal Process. Lett.*, 2006.

[12] C.-W. Hsin, T.-H. Chang, W.-K. Ma, & C.-Y. Chi, A linear fractional SDR approach to ML detection of higher-order QAM OSTBC in unknown channels," to appear in *IEEE Trans. SP*, 2010.

Linear Fractional SDR for 16-QAM Constellations

The idea is to use Charnes-Cooper transformation. Assume wlog that s_{PK} is known and that $s_{PK} = 1$. Then,

$$\boldsymbol{S}^\star = \arg\max \frac{\mathrm{tr}\{\boldsymbol{S}\boldsymbol{\mathcal{R}}\}}{\mathrm{tr}\{\boldsymbol{S}\}} \quad (*)$$

$$\text{s.t. } 1 \leq S_{ii} \leq 9, \quad i = 1, \ldots, KP-1$$

$$S_{KP,KP} = 1, \; \boldsymbol{S} \succeq \boldsymbol{0}$$

By letting $\boldsymbol{Z} = \boldsymbol{S}/\mathrm{tr}\{\boldsymbol{S}\}$, we prove that

Proposition

The following SDP

$$\boldsymbol{Z}^\star = \arg\max \mathrm{tr}\{\boldsymbol{Z}\boldsymbol{\mathcal{R}}\} \quad (\dagger)$$

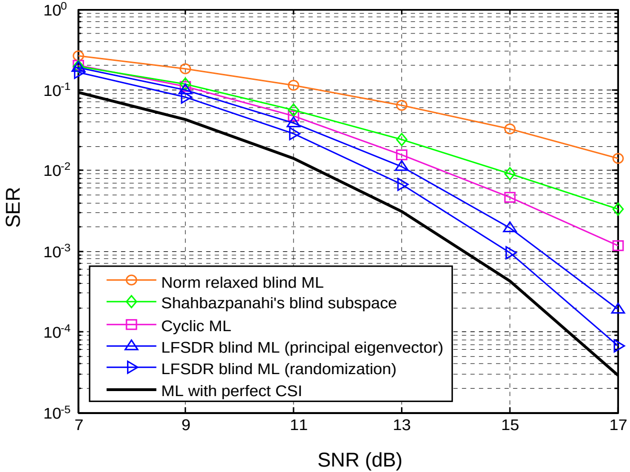
$$\text{s.t. } Z_{KP,KP} \leq Z_{ii} \leq 9Z_{KP,KP}, \quad i = 1, \ldots, KP-1$$

$$\mathrm{tr}\{\boldsymbol{Z}\} = 1, \; \boldsymbol{Z} \succeq \boldsymbol{0}$$

is equivalent to Problem $(*)$, in the sense that $\boldsymbol{S}^\star = \boldsymbol{Z}^\star / Z^\star_{KP,KP}$.

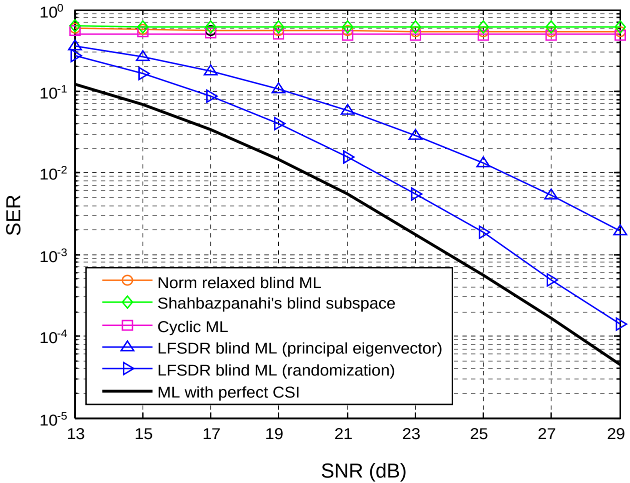
Simulation Results: Performance

3×4 16-QAM OSTBC, $M_r = 4$, $P = 8$.



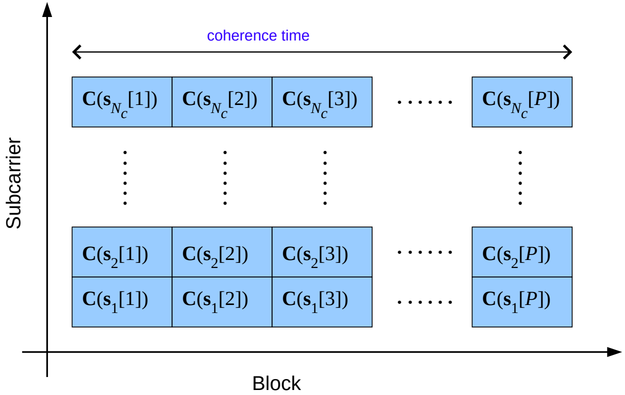
Simulation Results: Performance

3×4 16-QAM OSTBC, $M_r = 1$, $P = 8$.



