ELEG 2310B: Principles of Communication Systems **2021-22 First Term**

Handout 5: A Wider Class of AM Schemes

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Suggested Reading: Chapter 3 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wily & Sons Ltd; or Chapter 4 of B. P. Lathi and Z. Ding, *Modern Digital and Analog Communication Systems (4th Edition)*, Oxford University Press.

We consider several modifications of the previously studied AM scheme, namely, double sideband-suppressed carrier modulation, single sideband modulation and quadrature amplitude modulation. While the AM scheme is rarely seen in modern communication, we still see some AM concepts, particularly quadrature amplitude modulation, being used—that includes advanced digital communication systems.

1 Double Sideband-Suppressed Carrier Modulation

Recall that $m(t)$ denotes the message signal, and $c(t) = A_c \cos(2\pi f_c t)$ denotes the sinusoidal carrier wave. Also recall that $m(t)$ is assumed to be a baseband signal with bandwidth W Hz. In double sideband-suppressed carrier (DSB-SC) modulation, the modulated wave is given by

$$s(t) = m(t) \cdot c(t) \tag{1}$$

$$= A_c m(t) \cos(2\pi f_c t). \tag{2}$$

The difference between AM and DSB-SC modulation is that the DSB-SC modulated wave does not have the pure carrier component. Consequently, one hundred percent of the transmission power is spent on sending the message signal. The Fourier transform of the DSB-SC modulated signal $s(t)$ is simply

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]. \tag{3}$$

Figure 1 illustrates the corresponding amplitude spectrum. As can be seen in the figure, the transmission bandwidth of DSB-SC modulation is $2W$ Hz—the same as the AM transmission bandwidth.

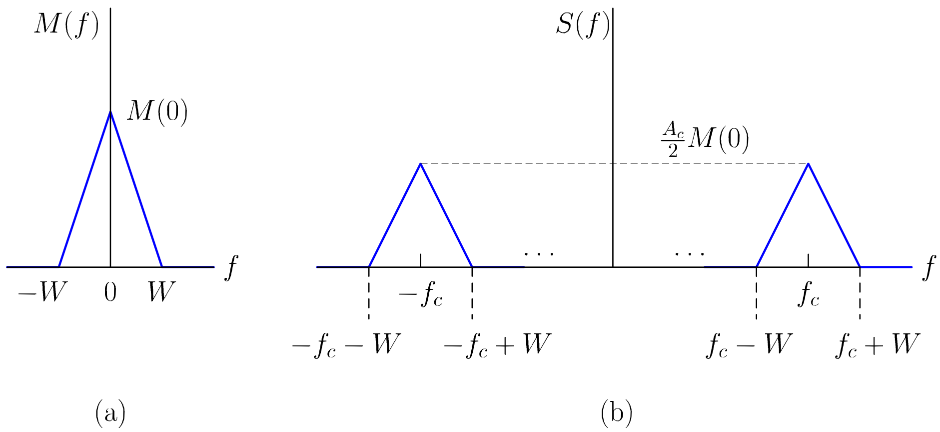

Figure 1: (a) Spectrum of the message signal. (b) Spectrum of the corresponding DSB-SC modulated signal.

The modulation process of DSB-SC modulation is similar to that of AM. Figure 2 shows a DSB-SC modulation process via the product modulator.

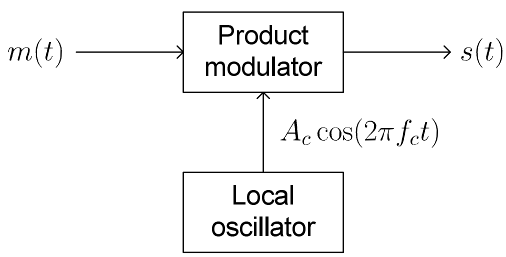

Figure 2: A modulator system diagram for DSB-SC modulation.

