WMMSE-based Multiuser MIMO Beamforming: A Practice-Oriented Design and LTE System Performance Evaluation

Jiaxian Pan and Wing-Kin Ma
Department of Electronic Engineering
The Chinese University of Hong Kong, Hong Kong
E-mail: jxpan@ee.cuhk.edu.hk, wkma@ieee.org

Xin Xia and Yuan Tian
Wireless Network Research Dept II, Chengdu, China
Huawei Technologies Co., Ltd.
E-mail: {tony.xiaxin, noc.tianyuan}@huawei.com

Abstract—**This paper deals with the multiuser MIMO downlink beamforming scenario. In this scenario, a popular approach for designing the beamformer is the weighted minimum mean square error (WMMSE) approach for sum rate maximization. This work considers the WMMSE method from an implementation perspective, where we take into account practical factors — such as linear receivers (rather than successive interference cancellation receivers, which are implicitly assumed in some existing works but are more complex to implement) and imperfect channel knowledge at the transmitter — for deployment in systems such as long term evolution (LTE). A modified WMMSE algorithm that incorporates such factors is proposed. We evaluate the performance of the proposed WMMSE algorithm using a time division duplexing (TDD) LTE simulation platform, and show that our practice-oriented design can improve the system throughput by $3 \sim 30\%$ in comparison with the conventional WMMSE algorithm.**

Index Terms—**WMMSE, sum rate maximization, linear receivers, imperfect CSIT, LTE.**

I. Introduction

Multiuser MIMO beamforming is a powerful technique to meet the rapidly growing data demands in wireless communications. Due to its ability to serve multiple users simultaneously, multiuser beamforming can significantly improve the system throughput and has been a feature in several standards, such as 802.11ac Wi-Fi and 4G long term evolution (LTE).

In general, we are interested in designing the multiuser beamformers such that the sum rate with respect to all users is maximized. Unfortunately, the sum rate maximization problem is NP-hard. Various methods [1]–[4] have been proposed to tackle the sum rate maximization problem. Among these works, the iteratively weighted minimum mean square error (WMMSE) method [3], [4] is a very promising approach. The difficulty in sum rate maximization mainly arises from the log determinant and matrix inverse operations in the achievable rate expression. By introducing two sets of auxiliary variables, the WMMSE method turns the sum rate maximization problem into a weighed mean square error minimization problem where the log determinant and matrix inverse operations become implicit. Block coordinate descent (BCD) is then applied to update the auxiliary variables and the beamformers in an alternating manner. Since all updates have closed-form solutions, WMMSE is a very convenient way to optimize the sum rate.

In this paper, we follow the WMMSE idea in [3], [4] to optimize the average sum rate for linear receivers and under imperfect channel state information at the transmitter (CSIT). The achievable rate expression adopted in the original works [3], [4] assumes implicitly that successive interference cancellation (SIC) receivers are used. The SIC receiver involves a complicated process at the receiver side; specifically, demodulation, channel decoding and re-encoding, re-modulation, and cancellation for each data stream. However, in wireless communications, users are usually powered by batteries and the use of computationally expensive receive algorithms may not be affordable. One simple alternative receiver is the linear receiver, where each data stream is demodulated by a linear filter first, and then channel decoding is applied. Therefore, when linear receivers are used, the rate expression and the subsequent WMMSE optimization algorithm should be redesigned accordingly. We also consider imperfect CSIT. In time division duplexing (TDD) systems, CSIT can be obtained by having users periodically sending uplink pilots to the base station to estimate the downlink channel. The channel uncertainty is mainly due to channel estimation errors and channel variations between uplink and downlink transmission. One reasonable performance measure with imperfect CSIT is the sum rate averaged over the channel uncertainty. When the distribution of channel uncertainty is known, the stochastic WMMSE [5] can be used. We consider the case where the distribution is not known, and use a simple approximation that uses only the second order statistics of the channel uncertainty.