Extension: OSTBC-OFDM

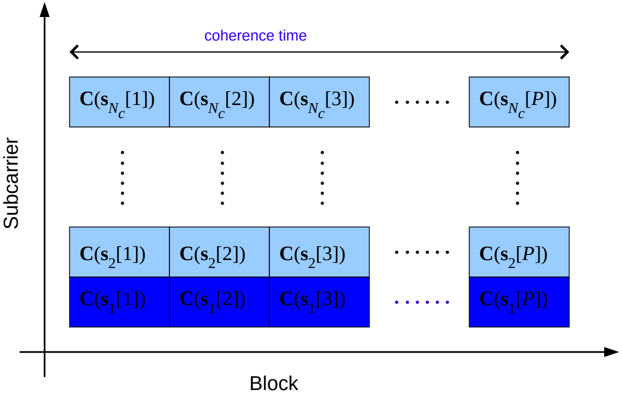
- In this scenario, each OFDM subchannel is a MIMO flat fading channel.



- A straightforward way would be to apply the aforedescribed blind ML methods to each subchannel independently.

Extension: OSTBC-OFDM

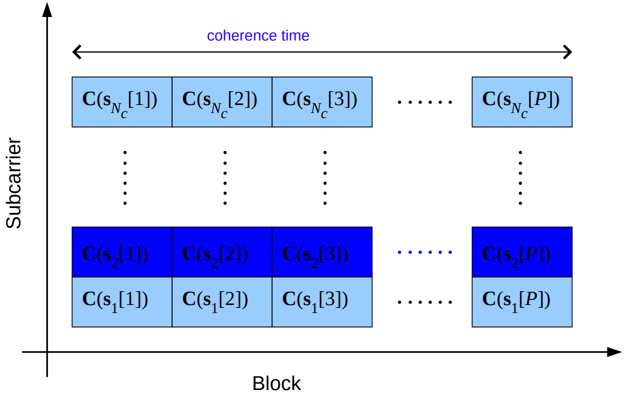
- In this scenario, each OFDM subchannel is a MIMO flat fading channel.



- A straightforward way would be to apply the aforedescribed blind ML methods to each subchannel independently.

Extension: OSTBC-OFDM

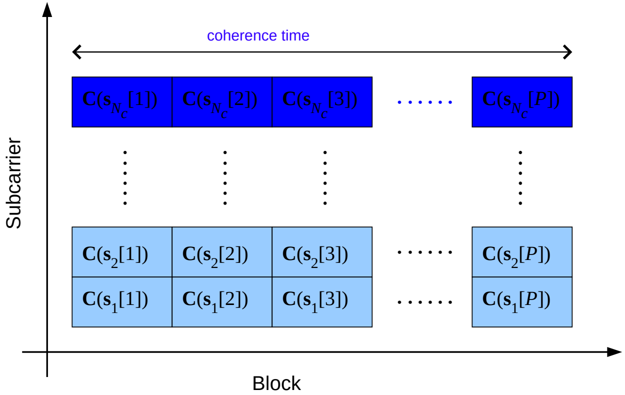
- In this scenario, each OFDM subchannel is a MIMO flat fading channel.



- A straightforward way would be to apply the aforedescribed blind ML methods to each subchannel independently.

Extension: OSTBC-OFDM

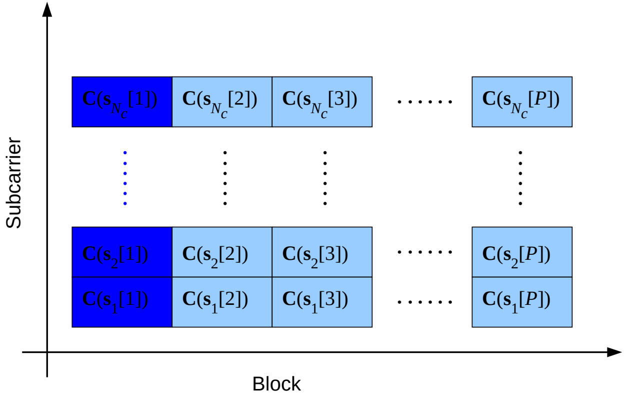
- In this scenario, each OFDM subchannel is a MIMO flat fading channel.



- A straightforward way would be to apply the aforedescribed blind ML methods to each subchannel independently.

Extension: OSTBC-OFDM
(Cont'd)

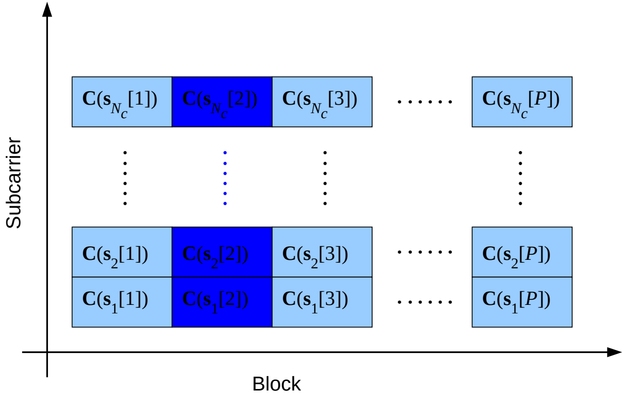
- We are interested in a subcarrier dependent approach.