Let us consider the demodulation of the DSB-SC modulation scheme. The envelope extraction approach used in AM does not work in DSB-SC modulation. Another technique, called *coherent detection* or *synchronous demodulation*, can be used to uniquely recover the message signal. To understand the idea, suppose that the receiver is able to locally generate the sinusoidal carrier wave, denoted herein by $c'(t) = A_c' \cos(2\pi f_c t)$. Consider the following multiplication process

$$\begin{aligned} v(t) &= s(t)c'(t) \\ &= A_c A_c' m(t) \left[\cos(2\pi f_c t)\right]^2 \\ &= A_c A_c' m(t) \left\{ \frac{1}{2}[1 + \cos(4\pi f_c t)] \right\}. \\ &= \frac{A_c A_c'}{2} m(t) + \frac{A_c A_c'}{2} m(t) \cos(4\pi f_c t). \end{aligned} \tag{4}$$

We examine the spectral content of $v(t)$ by considering its Fourier transform

$$V(f) = \frac{A_c A_c'}{2} \left\{ M(f) + \frac{1}{2} \left[M(f + 2f_c) + M(f - 2f_c) \right] \right\}. \tag{5}$$

Figure 3 illustrates $V(f)$. Our observation is that by applying an ideal lowpass filter with bandwidth W Hz on $v(t)$, we can remove the term $[M(f + 2f_c) + M(f - 2f_c)]$ in (5) and keep the term $M(f)$ at the same time. Consequently, the message signal can be recovered.

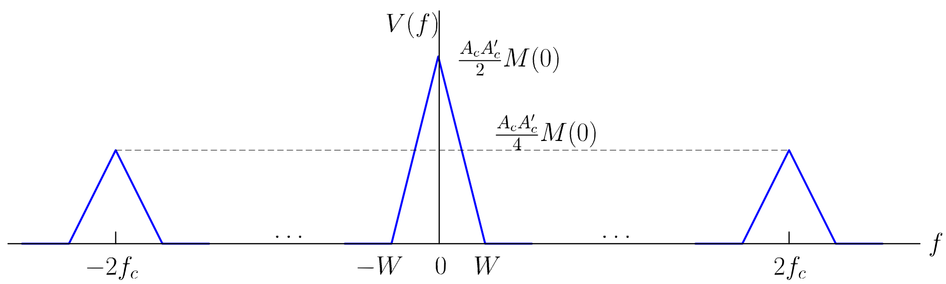

Figure 3: Spectrum of $v(t)$.

The process described above is an ideal case of coherent detection, where we assume that the receiver is able to perfectly generate the same carrier wave. In practice, the receiver can employ a local oscillator to generate the carrier locally. However, there may be some phase difference between the transmitter's and receiver's locally generated carriers. Figure 4 shows a DSB-SC demodulation process via coherent detection, where we model the receiver's local oscillator output as a phase-shifted version of the transmitter's local oscillator output; viz.,

$$c'(t) = A_c' \cos(2\pi f_c t + \phi)$$

with ϕ being the phase error. The signal at the product modulator output can be written as

$$\begin{aligned} v(t) &= s(t) \cdot [A_c' \cos(2\pi f_c t + \phi)] \\ &= A_c A_c' m(t) \cos(2\pi f_c t + \phi) \cos(2\pi f_c t) \\ &= \frac{1}{2} A_c A_c' \cos(\phi) m(t) + \frac{1}{2} A_c A_c' m(t) \cos(4\pi f_c t + \phi). \end{aligned} \tag{6}$$

By the same idea as before, we can use a lowpass filter to remove the term $m(t)\cos(4\pi f_c t + \phi)$ from (6). Specifically, the signal at the lowpass filter output is given by

$$v_0(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t). \tag{7}$$

We see from (7) that the magnitude of $v_0(t)$ depends on $\cos(\phi)$. We receive the largest magnitude with $v_0(t)$ when $\phi = 0$, i.e., perfect phase synchronism between the transmitter and receiver. However, the magnitude of $v_0(t)$ decreases as $|\phi|$ increases from 0 to $\pi/2$. The most unfavorable situation is $\phi = \pm\frac{\pi}{2}$, in which we have $v_0(t) = 0$.

