

Handout 3: Bandwidth

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Suggested Reading: Chapter 2 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wiley & Sons Ltd.

1 Bandwidth

Given a signal, the bandwidth is a measure of the extent of significant spectral content of the signal. There is more than one definition for bandwidth (which can sometimes be confusing); the reason behind is that it is difficult to impose a unified mathematical formula for measuring “significant” contents for all cases.

Let us start with the sinc pulse example in Handout 2, Section 2.4. Figure 1 shows the corresponding amplitude spectrum, which is $|G(f)| = \frac{A}{2W} \text{rect}\left(\frac{f}{2W}\right)$. It is easy to say about the bandwidth in this example. The spectral content is strictly limited within the frequency interval $[-W, W]$. Hence, we can say that the bandwidth, denoted by B_T here, is

$$B_T = W \text{ Hz.}$$

Note that we ignore negative frequencies in our evaluation of the bandwidth, since the portion of the spectrum over negative frequencies is just a mirror image of that over positive frequencies (true for all real-valued signals).

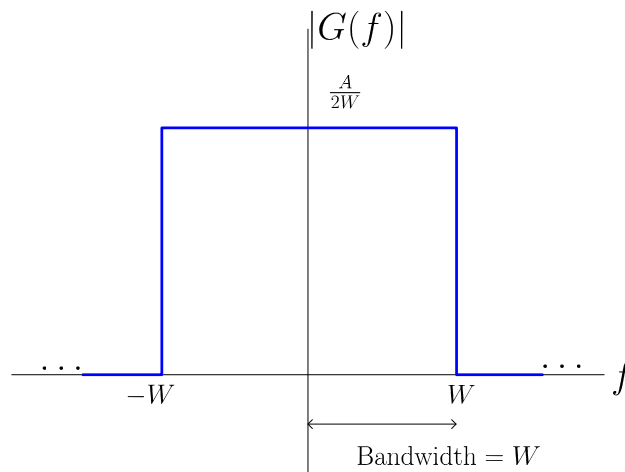


Figure 1: The amplitude spectrum of the sinc pulse $g(t) = A \text{sinc}(2Wt)$.

Next, consider the rectangular pulse example in Handout 2, Sections 1.1-1.2. Recall that the corresponding amplitude spectrum is $|G(f)| = AT|\text{sinc}(fT)|$. If we measure the bandwidth by finding the frequency duration under which all spectral components are contained, then the bandwidth

is infinity—the sinc function is *not* strictly limited. In order to obtain a meaningful bandwidth, we need to consider alternative bandwidth measures. One way is to measure a duration within which 90% (or perhaps 99%) of the energy of the spectrum lies. This “90% bandwidth” measure is however hard to lead to a simple result, although it can be done numerically. A rough, but common way to measure the bandwidth in this example is to consider the width of the mainlobe of the sinc function—we can at least see from Figure 2 that the mainlobe is where a significant portion of spectral components lies (although you can argue with me regarding how significant is significant!). By such measure, we *may* say that the bandwidth is

$$B_T = \frac{1}{T} \text{ Hz.}$$

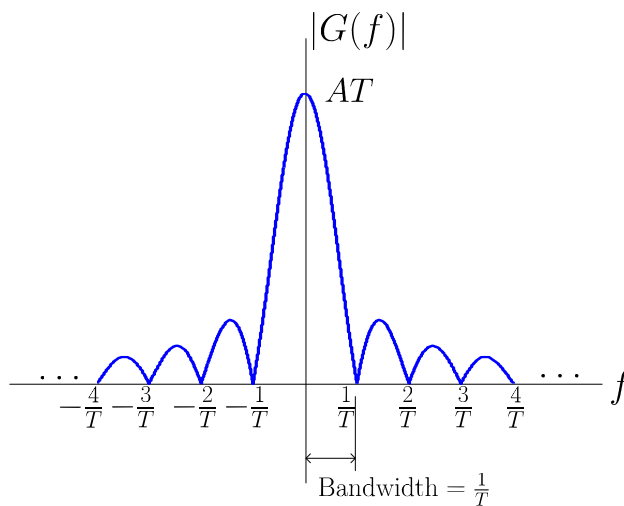


Figure 2: The amplitude spectrum of the rectangular pulse $g(t) = A\text{rect}\left(\frac{t}{T}\right)$.