- The advantage is the ability to accommodate faster fading.

Extension: OSTBC-OFDM
(Cont'd)

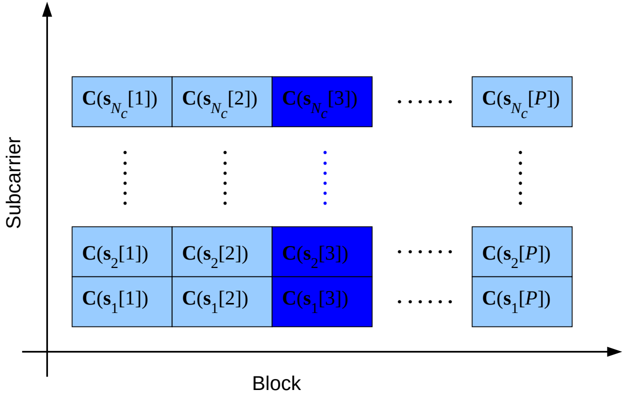
- We are interested in a subcarrier dependent approach.



- The advantage is the ability to accommodate faster fading.

Extension: OSTBC-OFDM
(Cont'd)

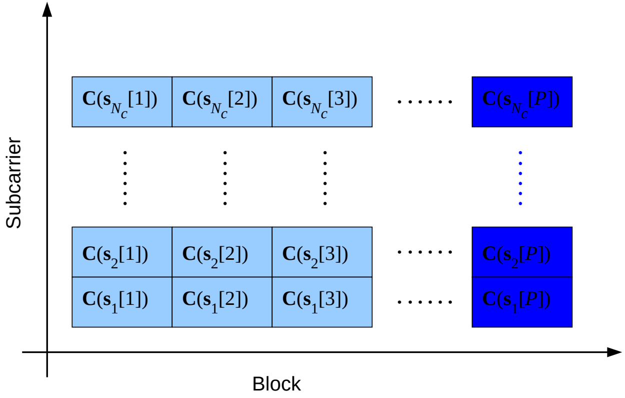
- We are interested in a subcarrier dependent approach.



- The advantage is the ability to accommodate faster fading.

Extension: OSTBC-OFDM
(Cont'd)

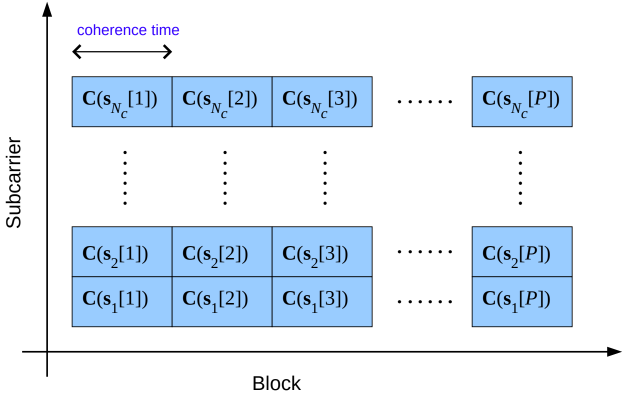
- We are interested in a subcarrier dependent approach.



- The advantage is the ability to accommodate faster fading.

Extension: OSTBC-OFDM
(Cont'd)

- We are interested in a subcarrier dependent approach.



- The advantage is the ability to accommodate faster fading.

Extension: OSTBC-OFDM (Cont'd)

- Let N_c be the OFDM size. The received signal for subchannel n is:

$$\mathbf{Y}_n = \mathbf{H}_n \mathbf{C}(\mathbf{s}_n) + \mathbf{V}_n, \qquad n = 1, \ldots, N_c$$

where $\mathbf{H}_n$ is MIMO frequency response at subcarrier n.

- $\mathbf{H}_n$ can be parametrized by their FIR coefficients. Specifically,

$$\mathbf{H}_n = \boldsymbol{\mathcal{H}}(\mathbf{I}_{M_t} \otimes \mathbf{f}_n)$$

where

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \mathbf{h}_{11}^T & \cdots & \mathbf{h}_{M_t,1}^T \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{1,M_r}^T & \cdots & \mathbf{h}_{M_t,M_r}^T \end{bmatrix} \in \mathbb{C}^{M_r \times LM_t},$$

$\mathbf{h}_{m,i} = [\ h_{m,i}[0], h_{m,i}[1], \ldots, h_{m,i}[L-1]\ ]^T$ is the impulse response between the mth tx & ith rx antennas, L denotes the channel length, & $\mathbf{f}_n = \frac{1}{N_c}[\ 1, e^{-j\frac{2\pi}{N_c}(n-1)}, \ldots, e^{-j\frac{2\pi}{N_c}(n-1)(L-1)}\ ]^T$.