The demodulation process described above suggests that phase synchronism plays an essential role to coherent detection. There are a variety of practical reasons that cause the phase error. For

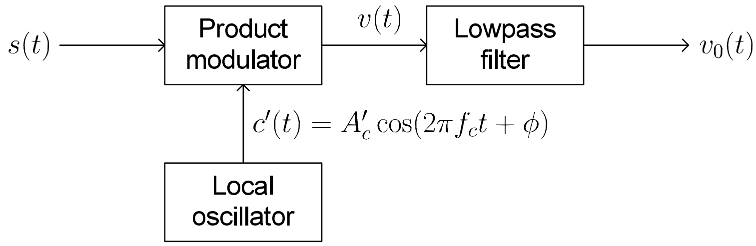

Figure 4: A demodulator system diagram for DSB-SC modulation.

example, it is hard to build a pair of analog local oscillator circuits that have perfect synchronism in timing[1]. Also, the channel itself may introduce phase shifts to the modulated signal at the receiver side[2]. On the other hand, there are techniques that can perform phase synchronization by estimating the phase difference ϕ from the incoming signal $s(t)$; this is particularly true for modern digital communication systems. Phase synchronization is beyond the scope of this course. Simply speaking (but also roughly speaking), we may assume that the phase synchronization problem can be compensated.

We should note that in AM, demodulation via envelope extraction is generally called a *noncoherent detection* approach. The reason is that envelope extraction does not require phase synchronism.

2 Single Sideband Modulation

Single sideband (SSB) modulation considers transmitting either the upper sideband or the lower sideband. The motivation behind is that there is a direct relationship between the upper and lower sidebands, and thus transmitting both the upper and lower sidebands would be a waste of the transmission bandwidth.

A powerful way to understand SSB modulated signals would be to use an analysis technique based on the *Hilbert transform*. Here, we employ a simpler approach where we use frequency domain representation to describe SSB modulation. Figure 5.(a) shows an SSB modulator system diagram. A bandpass filter is applied to remove either the upper sideband or the lower sideband. Now, suppose that we use the upper sideband to transmit. The bandpass filter should have its frequency response given by

$$H(f) = \begin{cases} 1, & f_c \leq f \leq f_c + W \text{ or } -f_c - W \leq f \leq -f_c \\ 0, & \text{otherwise} \end{cases}$$

Consequently, the bandpass filter output $s(t)$ has its Fourier transform taking the form in Figure 6. The signal $s(t)$ is used as the SSB-modulated signal. As can be seen in Figure 6, the SSB transmission bandwidth is W Hz.

[1]Sometimes there may even be frequency synchronism problems with circuits.

[2]For example, consider a channel that introduces some time delay owing to time required for RF wave propagation. The modulated wave picked up at the receiver may be modeled as $\tilde{s}(t) = s(t - \tau)$, where $\tau > 0$ is the time delay. Assuming that $m(t)$ changes slowly relative to τ such that $m(t-\tau) \simeq m(t)$, we have $\tilde{s}(t) = m(t-\tau)\cos(2\pi f_c(t-\tau)) \simeq m(t)\cos(2\pi f_c t + \phi)$ where $\phi = -2\pi f_c \tau$.