We will propose a modified WMMSE algorithm that handles the aforementioned aspects. Our interest also lies in testing the proposed algorithm under realistic environments, thereby paving the way for real-world implementation. We build a TDD LTE simulation platform based on the LTE physical layer specifications [6]–[8] to evaluate the performance of the proposed algorithm. Instead of using the information-theoretic achievable rates (those under the Gaussian codebook assumption) to measure the performances, the built platform

978-1-4799-1931-4/15/$31.00 ©2015 IEEE

faithfully follows the physical layer procedure to evaluate the system performances: all users periodically transmit uplink pilots for the base station to estimate the downlink channels via channel reciprocity. The base station encodes all data streams by Turbo codes and beamforms the coded data to the intended users via MIMO-OFDM. Each user then performs downlink channel estimation, MMSE demodulation, and Turbo decoding. The resulting sum throughput — that is, the total number of correctly detected data bits over physical layer — is used as the performance metric. We examine the sum throughput of the proposed algorithm under various settings, including different number of users, uplink pilot periods and signal to noise ratio (SNR). Simulation results show that the proposed algorithm can outperform the conventional WMMSE method.

II. System Model and Background

We consider a frequency-flat multiuser MIMO downlink model. The transmitting signal at the base station is

$$\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{V}_k \boldsymbol{s}_k, \tag{1}$$

where $\boldsymbol{s}_k \in \mathbb{C}^{D_k}$ is a multi-stream data vector for user k, $\boldsymbol{V}_k = [\boldsymbol{v}_{k,1}, \ldots, \boldsymbol{v}_{k,D_k}] \in \mathbb{C}^{N \times D_k}$ is the corresponding beamforming matrix, and K is the number of users. Here, D_k is the number of data streams for user k, N is the number of transmitting antennas of the base station, and each element of s_k is assumed to have zero mean and unit variance.

The received signal at user k is given by

$$\boldsymbol{y}_k = \boldsymbol{H}_k \left(\sum_{k'=1}^{K} \boldsymbol{V}_{k'} \boldsymbol{s}_{k'} \right) + \boldsymbol{\nu}_k, \tag{2}$$

where $\boldsymbol{H}_k \in \mathbb{C}^{M_k \times N}$ is the channel at user k, and $\boldsymbol{\nu}_k \in \mathbb{C}^{M_k}$ the noise. Here, M_k is the number of receive antennas of user k, and the noise vector $\boldsymbol{\nu}_k$ is i.i.d. circular complex Gaussian distributed with mean zero and variance σ_k^2.

In the literature, it is common to model users' data rates by the following achievable rate formula

$$R_k = \log\det\left(\boldsymbol{I} + \boldsymbol{V}_k^H \boldsymbol{H}_k^H (\boldsymbol{J}_k - \boldsymbol{H}_k \boldsymbol{V}_k \boldsymbol{V}_k^H \boldsymbol{H}_k)^{-1} \boldsymbol{H}_k \boldsymbol{V}_k\right) \tag{3}$$

where R_k denotes the rate of user k, and

$$\boldsymbol{J}_k = \sum_{k'} \boldsymbol{H}_k \boldsymbol{V}_{k'} \boldsymbol{V}_{k'}^H \boldsymbol{H}_k^H + \sigma_k^2 \boldsymbol{I} \tag{4}$$

is the covariance matrix of received signal of user k. For example, the original WMMSE method [2], [4] for sum rate maximization adopts the above rate model. While the rate expression in (3) is popularly used, upon a closer look, it assumes that successive interference cancellation (SIC) receivers are used [9]. In mobile communications, users are usually power limited and cannot afford this kind of expensive SIC receivers. One practical solution is to use linear receivers.

Another practical consideration is that the channel state information at the transmitter (CSIT) is not perfect. Following a conventional way of modeling imperfect CSIT, we write

$$\boldsymbol{H}_k = \bar{\boldsymbol{H}}_k + \boldsymbol{\Delta}_k, \tag{5}$$

where $\bar{\boldsymbol{H}}_k$ is the channel mean and $\boldsymbol{\Delta}_k$ represents channel uncertainty. We assume that the base station knows $\bar{\boldsymbol{H}}_k$, and $\boldsymbol{\Delta}_k$ has zero mean and covariance matrix $\boldsymbol{\Theta}_k = \mathbb{E}\{\text{vec}(\boldsymbol{\Delta}_k)\text{vec}(\boldsymbol{\Delta}_k)^H\}$.