Extension: OSTBC-OFDM
(Cont'd)

- The received signal model can therefore be rewritten as

$$\mathbf{Y}_n = \boldsymbol{\mathcal{H}}(\mathbf{I}_{M_t} \otimes \mathbf{f}_n)\mathbf{C}(\mathbf{s}_n) + \mathbf{V}_n.$$

- By letting $\boldsymbol{\mathcal{Y}} = [\ \mathbf{Y}_1, \ldots, \mathbf{Y}_{N_c}\ ]$, $\mathbf{G}_n(\mathbf{s}_n) = (\mathbf{I}_{M_t} \otimes \mathbf{f}_n)\mathbf{C}(\mathbf{s}_n)$, & $\boldsymbol{\mathcal{G}}(\boldsymbol{s}) = [\ \mathbf{G}_1(\mathbf{s}_1), \ldots, \mathbf{G}_{N_c}(\mathbf{s}_{N_c})\ ]$, we obtain

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{G}}(\boldsymbol{s}) + \boldsymbol{\mathcal{V}}$$

- Like the flat fading case, $\boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{\mathcal{G}}^H(\boldsymbol{s})$ is shown to be independent of any $\boldsymbol{s} \in \{\pm\}^{KN_c}$.

- Hence, the implementation techniques in the flat fading case are applicable to the OSTBC-OFDM case here.[13,14]

[13] T. Cui and C. Tellambura, "Joint data detection and channel estimation for OFDM systems," *IEEE Trans. Commun.*, Apr. 2006.

[14] T.-H. Chang, W.-K. Ma, & C.-Y. Chi, "ML detection of OSTBC-OFDM in unknown block fading channels," *IEEE Trans. SP*, Apr. 2008.

Extension: OSTBC-OFDM
(Cont'd)

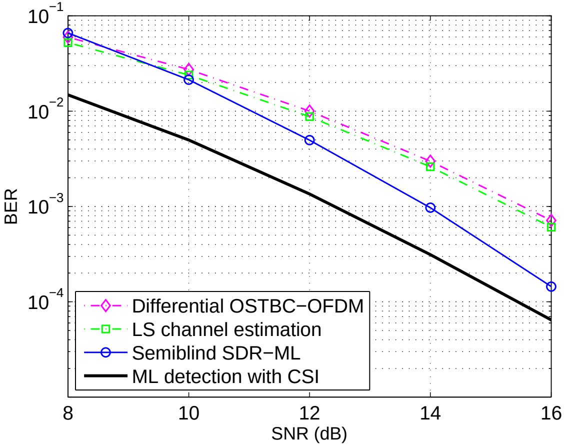
QPSK Alamouti code, $M_r = 2$, $L = 4$, $N_c = 32$, $15,000$ trials.

'LS channel estimation'

Use L pilot codes (min. requirement) to estimate the channel, followed by coherent detection.

'Semiblind SDR-ML'

Use SDR to solve the semiblind ML problem. Only 1 pilot code is used.



Extension: OSTBC-OFDM
(Cont'd)

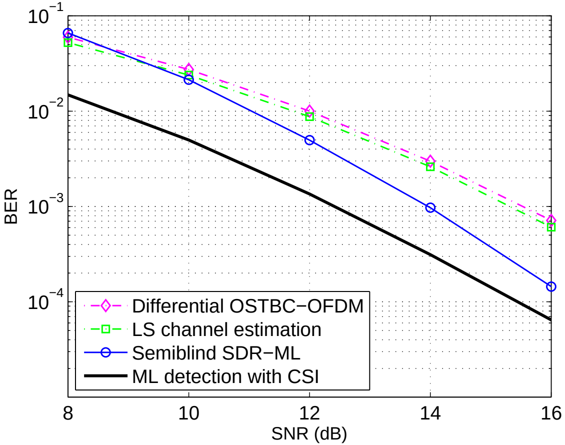
QPSK Alamouti code, $M_r = 2$, $L = 4$, $N_c = 32$, $15,000$ trials.

'LS channel estimation'

Use L pilot codes (min. requirement) to estimate the channel, followed by coherent detection.

'Semiblind SDR-ML'

Use SDR to solve the semiblind ML problem. Only 1 pilot code is used.



1. Background
   - Orthogonal space-time block codes (OSTBCs): The basics
   - Detection without channel state information (CSI)

2. Blind Maximum-Likelihood (ML) OSTBC detection: Implementations
   - Flat fading channels
   - Extension: 16-QAM constellations
   - Extension: OSTBC-OFDM for frequency selective block fading channels

3. Blind Identifiability and Code Constructions
   - Probability 1 identifiability
   - Perfect identifiability and code constructions

Problem Statement

- For ease of exposition of the problem, assume that noise is absent, & that $P = 1$.
- The signal model can be notationally simplified to

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s})$$

- Identifiability problem statement: Does there exist another channel and symbols, denoted by $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}})? \quad (6)$$

- An obvious case is when $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} = \{-\mathbf{s}, -\mathbf{H}\}$. But this sign ambiguity can be easily fixed, say by using 1 pilot bit.
- $\mathbf{s}$ (or $\mathbf{H}$) is said to be uniquely identifiable up to a sign (UI-± 1) if (6) cannot be satisfied for any $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} \neq \{\pm\mathbf{s}, \pm\mathbf{H}\}$.