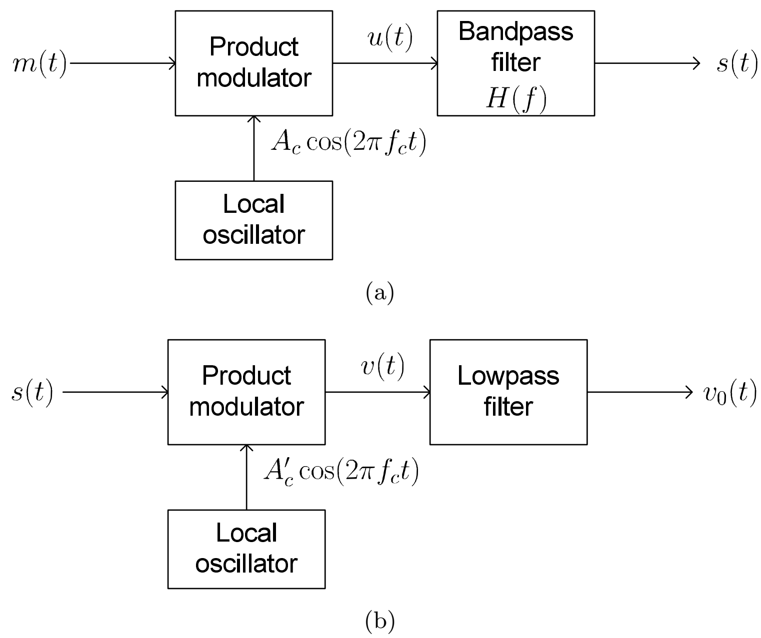

Figure 5: The modulation and demodulation process of SSB modulation. (a) Modulation. (b) Demodulation.

The SSB-modulated signals can be demodulated by the same way as in coherent detection in DSB-SC modulation. An SSB demodulator system diagram is shown in Figure 5.(b); it is the same as the DSB-SC coherent detector in Figure 4, and we assume perfect phase synchronism for simplicity. The coherent detection process can be understood by looking at the signals in frequency domain. Let

$$M_+(f) = \begin{cases} M(f), & f \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$M_-(f) = \begin{cases} 0, & f \geq 0 \\ M(f), & \text{otherwise} \end{cases}$$

Note that we may have $M_+(f) + M_-(f) = M(f)$[3]. The Fourier transform of the SSB-modulated signal $s(t)$ may be expressed as

$$S(f) = \frac{A_c}{2} \left[M_-(f + f_c) + M_+(f - f_c) \right].$$

[3]There is some subtlety with the condition $M_+(f) + M_-(f) = M(f)$. Simply speaking, if we assume that the message signal has zero DC component, i.e., $M(0) = 0$, then we do have $M_+(f) + M_-(f) = M(f)$. The zero DC assumption appears to be reasonable for audio message signals.

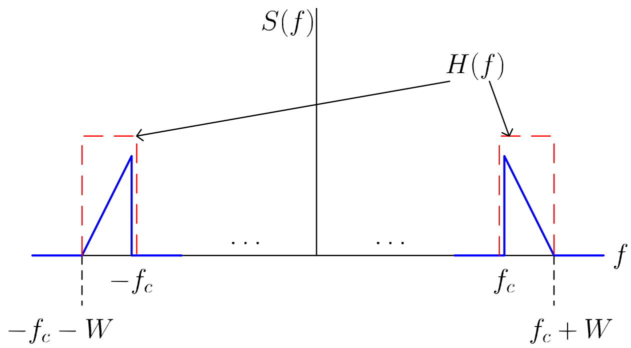

Figure 6: Spectrum of an SSB-modulated signal. The upper sideband is chosen for transmission.

It follows that the Fourier transform of the product modulator output $v(t)$ is

$$\begin{aligned} V(f) &= \frac{A_c'}{2}\left[S(f-f_c)+S(f+f_c)\right] \\ &= \frac{A_cA_c'}{4}\left\{[M_-(f)+M_+(f-2f_c)]+[M_-(f+2f_c)+M_+(f)]\right\} \\ &= \frac{A_cA_c'}{4}\left[M_-(f+2f_c)+M(f)+M_+(f-2f_c)\right] \end{aligned} \tag{8}$$

(You may want to sketch $V(f)$ to get a better understanding of what has happened). By applying a lowpass filter to retrieve only the term $M(f)$ from $V(f)$, we obtain $V_0(f) = \frac{A_cA_c'}{4}M(f)$, or in time domain form, $v_0(t) = \frac{A_cA_c'}{4}m(t)$.

It should be noted that in analog TV broadcast, the concept of SSB is employed to perform carrier modulation. More precisely, analog TV broadcast uses a scheme called *vestigal sideband modulation*, which is a more practical version of SSB.