III. WMMSE for Linear Receivers and Imperfect CSIT

In this section, we follow the idea of WMMSE [3], [4] to optimize the sum rate for linear receivers and imperfect CSIT. As mentioned previously, our design philosophy is more from a practical implementation viewpoint, where simple and efficient algorithms are what we desire.

A. Rate Expression for Linear Receivers and Imperfect CSIT

If user k is equipped with a linear receiver, then the achievable rate of user k should be modified as

$$R_k = \sum_{d_k=1}^{D_k} R_{k,d_k}, \tag{6}$$

where R_{k,d_k} is the achievable rate of stream d_k of user k and is given by

$$R_{k,d_k} = \log(1 + \boldsymbol{v}_{k,d_k}^H \boldsymbol{H}_k^H (\boldsymbol{J}_k - \boldsymbol{H}_k \boldsymbol{v}_{k,d_k} \boldsymbol{v}_{k,d_k}^H \boldsymbol{H}_k^H)^{-1} \boldsymbol{H}_k \boldsymbol{v}_{k,d_k}). \tag{7}$$

With imperfect CSIT, one reasonable performance measure is the average data rate with respect to the channel:

$$\bar{R}_{k,d_k} = \mathbb{E}_{\boldsymbol{H}_k}\{R_{k,d_k}\}. \tag{8}$$

However, (8) does not admit a simple explicit expectation, due mainly to the logarithm and the inverse in (7). One method is to use stochastic programming [5] to avoid computing $\bar{R}_{k,d_k}$ explicitly, although it still requires knowledge of the distribution of $\boldsymbol{\Delta}_k$. Moreover, from a practical implementation viewpoint, simple remedies, even in the form of heuristic, would be preferred. Hence, we adopt the following approximation:

$$\begin{aligned}\hat{R}_{k,d_k} = \log(1 + &\boldsymbol{v}_{k,d_k}^H \bar{\boldsymbol{H}}_k^H \times \\ &(\mathbb{E}\{\boldsymbol{J}_k - \boldsymbol{H}_k \boldsymbol{v}_{k,d_k} \boldsymbol{v}_{k,d_k}^H \boldsymbol{H}_k^H\})^{-1} \bar{\boldsymbol{H}}_k \boldsymbol{v}_{k,d_k}),\end{aligned} \tag{9}$$

which can be shown to be

$$\hat{R}_{k,d_k} = \log(1 + \boldsymbol{v}_{k,d_k}^H \bar{\boldsymbol{H}}_k^H (\bar{\boldsymbol{J}}_k - \bar{\boldsymbol{H}}_k \boldsymbol{v}_{k,d_k} \boldsymbol{v}_{k,d_k}^H \bar{\boldsymbol{H}}_k^H)^{-1} \bar{\boldsymbol{H}}_k \boldsymbol{v}_{k,d_k}) \tag{10}$$

where

$$\begin{aligned}\bar{\boldsymbol{J}}_k = \sum_{k', d_{k'}} &(\boldsymbol{v}_{k',d_{k'}}^* \otimes \boldsymbol{I})^H \boldsymbol{\Theta}_k (\boldsymbol{v}_{k',d_{k'}}^* \otimes \boldsymbol{I}) \\ &+ \bar{\boldsymbol{H}}_k \boldsymbol{v}_{k',d_{k'}} \boldsymbol{v}_{k',d_{k'}}^H \bar{\boldsymbol{H}}_k^H + \sigma_k^2 \boldsymbol{I}.\end{aligned} \tag{11}$$

It can be seen that $\hat{R}_{k,d_k}$ only depends on $\bar{\boldsymbol{H}}_k$ and $\boldsymbol{\Theta}_k$. Therefore, $\hat{R}_{k,d_k}$ can be computed easily if both $\bar{\boldsymbol{H}}_k$ and $\boldsymbol{\Theta}_k$ are known.

B. WMMSE Formulation

Having derived approximate rate expressions for linear receivers and under imperfect CSIT (cf. (10)), we turn our attention to the beamformer design. The problem is to maximize the average sum rate (more precisely, $\sum_{k,d_k} \hat{R}_{k,d_k}$, where $\hat{R}_{k,d_k}$ is shown in (10)), subject to a total power constraint $\sum_{k,d_k} \|\boldsymbol{v}_{k,d_k}\|_2^2 \leq P_T$, where P_T is the total available power. We use WMMSE to deal with the problem.