Problem Statement

- For ease of exposition of the problem, assume that noise is absent, & that $P = 1$.
- The signal model can be notationally simplified to

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s})$$

- Identifiability problem statement: Does there exist another channel and symbols, denoted by $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}})? \tag{6}$$

- An obvious case is when $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} = \{-\mathbf{s}, -\mathbf{H}\}$. But this sign ambiguity can be easily fixed, say by using 1 pilot bit.
- $\mathbf{s}$ (or $\mathbf{H}$) is said to be uniquely identifiable up to a sign (UI-± 1) if (6) cannot be satisfied for any $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} \neq \{\pm\mathbf{s}, \pm\mathbf{H}\}$.

Problem Statement

- For ease of exposition of the problem, assume that noise is absent, & that $P = 1$.
- The signal model can be notationally simplified to

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s})$$

- Identifiability problem statement: Does there exist another channel and symbols, denoted by $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}})? \tag{6}$$

- An obvious case is when $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} = \{-\mathbf{s}, -\mathbf{H}\}$. But this sign ambiguity can be easily fixed, say by using 1 pilot bit.
- $\mathbf{s}$ (or $\mathbf{H}$) is said to be uniquely identifiable up to a sign (UI-± 1) if (6) cannot be satisfied for any $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} \neq \{\pm \mathbf{s}, \pm \mathbf{H}\}$.

Problem Statement

- For ease of exposition of the problem, assume that noise is absent, & that $P = 1$.
- The signal model can be notationally simplified to

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s})$$

- Identifiability problem statement: Does there exist another channel and symbols, denoted by $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}})? \tag{6}$$

- An obvious case is when $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} = \{-\mathbf{s}, -\mathbf{H}\}$. But this sign ambiguity can be easily fixed, say by using 1 pilot bit.
- $\mathbf{s}$ (or $\mathbf{H}$) is said to be uniquely identifiable up to a sign (UI-± 1) if (6) cannot be satisfied for any $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} \neq \{\pm\mathbf{s}, \pm\mathbf{H}\}$.

Problem Statement

- For ease of exposition of the problem, assume that noise is absent, & that $P = 1$.
- The signal model can be notationally simplified to

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s})$$

- Identifiability problem statement: Does there exist another channel and symbols, denoted by $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}})? \tag{6}$$

- An obvious case is when $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} = \{-\mathbf{s}, -\mathbf{H}\}$. But this sign ambiguity can be easily fixed, say by using 1 pilot bit.
- $\mathbf{s}$ (or $\mathbf{H}$) is said to be uniquely identifiable up to a sign (UI-± 1) if (6) cannot be satisfied for any $\{\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\} \neq \{\pm\mathbf{s}, \pm\mathbf{H}\}$.

Strictly Non-Rotatable OSTBCs

Definition

An OSTBC $\mathbf{C}(.)$ is said to be strictly non-rotatable if there does not exist a matrix $\mathbf{Q} \in \mathbb{C}^{M_t \times M_t}$ such that

$$\mathbf{QC}(\mathbf{s}) = \mathbf{C}(\tilde{\mathbf{s}}) \tag{7}$$

for any $\mathbf{s}, \tilde{\mathbf{s}} \in \{\pm 1\}^K$, $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$.

- Suppose that (7) can be satisfied for some $\mathbf{s}, \tilde{\mathbf{s}}$.
- It can be shown that $\mathbf{Q}$ is unitary.
- Then,

$$\mathbf{HC}(\mathbf{s}) = \mathbf{HQ}^H\mathbf{QC}(\mathbf{s}) = (\mathbf{HQ}^H)\mathbf{C}(\tilde{\mathbf{s}})$$

where we are faced with a rotational ambiguity.

An Example of Rotatable Codes

Example

Consider the BPSK Alamouti code

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix}.$$

We note that

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix} = \begin{bmatrix} -s_2 & -s_1 \\ s_1 & -s_2 \end{bmatrix} = \mathbf{C}((-s_2, s_1))$$

Probability 1 Identifiability

To achieve unique identifiability, it is necessary to employ strictly non-rotatable codes. But is that sufficient?

We show that[15]

Theorem

Suppose that $\mathbf{H}$ is Gaussian distributed, & that at least one row of $\mathbf{H}$ has positive def. covariance matrix. Then, $\mathbf{s}$ is UI-± 1 with probability 1 if $\mathbf{C}(.)$ is strictly non-rotatable.

Corollary

For i.i.d. Gaussian channels, $\mathbf{s}$ is UI-± 1 with probability 1 if $\mathbf{C}(.)$ is strictly non-rotatable.

Note that these results are true even for $M_r = 1$.

---

[15] W.-K. Ma, "Blind ML detection of OSTBCs: Identifiability and code constructions," *IEEE Trans. SP*, July 2007.

