ELEG 2310B: Principles of Communication Systems **2021-22 First Term**

Handout 6: Frequency Translation

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Suggested Reading: Chapters 3 and 4.6 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wily & Sons Ltd; or Chapter 3.7 of R. E. Ziemer and W. H. Tranter, *Principles of Communications: System Modulation and Noise (5th Edition), John Wiley and Sons, Inc.*

From a Fourier transform viewpoint, the processes in amplitude modulation schemes can be viewed as those of frequency-translating a certain signal to a desired frequency. This handout describes some more aspects that are related to the concept of frequency translation.

1 Frequency Division Multiplexing

Multiplexing is generally referred to as a technique for transmitting a multitude of message signals over the same channel. In *frequency division multiplexing (FDM)*, a frequency band is divided into a number of frequency subbands, and we assign each message signal to occupy one subband. Figure 1 shows a generic system diagram for FDM. At the transmitter, each message signal is modulated to its assigned subband by a suitably chosen carrier frequency. Figure 2 illustrates the spectrum of an FDM transmitted signal. To avoid interference between subbands, the widths of the subbands must be chosen such that the modulated signals are non-overlapping in frequency. At the receiver, the modulated signals of the various message signals must first be separated from one another. This can easily be done by applying an appropriate bandpass filter for each subband (at least in principle). Then, a standard demodulator can be applied to recover the message signal for each subband.

The key concepts we wish to emphasize here are i) frequency translation of the various message signals to spectrally non-overlapping subbands, and ii) the idea of dividing a channel into independent resource blocks in frequency domain. In that regard, the specific modulation scheme used within each subband is not as crucial, as far as the modulation scheme modulates the message signal to the assigned subband properly and the bandwidth of the resulting modulated signal is no greater than the subband bandwidth. In fact, in principle there is flexibility for each subband to employ a different modulation scheme.

FDM has been used in early long-haul voice telephone systems (obsolete today), where a number of modulated voice signals are multiplexed and transmitted over a trunk line[1]. The principle of FDM has also been used in 1G mobile systems under the name of *frequency division multiple access (FDMA)*, where mobile users share the same channel via the same frequency division concept as discussed above.

[1] A specification is as follows. The transmission line has a transmission band of 60 kHz to 108 kHz. SSB is used to modulate voice signals of bandwidth 4 kHz. A simple calculation reveals that we can multiplex at most 12 voice signals over the line.

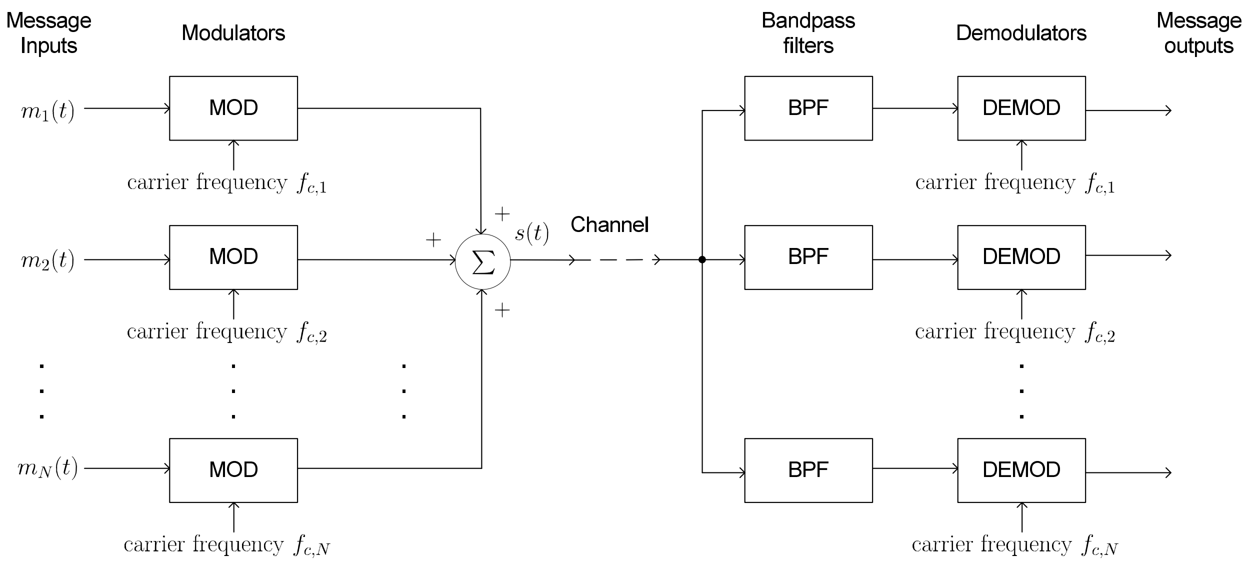

Figure 1: Block diagram of an FDM system.