The main idea of WMMSE [3], [4] is to use the following two identities to turn the rate expression into a form that can be easily optimized.

Fact 1. For any positive definite $\boldsymbol{E} \in \mathbb{C}^{P\times P}$, we have

$$\log\det \boldsymbol{E} = \min_{\boldsymbol{W}\succeq \boldsymbol{0}} \text{tr}\boldsymbol{E}\boldsymbol{W} - \log\det \boldsymbol{W} + \text{constant} \tag{12}$$

where $\boldsymbol{W} \succeq \boldsymbol{0}$ means positive semidefinite.

Fact 2. For any positive definite $\boldsymbol{J} \in \mathbb{C}^{P\times P}$, $\boldsymbol{H} \in \mathbb{C}^{P\times Q}$, and $\boldsymbol{V} \in \mathbb{C}^{Q\times T}$, we have

$$-\text{tr}(\boldsymbol{V}^H\boldsymbol{H}^H\boldsymbol{J}^{-1}\boldsymbol{H}\boldsymbol{V}) = \min_{\boldsymbol{U}} \text{tr}(-2\Re\boldsymbol{U}^H\boldsymbol{H}\boldsymbol{V} + \boldsymbol{U}^H\boldsymbol{J}\boldsymbol{U}), \tag{13}$$

where $\boldsymbol{U} \in \mathbb{C}^{P\times T}$.

Firstly, let us use the matrix inversion lemma to rewrite $\hat{R}_{k,d_k}$ in (10) as follows

$$\hat{R}_{k,d_k} = -\log(1 - \boldsymbol{v}_{k,d_k}^H\bar{\boldsymbol{H}}_k^H\bar{\boldsymbol{J}}_k^{-1}\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k}). \tag{14}$$

Then, by (12) and introducing a variable $w_{k,d_k} \geq 0$, $\hat{R}_{k,d_k}$ (up to a constant) is equal to

$$\hat{R}_{k,d_k} = -\min_{w_{k,d_k}\geq 0} w_{k,d_k}(1-\boldsymbol{v}_{k,d_k}^H\bar{\boldsymbol{H}}_k^H\bar{\boldsymbol{J}}_k^{-1}\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k}) - \log w_{k,d_k}$$

Further, using (13) and introducing a variable $\boldsymbol{u}_{k,d_k} \in \mathbb{C}^{M_k}$ lead to the following equivalent form

$$\hat{R}_{k,d_k} = -\min \left\{ \begin{array}{r} w_{k,d_k}(1 - 2\Re\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k} + \boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{J}}_k\boldsymbol{u}_{k,d_k}) \\ -\log w_{k,d_k} \end{array} \right\}$$
$$\text{s.t. } w_{k,d_k} \geq 0, \boldsymbol{u}_{k,d_k} \in \mathbb{C}^{M_k}.$$

Therefore, the average sum rate maximization problem is equivalent to

$$\begin{aligned} \min \sum_{k,d_k} & \left\{ \begin{array}{r} w_{k,d_k}(1 - 2\Re\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k} + \boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{J}}_k\boldsymbol{u}_{k,d_k}) \\ -\log w_{k,d_k} \end{array} \right\} \\ \text{s.t. } & \sum_{k,d_k} \|\boldsymbol{v}_{k,d_k}\|_2^2 \leq P_T, \quad w_{k,d_k} \geq 0, \\ & \boldsymbol{v}_{k,d_k} \in \mathbb{C}^N, \boldsymbol{u}_{k,d_k} \in \mathbb{C}^{M_k}, w_{k,d_k} \in \mathbb{R} \text{ for all } k \text{ and } d_k, \end{aligned} \tag{15}$$

where $\bar{\boldsymbol{J}}_k$ is given in (11). The advantage of (15) is that (15) is convex in anyone of the three sets of variables $\{w_{k,d_k}\}, \{\boldsymbol{u}_{k,d_k}\}, \{\boldsymbol{v}_{k,d_k}\}$ when the other two are fixed. Therefore, (15) is particularly suitable for BCD optimization.