Strict Non-Rotatability of the Existing Codes

(T, M_t, K)	strictly non-rotatable?
$(2,2,2)$	no
$(4,3,4)$	yes
$(4,4,4)$	no
$(8,5,8)$	yes
$(8,6,8)$	no
$(8,7,8)$	yes
$(8,8,8)$	no

Real OSTBCs.

$(T, M_t, K/2)$	strictly non-rotatable?
$(2,2,2)$	no
$(4,3,3)$	no
$(4,4,3)$	no
$(15,5,10)$	not known
$(8,5,4)$	yes
$(8,6,4)$	yes
$(8,7,4)$	no
$(8,8,4)$	no

Complex OSTBCs.

- It appears that many existing OSTBCs are not strictly non-rotatable.
- A natural question that arises is whether or not we can design our own blindly identifiable codes. This leads to the nonintersecting subspace OSTBCs to be presented next.

Non-Intersecting Subspace OSTBCs

Let us use the notation $\mathcal{R}(\mathbf{A})$ to denote the range space of $\mathbf{A}$.

Definition

An OSTBC is said to be a non-intersecting subspace (NIS) OSTBC if

$$\mathcal{R}(\mathbf{C}^T(\mathbf{s})) \cap \mathcal{R}(\mathbf{C}^T(\tilde{\mathbf{s}})) = \{\mathbf{0}\} \quad (8)$$

for every $\mathbf{s}, \tilde{\mathbf{s}} \in \{\pm 1\}^K$, $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$.

- The NIS property in (8) has been used in noncoherent space-time coding for achieving the max. noncoherent diversity[16,17] in i.i.d. Gaussian channels.

[16] F. E. Oggier, N. J. A. Sloane, S. N. Diggavi, and A. R. Calderbank, "Nonintersecting subspaces based on finite alphabets," *IEEE Trans. IT*, 2005.

[17] M. Brehler & M. K. Varanasi, "Asymptotic error probability analysis of quadratic receivers in Rayleigh fading channels with applications to a unified analysis of coherent and noncoherent space-time receivers," *IEEE Trans. IT*, 2001.

Perfect Identifiability

NIS-OSTBCs are 'perfect' from a blind identifiability standpoint:

Theorem

$\mathbf{s}$ is UI-± 1 for any nonzero $\mathbf{H}$, if and only if $\mathbf{C}(.)$ is an NIS-OSTBC.

There is a price for using NIS-OSTBCs, however.

Lemma

Suppose that $\mathbf{C}(.)$ is a real or complex GOD. If $\mathbf{C}(.)$ is also an NIS-OSTBC, then it does not achieve the full rate; i.e., $K < T$ for real GODs and $K/2 < T$ for complex GODs.

An additional property:

Property

All NIS-OSTBCs have $2M_t \leq T$.

Perfect Identifiability

NIS-OSTBCs are ‘perfect’ from a blind identifiability standpoint:

Theorem

$\mathbf{s}$ is UI-± 1 for any nonzero $\mathbf{H}$, if and only if $\mathbf{C}(.)$ is an NIS-OSTBC.

There is a price for using NIS-OSTBCs, however.

Lemma

Suppose that $\mathbf{C}(.)$ is a real or complex GOD. If $\mathbf{C}(.)$ is also an NIS-OSTBC, then it does not achieve the full rate; i.e., $K < T$ for real GODs and $K/2 < T$ for complex GODs.

An additional property:

Property

All NIS-OSTBCs have $2M_t \leq T$.

Perfect Identifiability

NIS-OSTBCs are ‘perfect’ from a blind identifiability standpoint:

Theorem

$\mathbf{s}$ is UI-± 1 for any nonzero $\mathbf{H}$, if and only if $\mathbf{C}(.)$ is an NIS-OSTBC.

There is a price for using NIS-OSTBCs, however.

Lemma

Suppose that $\mathbf{C}(.)$ is a real or complex GOD. If $\mathbf{C}(.)$ is also an NIS-OSTBC, then it does not achieve the full rate; i.e., $K < T$ for real GODs and $K/2 < T$ for complex GODs.

An additional property:

Property

All NIS-OSTBCs have $2M_t \leq T$.

A Seemingly Heuristic Code Construction

Procedure

1. Given an OSTBC with even no. of bits.
2. Concatenate two OSTBCs into one.
3. Drop one bit.

$$\mathbf{C}_e(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & s_4 & -s_3 \\ -s_3 & -s_4 & s_1 & s_2 \end{bmatrix}$$

A Seemingly Heuristic Code Construction

Procedure

1. Given an OSTBC with even no. of bits.
2. Concatenate two OSTBCs into one.
3. Drop one bit.

$$\left[\underbrace{\begin{matrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & s_4 & -s_3 \\ -s_3 & -s_4 & s_1 & s_2 \end{matrix}}_{\text{one } \mathbf{C}_e(.)} \quad \underbrace{\begin{matrix} s_5 & s_6 & s_7 & s_8 \\ -s_6 & s_5 & s_8 & -s_7 \\ -s_7 & -s_8 & s_5 & s_6 \end{matrix}}_{\text{another}}\right]$$