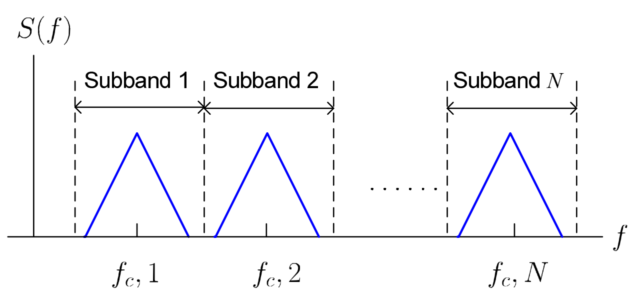

Figure 2: Spectrum of the FDM signal.

2 Frequency Translation

Consider the following problem: Given a bandpass signal $s_1(t)$ whose center frequency is f_1, transform $s_1(t)$ such that the transformed signal, denoted by $s_2(t)$, has its center frequency at f_2. This process is called a *frequency translation* process, and is a generalized form of some modulation and demodulation processes we encountered previously. For example, in DSB-SC modulation, we have a message signal $s_1(t) = m(t)$ and we wish to frequency translate $s_1(t)$ to the carrier frequency f_c so that a modulated wave $s_2(t) = m(t)\cos(2\pi f_c t)$ can be obtained. Also, in DSB-SC demodulation, we have a modulated wave $s_1(t) = m(t)\cos(2\pi f_c t)$ and our task is to shift the center frequency down to 0 Hz such that $s_2(t) = m(t)$.

Frequency translation can be accomplished by the mixer system shown in Figure 3. The idea is similar to that of the modulation and demodulation processes for DSB-SC. The key issue here is to choose the frequency of the local oscillator, denoted by f_ℓ, and design the bandpass filter. To

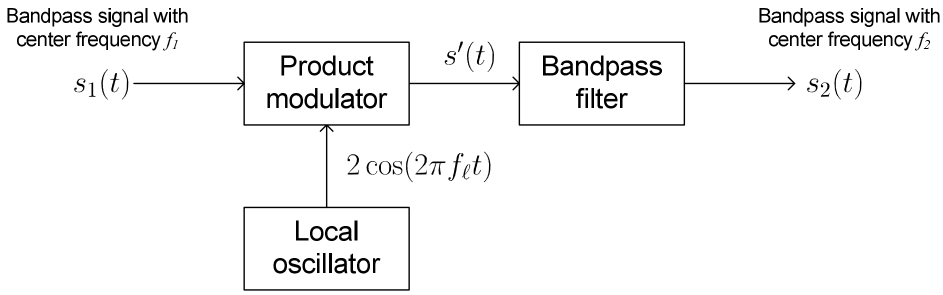

Figure 3: Block diagram of a mixer.

simplify our study, we will concentrate on the special case of DSB-SC modulated signals, i.e.,

$$s_1(t) = m(t)\cos(2\pi f_1 t),$$

although we should note that the same idea is applicable to general bandpass signals (e.g., QAM). The product modulator output of the mixer is

$$\begin{aligned} s'(t) &= s_1(t)[2\cos(2\pi f_\ell t)] \\ &= 2m(t)\cos(2\pi f_1 t)\cos(2\pi f_\ell t) \\ &= m(t)\cos(2\pi(f_1 + f_\ell)t) + m(t)\cos(2\pi(f_1 - f_\ell)t). \end{aligned} \tag{1}$$

There are two cases to consider. The first case is when the target center frequency f_2 is greater than the original center frequency f_1, i.e., $f_2 \geq f_1$. In this case, we choose f_ℓ such that $f_2 = f_1 + f_\ell$. The result is simply

$$f_\ell = f_2 - f_1.$$

By designing the bandpass filter such that the term $m(t)\cos(2\pi(f_1 + f_\ell)t)$ in (1) is kept and the term $m(t)\cos(2\pi(f_1 - f_\ell)t)$ in (1) is rejected, we obtain the desired result $s_2(t) = m(t)\cos(2\pi f_2 t)$ at the bandpass filter output (Note that we have made a subtle assumption, namely, that $f_\ell \geq W$ where W is the message signal bandwidth. Draw the spectrum of $s'(t)$ and you will see why).