C. BCD Update

If BCD is directly applied to problem (15), however, the update of $\{\boldsymbol{v}_{k,d_k}\}$ is to solve a quadratic program with a quadratic constraint. As shown in [3], this quadratic program can be solved by a water-filling algorithm which involves a singular value decomposition (SVD) and a bisection procedure. From a real-time implementation viewpoint, calling SVD for each iteration may not be efficient. This motivates us to find an alternative that avoids SVD. Consider the following optimization problem

$$\begin{aligned} \min \sum_{k,d_k} & \left\{ \begin{array}{r} w_{k,d_k}(1 - 2\Re\beta\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k} + \beta^2\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{J}}_k\boldsymbol{u}_{k,d_k}) \\ -\log w_{k,d_k} \end{array} \right\} \\ \text{s.t. } & \sum_{k,d_k} \|\boldsymbol{v}_{k,d_k}\|_2^2 \leq P_T, w_{k,d_k} \geq 0, w_{k,d_k} \in \mathbb{R}, \\ & \boldsymbol{v}_{k,d_k} \in \mathbb{C}^N, \boldsymbol{u}_{k,d_k} \in \mathbb{C}^{M_k}, \text{ for all } k \text{ and } d_k, \\ & \beta \in \mathbb{R}, \end{aligned} \tag{16}$$

where we introduce an additional variable β. It is easy to see that (15) and (16) are equivalent.

The updates of $\{\boldsymbol{u}_{k,d_k}\}$ and $\{w_{k,d_k}\}$ can be easily derived by equating the partial derivatives to zeros; they are given by

$$\boldsymbol{u}_{k,d_k} = \beta^{-1}\boldsymbol{J}_k^{-1}\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k}, \tag{17}$$

$$w_{k,d_k} = (1 - \beta\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k})^{-1}. \tag{18}$$

The simultaneous update of $\{\boldsymbol{v}_{k,d_k}\}$ and β is to solve the problem

$$\begin{aligned} \min\ & \beta^2\eta - \sum_{k,d_k} 2w_{k,d_k}\beta\Re\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k\boldsymbol{v}_{k,d_k} + \beta^2\boldsymbol{v}_{k,d_k}^H\bar{\boldsymbol{J}}\boldsymbol{v}_{k,d_k} \\ \text{s.t. } & \sum_{k,d_k} \|\boldsymbol{v}_{k,d_k}\|_2^2 \leq P_T, \\ & \beta \in \mathbb{R}, \ \boldsymbol{v}_{k,d_k} \in \mathbb{C}^N \text{ for all } k \text{ and } d_k, \end{aligned} \tag{19}$$

where $\bar{\boldsymbol{J}} = \sum_{k,d_k} w_{k,d_k}((\boldsymbol{I}\otimes\boldsymbol{u}_{k,d_k})^T\boldsymbol{\Theta}_k^T(\boldsymbol{I}\otimes\boldsymbol{u}_{k,d_k})^* + \bar{\boldsymbol{H}}_k^H\boldsymbol{u}_{k,d_k}\boldsymbol{u}_{k,d_k}^H\bar{\boldsymbol{H}}_k)$ and $\eta = \sum_{k,d_k} w_{k,d_k}\sigma_k^2\|\boldsymbol{u}_{k,d_k}\|_2^2$. We invoke the following lemma.

Lemma 1 ([10, Appendix D.1]). *Consider the optimization problem*

$$\begin{aligned} \min\ & \beta^2\eta - 2\beta \textit{tr}\Re\boldsymbol{F}^H\boldsymbol{V} + \beta^2\textit{tr}\boldsymbol{V}^H\boldsymbol{J}\boldsymbol{V} \\ \textit{s.t.}\ & \textit{tr}\boldsymbol{V}\boldsymbol{V}^H \leq P_T, \ \beta \in \mathbb{R}, \end{aligned}$$

where $\eta \geq 0$, $\boldsymbol{J}$ is positive definite, and $\boldsymbol{F}$ is nonzero. The optimal solution of $\boldsymbol{V}$ is

$$\boldsymbol{V} = \beta^{-1}\left(\boldsymbol{J} + \frac{\eta}{P_T}\boldsymbol{I}\right)^{-1}\boldsymbol{F} \tag{20}$$

with β set to satisfy the constraint with equality.