A Seemingly Heuristic Code Construction

Procedure

1. Given an OSTBC with even no. of bits.
2. Concatenate two OSTBCs into one.
3. Drop one bit.

$$\begin{bmatrix} s_1 & s_2 & s_3 & 0 & s_5 & s_6 & s_7 & s_8 \\ -s_2 & s_1 & 0 & -s_3 & -s_6 & s_5 & s_8 & -s_7 \\ -s_3 & 0 & s_1 & s_2 & -s_7 & -s_8 & s_5 & s_6 \end{bmatrix}$$

A Seemingly Heuristic Code Construction

Procedure

1. Given an OSTBC with even no. of bits.
2. Concatenate two OSTBCs into one.
3. Drop one bit.

$$\begin{bmatrix} s_1 & s_2 & s_3 & 0 & s_5 & s_6 & s_7 & s_8 \\ -s_2 & s_1 & 0 & -s_3 & -s_6 & s_5 & s_8 & -s_7 \\ -s_3 & 0 & s_1 & s_2 & -s_7 & -s_8 & s_5 & s_6 \end{bmatrix}$$

The constructed code satisfies the 2 necessary NIS conditions. But, is it sufficient that the code is NIS?

A Seemingly Heuristic Code Construction

Procedure

1. Given an OSTBC with even no. of bits.
2. Concatenate two OSTBCs into one.
3. Drop one bit.

$$\begin{bmatrix} s_1 & s_2 & s_3 & 0 & s_5 & s_6 & s_7 & s_8 \\ -s_2 & s_1 & 0 & -s_3 & -s_6 & s_5 & s_8 & -s_7 \\ -s_3 & 0 & s_1 & s_2 & -s_7 & -s_8 & s_5 & s_6 \end{bmatrix}$$

The constructed code satisfies the 2 necessary NIS conditions. But, is it sufficient that the code is NIS?

Answer: Yes!

NIS Code Constructions

A formal description of the above described procedure:

Construction I

1. **given** an OSTBC $\mathbf{C}_e(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k$ where K is even.
2. **set** $\mathbf{C}_o(\mathbf{s}) = \sum_{k=1}^{K-1} s_k \mathbf{X}_k$.
3. **output** $\mathbf{C}_{new}(\mathbf{s}) = [\ \mathbf{C}_o(\mathbf{s}_1)\ \mathbf{C}_e(\mathbf{s}_2)\ ]$ as the new code, where $\mathbf{s} = [\ \mathbf{s}_1^T\ \mathbf{s}_2^T\ ]^T$.

Theorem

Given any BPSK/QPSK OSTBC function $\mathbf{C}_e : \mathbb{R}^K \rightarrow \mathbb{C}^{M_t \times T}$ where K is even, the code generated by Construction I is an NIS-OSTBC.

Some Constructed NIS-OSTBCs

- From the QPSK Alamouti code

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}$$

we can construct an QPSK 2-transmitter NIS-OSTBC

$$\begin{bmatrix} u_1 & -s_2 & u_3 & -u_4^* \\ s_2 & u_1^* & u_4 & u_3^* \end{bmatrix}.$$

- Likewise, from

$$\begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \end{bmatrix}$$

we can construct an QPSK 3-transmitter NIS-OSTBC

$$\begin{bmatrix} u_1 & -u_2^* & -s_3 & 0 & u_4 & -u_5^* & -u_6^* & 0 \\ u_2 & u_1^* & 0 & -s_3 & u_5 & u_4^* & 0 & -u_6^* \\ s_3 & 0 & u_1^* & u_2^* & u_6 & 0 & u_5^* & u_4^* \end{bmatrix}.$$

Some Constructed NIS-OSTBCs

- From the QPSK Alamouti code

$$\begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix}$$

we can construct an QPSK 2-transmitter NIS-OSTBC

$$\begin{bmatrix} u_1 & -s_2 & u_3 & -u_4^* \\ s_2 & u_1^* & u_4 & u_3^* \end{bmatrix}.$$

- Likewise, from

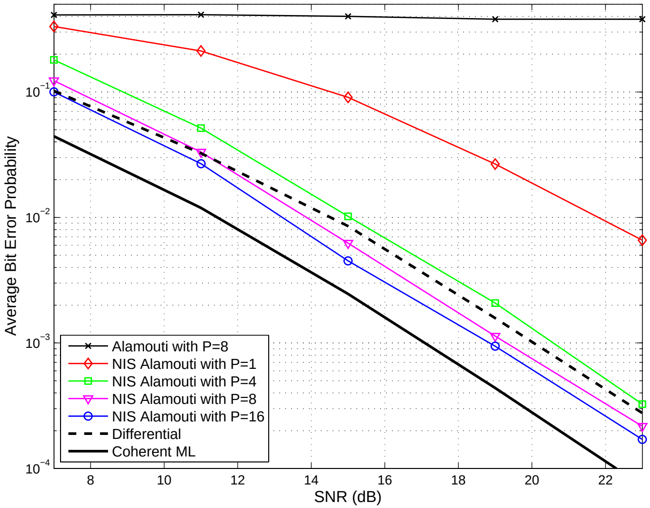
$$\begin{bmatrix} u_1 & -u_2^* & -u_3^* & 0 \\ u_2 & u_1^* & 0 & -u_3^* \\ u_3 & 0 & u_1^* & u_2^* \end{bmatrix}$$