The second case is when $f_1 \geq f_2$. The idea is the same as above. We choose f_ℓ such that $f_2 = f_1 - f_\ell$, or simply

$$f_\ell = f_1 - f_2.$$

Then, we design the bandpass filter such that the term $m(t)\cos(2\pi(f_1 + f_\ell)t)$ in (1) is rejected and the term $m(t)\cos(2\pi(f_1 - f_\ell)t)$ in (1) is kept.

Frequency translation is important in enabling any desired frequency up-conversion and down-conversion of a signal. Particularly, practical modulation and demodulation systems may have more than one frequency up-conversion and down-conversion stages, owing to practical implementation constraints. One well-known example is the *superheterodyne receiver* architecture, to be described next.

3 The Superheterodyne Receiver

The superheterodyne receiver is a special type of receiver that is quite widely used in communication systems, especially, in broadcast radio receivers. In DSB-SC, for example, coherent demodulation is nothing more than a one-stage frequency translation process—but only in theory. The superheterodyne receiver, shown in Figure 4, has two stages. In particular, we perform frequency down-conversion of the received modulated signal to a predetermined *intermediate frequency* (IF), and then demodulation is carried over the IF band. The reason of doing so has several practical factors taken into consideration. Simply speaking, a receiver is generally supposed to be tunable with the carrier frequency f_c. It may be difficult to build a sharp bandpass filter for a very high and tunable center frequency. The superheterodyne receiver overcomes this difficulty by down-converting the carrier frequency to the IF. Specifically, let f_c and f_{IF} denote the carrier frequency and the IF, respectively. The IF f_{IF} is chosen to be low, and is fixed irrespective of any change of f_c. A mixer with a local oscillator frequency f_{LO} is used to carry out the frequency down-conversion. By the frequency translation concepts discussed in the last section, the value of f_{LO} should be chosen as

$$f_{\mathrm{LO}} = f_c - f_{\mathrm{IF}}.$$

Note that f_{LO} varies with the desired f_c; in other words, f_{LO} is tunable.[2] Since the IF is low and always fixed, good amplification and bandpass filtering (in terms of high selectivity) can be performed over the IF band. Note that the RF section still performs bandpass filtering (in a tunable and less selective manner), although the idea is to let the IF section do the main task.

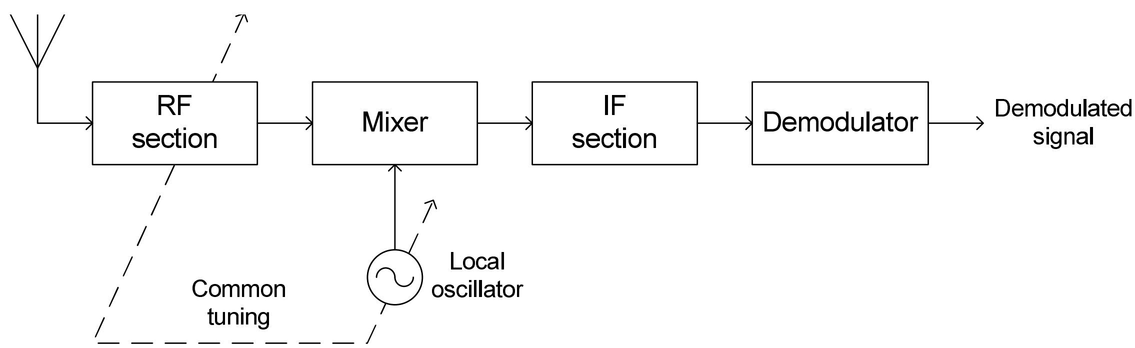

Figure 4: Block diagram of a superheterodyne receiver.

The superheterodyne receiver architecture is elegant and practical. However, the two-stage architecture also results in the so-called *image interference* effects. This issue may be discussed in class.

[2]As an additional note, there are practical receivers that would use a local oscillator frequency different from the abovementioned, namely, $f_{\mathrm{LO}} = f_c - f_{\mathrm{IF}}$. For example, receivers in AM broadcast may use $f_{\mathrm{LO}} = f_c + f_{\mathrm{IF}}$ for some rather practical circuitry reasons. It can be proven that the choice $f_{\mathrm{LO}} = f_c + f_{\mathrm{IF}}$ works at least for AM signals.