Note that a similar result was used in [4]. By Lemma 1, the optimal solution of (19) is given by

$$\boldsymbol{v}_{k,d_k} = \beta^{-1}\left(\bar{\boldsymbol{J}} + \frac{\eta}{P_T}\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{H}}_k^H\boldsymbol{u}_{k,d_k}w_{k,d_k}, \tag{21}$$

and β is chosen to satisfy the power constraint with equality. Therefore, the BCD optimization of WMMSE formulation for the average sum rate maximization problem can be easily carried out, with each update having a closed-form solution.

IV. Simulation Results under an LTE Simulation Platform

In this section, we will demonstrate the performances of the proposed algorithm.

A. LTE Simulation Platform

A TDD-LTE simulation platform is built based on the physical layer specification [6]–[8] of the LTE standard. We briefly introduce the simulation platform and settings. As shown in Fig. 1, in LTE a radio frame of length 10ms consists of 10 subframes of length 1ms each. Each subframe is a uplink subframe, downlink subframe, or special subframe, which are represented by U, D and S in the figure. In uplink subframes, users transmit periodically uplink pilots (sounding reference signal (SRS)) for the base station to estimate the downlink channel. In each downlink subframe, the base station independently encodes each data stream of each user by a Turbo code and then modulation is applied. The Turbo code rate and constellation are determined by the outer loop link adaption (OLLA) algorithm [11] so that it achieves a block error rate specification of 0.1. Data symbols of each data stream, together with the corresponding downlink pilot (UE-specific reference signal, UE-RS), are allocated to a resource grid of 72 subcarriers in the frequency domain and 14 OFDM symbols in the time domain. The same beamformer is used for consecutive 12 subcarriers and all OFDM symbols within one subframe. All resource grids are then beamformed and transmitted. The channel model is 3GPP SCME [12], [13]. Each user performs channel estimation by the UE-RS, demodulation by MMSE detection, and channel decoding. The moving speeds of all users are 1m/s. The channel uncertainty $\mathbf{\Theta}_k$ is estimated by considering the uplink channel estimation error and the delay between uplink channel estimation and downlink transmission.

The performance metric is not the information-theoretic average sum rate $\sum_{k,d_k} \bar{R}_{k,d_k}$ (cf. (7) and (8)). We evaluate the system performance using the so-called *system throughput*, defined as follows.

$$\text{System throughput} = \sum_{i=1}^{N_{\text{subframe}}} \sum_{k=1}^{K} \sum_{d_k=1}^{D_K} B_{k,d_k,i} \mathbb{1}_{k,d_k,i}, \quad (22)$$

where $N_{\text{subframe}} = 1000$ is the number of subframes simulated, $B_{k,d_k,i}$ is the number of data bits of stream d_k of user k at subframe i, and $\mathbb{1}_{k,d_k,i} = 1$ if all bits of stream d_k of user k at subframe i are correctly detected and $\mathbb{1}_{k,d_k,i} = 0$ otherwise. Simply speaking, we use the total number of successfully transmitted data bits over a real physical-layer chain as our performance metric.

We will compare the proposed algorithm (denoted as "proposed") with the conventional WMMSE (denoted as "WMMSE") and the simple zero-forcing (ZF) algorithm.

B. Performance

Fig. 2 shows the system throughput under the following settings. The base station has $N = 8$ antennas. There are $K = 2$ users and each user has two antennas and receives two streams of data. Uplink pilot period is 10 subframes and SNR=10dB. We use this setting as a reference setting and will compare performances of other settings with this one. We can see from Fig. 2 that WMMSE is better than ZF beamforming at low and high downlink SNRs, but may be worse than ZF at medium downlink SNRs. The proposed algorithm can outperform WMMSE by $0.5 \sim 1$ Mbits.

We consider settings where the CSIT is more inaccurate. In Fig. 3, we change the uplink SNR to 0dB. At medium to high downlink SNRs (≥ 10dB), the effect of inaccurate CSIT starts to kick in and the throughputs of all beamformers at 0dB uplink SNR are less than those at 10dB uplink SNR. This effect is more significant at high downlink SNRs; for example, the throughput losses are around 30% at downlink SNR=35dB. WMMSE is very close to ZF and the proposed algorithm still achieves a gain of $0.5 \sim 1$ Mbits over WMMSE. In Fig. 4, we change the uplink pilot period to 40 subframes and the uplink SNR is 10dB, which leads to more inaccurate CSIT. The throughputs of all the beamformers are seen to further drop. The gap between the proposed algorithm and WMMSE also shrinks.