we can construct an QPSK 3-transmitter NIS-OSTBC

$$\begin{bmatrix} u_1 & -u_2^* & -s_3 & 0 & u_4 & -u_5^* & -u_6^* & 0 \\ u_2 & u_1^* & 0 & -s_3 & u_5 & u_4^* & 0 & -u_6^* \\ s_3 & 0 & u_1^* & u_2^* & u_6 & 0 & u_5^* & u_4^* \end{bmatrix}.$$

Simulation Results

QPSK Alamouti, $M_r = 1$, SDR-ML was employed.



Concurrent Works in Identifiability & Code Design

Some further extensions of blind ML OSTBC:

- NIS OSTBC designs for M-ary PSK [18]: use two different (coprime) PSK constellations for the Alamouti codes, and NIS will be achieved.
- Probability-1 identifiability of OSTBC-OFDM and scheme design[19]: Such identifiability can be achieved with 1 pilot code, or even 1 pilot bit with proper code assignment.
- Perfect identifiability of OSTBC-OFDM and scheme design[20]: More stringent than probability-1 identifiability. You need at least L pilot bits (where L is the channel length).

[18] J.-K. Zhang and W.-K. Ma, “Full diversity blind Alamouti space-time block codes for unique identification of flat fading channels,” *IEEE Trans. SP*, 2009.

[19] T.-H. Chang, W.-K. Ma, & C.-Y. Chi, “ML detection of OSTBC-OFDM in unknown block fading channels,” *IEEE Trans. SP*, Apr. 2008.

[20] T.-H. Chang, W.-K. Ma, & C.-Y. Chi, “On Perfect Channel Identifiability of Semiblind ML Detection of Orthogonal Space-Time Block Coded OFDM,” in *ICASSP* 2009.

Concurrent Works in Identifiability & Code Design

It was also found that in second-order statistics (SOS)-based blind channel estimation, OSTBCs provide many advantages.

- Identifiability analysis for SOS-based blind receivers[21]
- Achieving SOS-based blind identifiability by unequal symbol power loading[22].
- Time-varying STBC transmissions for enhancing SOS-based blind identifiability[23].

[21] J. Vía & I. Santamaria, "On the blind identifiability of MIMO-OSTBC channels based on second-order statistics," *IEEE Trans. IT*, 2008.

[22] S. Shahbazpanahi, A. Gershman, & J. Manton, "Closed-form blind MIMO channel estimation for OSTBCs," *IEEE Trans. SP*, Dec. 2005.

[23] J. Vía, I. Santamaria, & J. Pérez, "Code combination for blind channel estimation in general MIMO-STBC systems," *EURASIP J. Advances SP*, 2009.

Conclusion and Discussion

- We show that OSTBCs are 'good' codes for blind or noncoherent space-time coding.
- Its ML receiver can be effectively implemented, by exploiting the code structures.
- There exist OSTBCs that provide very attractive blind identifiability conditions, though they do not fall into the conventional class of OSTBCs and judicious code constructions are required.
- Code designs with provable identifiability results remain an open direction.

Thank you!

Conclusion and Discussion

- We show that OSTBCs are 'good' codes for blind or noncoherent space-time coding.
- Its ML receiver can be effectively implemented, by exploiting the code structures.
- There exist OSTBCs that provide very attractive blind identifiability conditions, though they do not fall into the conventional class of OSTBCs and judicious code constructions are required.
- Code designs with provable identifiability results remain an open direction.

Thank you!

Conclusion and Discussion

- We show that OSTBCs are ‘good’ codes for blind or noncoherent space-time coding.
- Its ML receiver can be effectively implemented, by exploiting the code structures.
- There exist OSTBCs that provide very attractive blind identifiability conditions, though they do not fall into the conventional class of OSTBCs and judicious code constructions are required.
- Code designs with provable identifiability results remain an open direction.

Thank you!

Conclusion and Discussion

- We show that OSTBCs are 'good' codes for blind or noncoherent space-time coding.
- Its ML receiver can be effectively implemented, by exploiting the code structures.
- There exist OSTBCs that provide very attractive blind identifiability conditions, though they do not fall into the conventional class of OSTBCs and judicious code constructions are required.
- Code designs with provable identifiability results remain an open direction.

Thank you!

Conclusion and Discussion

- We show that OSTBCs are ‘good’ codes for blind or noncoherent space-time coding.
- Its ML receiver can be effectively implemented, by exploiting the code structures.
- There exist OSTBCs that provide very attractive blind identifiability conditions, though they do not fall into the conventional class of OSTBCs and judicious code constructions are required.
- Code designs with provable identifiability results remain an open direction.

Thank you!