The previous two examples are *lowpass* or *baseband signals*. Baseband signals have their significant frequency components centering around $f = 0$, the DC. In the RF pulse example in Handout 2, Section 2.3, this is not the case. Figure 3 shows the amplitude spectrum. It can be seen that the significant spectral content lies in a frequency interval, or frequency band, that is far from $f = 0$ (recall the assumption that $f_c \gg \frac{1}{T}$). We will call this kind of signals *bandpass* signals in the sequel. Hence, in this case, it is more appropriate to measure the bandwidth by first determining the frequency band within which the significant spectral content lies, and then by evaluating the width of the band. Following the argument in the previous rectangular pulse example, we determine the frequency band of the RF pulse as $[f_c - 1/T, f_c + 1/T]$ (negative frequencies are ignored, again). Since the width of $[f_c - 1/T, f_c + 1/T]$ is $2/T$, we have

$$B_T = \frac{2}{T} \text{ Hz}$$

as the bandwidth for the RF pulse.

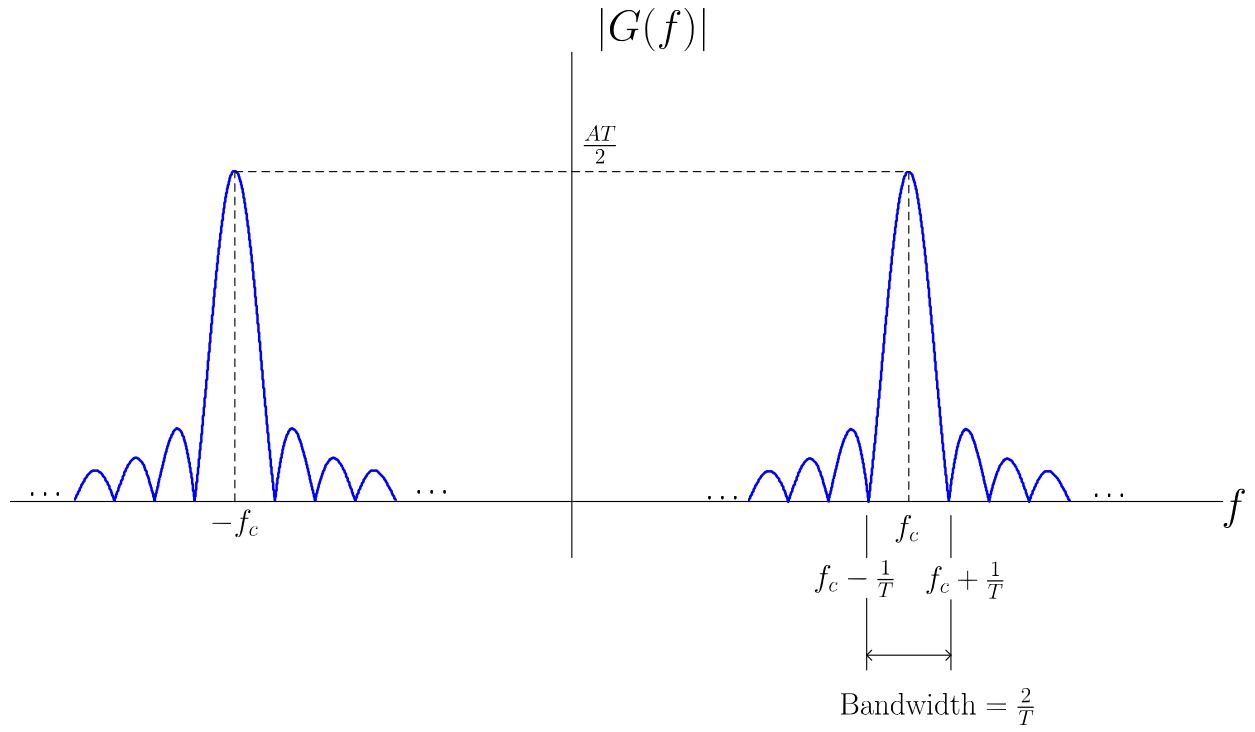


Figure 3: The amplitude spectrum of the RF pulse $g(t) = A \cdot \text{rect}\left(\frac{t}{T}\right) \cdot \cos(2\pi f_c t)$.