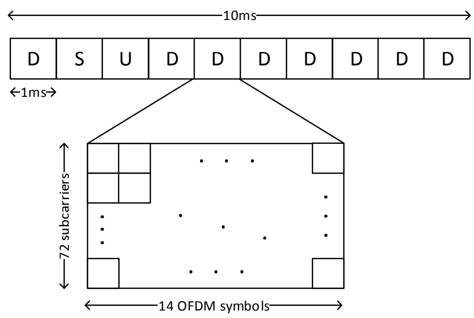

Fig. 1. Frame Structure in LTE

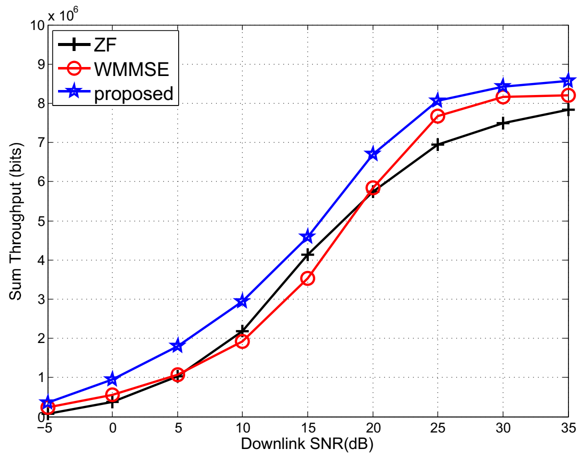

Fig. 2. Sum Throughput in LTE Simulations. $N = 8$, $K = 2$, $D_1 = D_2 = 2$, $M_1 = M_2 = 2$. Uplink pilot period = 10 subframes and SNR=10dB.

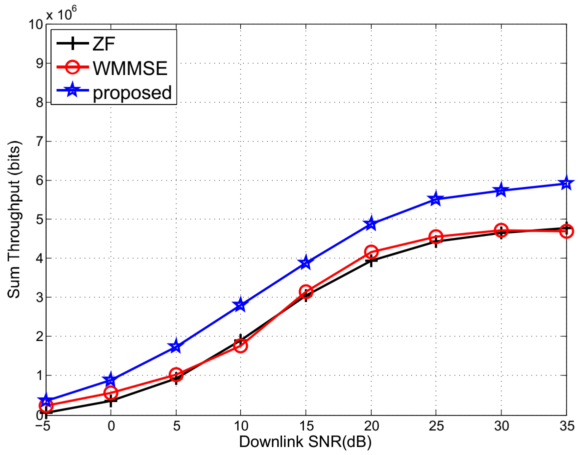

Fig. 3. Sum Throughput in LTE Simulations. $N = 8$, $K = 2$, $D_1 = D_2 = 2$, $M_1 = M_2 = 2$. Uplink pilot period = 10 subframes and SNR=0dB.

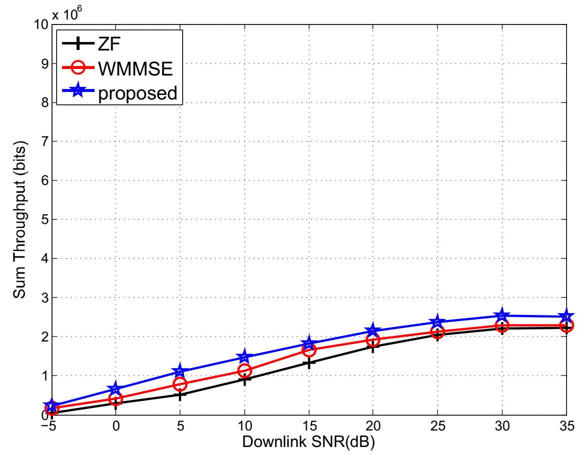

Fig. 4. Sum Throughput in LTE Simulations. $N = 8$, $K = 2$, $D_1 = D_2 = 2$, $M_1 = M_2 = 2$. Uplink pilot period = 40 subframes and SNR=10dB.