3 Quadrature Amplitude Modulation

Quadrature amplitude modulation (QAM), also called quadrature-carrier multiplexing, is a scheme that transmits two independent message signals over the same carrier. Let $m_1(t)$ and $m_2(t)$ denote the two different message signals. The QAM signal is given by

$$s(t) = A_cm_1(t)\cos(2\pi f_ct) + A_cm_2(t)\sin(2\pi f_ct). \tag{9}$$

The modulation process may be viewed as a process of putting $A_cm_1(t)$ as the in-phase component of the bandpass signal $s(t)$, and $A_cm_2(t)$ as the quadrature-phase component. The transmission bandwidth of the QAM signal can easily be shown to be $2W$ Hz; note that we do so with *two* message signals, rather than one.

Figure 7.(a) shows a typical modulation system diagram for QAM. The process requires two product modulators that are supplied with two carrier waves of the same frequency but differing by -90 degrees in phase. Figure 7.(a) shows the demodulation process. Two separate coherent detectors are used to recover $m_1(t)$ and $m_2(t)$ respectively. By following the same steps as in the coherent detection process for DSB-SC modulation in Section 1, one can show that the two message

signals can be uniquely recovered. Detailed derivations may be revealed in class. The underlying assumption for the perfect recovery statement above is that there is no phase error and that the $-90°$ phase shifter performs $-90°$ phase shift in a perfect manner (what if it doesn't?).

QAM is quite commonly seen in digital communication systems.

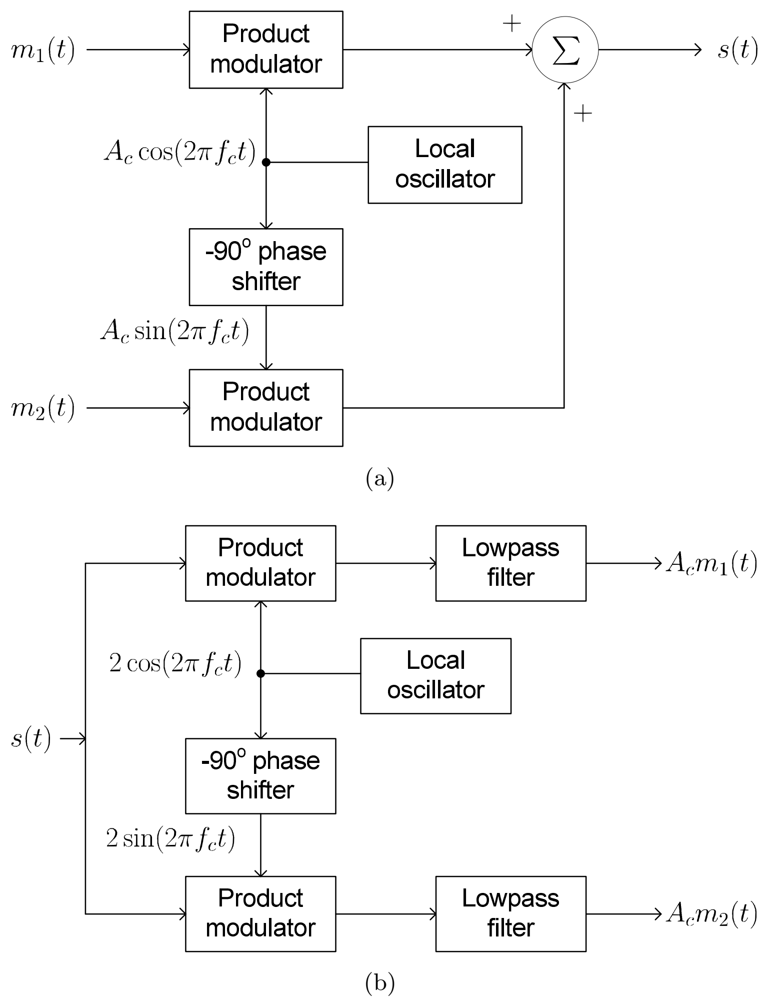

Figure 7: The modulation and demodulation process of QAM. (a) Modulation. (b) Demodulation.