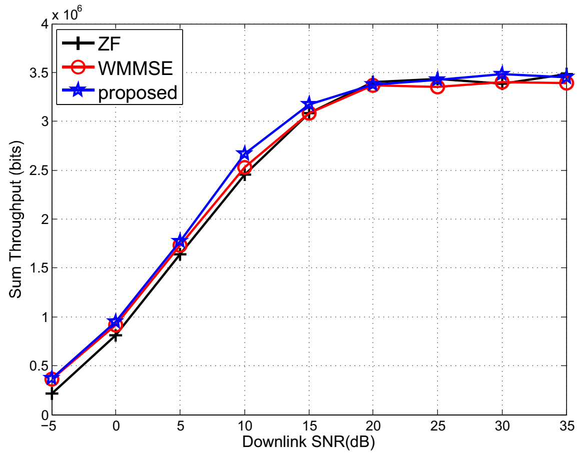

Fig. 5. Sum Throughput in LTE Simulations. $N = 8$, $K = 4$, $D_k = 1$, $M_k = 2$ for all k. Uplink pilot period = 10 subframes and SNR=10dB.

Fig. 5 shows the throughput of a four-user case where each user receives one data stream. Note the difference in scale between Fig. 5 and Fig. 2. The performance gain of the proposed algorithm is around 3% at some SNRs, though it is not as much as that in the two-user case. The throughputs of all beamformers are also much less than those in the two-user case. This may be caused by 1) the fact that the accuracy of CSIT does not improve as the downlink SNR increases; 2) that the downlink pilot UE-RS in LTE of a user maybe interfered by the downlink pilots and data of other users. When there are many users, the quality of downlink channel estimation could deteriorate seriously.

V. Conclusion

In this paper, we derived a WMMSE algorithm for linear receivers and under imperfect CSIT. The design principle follows a practice-oriented approach for paving the way for real implementations. Simulation results under an LTE simulation platform showed that the proposed algorithm achieves a higher system throughput than the conventional WMMSE.

References

[1] M. Codreanu, A. Tolli, M. Juntti, and M. Latva-aho, "MIMO downlink weighted sum rate maximization with power constraints per antenna groups," in *IEEE 65th Vehicular Technology Conference(VTC2007-Spring)*. IEEE, 2007, pp. 2048–2052.

[2] S. Shi, M. Schubert, and H. Boche, "Rate optimization for multiuser MIMO systems with linear processing," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4020–4030, 2008.

[3] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, 2011.

[4] S. S. Christensen, R. Agarwal, E. Carvalho, and J. M. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, 2008.

[5] H. Baligh, M. Hong, W.-C. Liao, Z.-Q. Luo, M. Razaviyayn, M. Sanjabi, and R. Sun, "Cross-layer provision of future cellular networks: A WMMSE-based approach," *IEEE Signal Process Mag.*, vol. 31, no. 6, pp. 56–68, Nov 2014.

[6] "3GPP TS 36.211, Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channel and Modulation," 2014.

[7] "3GPP TS 36.212, Evolved Universal Terrestrial Radio Access (E-UTRA); Multiplexing and channel coding ," 2014.

[8] "3GPP TS 36.213, Evolved Universal Terrestrial Radio Access (E-UTRA); Physical layer procedures," 2014.

[9] D. Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge university press, 2005.

[10] F. A. Dietrich, *Robust Signal Processing for Wireless Communications*, vol. 2 of *Foundations in Signal Processing, Communications and Networking*, Springer, 2008.

[11] A Müller and P. Frank, "Cooperative interference prediction for enhanced link adaptation in the 3GPP LTE uplink," in *IEEE 71st Vehicular Technology Conference (VTC 2010-Spring)*, 2010, pp. 1–5.

[12] D. S. Baum, J. Salo, M. Milojevic, P. Kyösti, and J. Hansen, "Matlab Implementation of the 3GPP Spatial Channel Model Extended (SCME)," 2006.

[13] D. S. Baum, J. Hansen, and J. Salo, "An interim channel model for beyond-3G systems: extending the 3GPP spatial channel model (SCM)," in *IEEE 61st Vehicular Technology Conference (VTC 2005-Spring)*, May 2005, vol. 5, pp. 3132–3136 Vol. 5.