ELEG 2310B: Principles of Communication Systems **2021-22 First Term**

Handout 14: Digital Passband Transmission

Instructor: Wing-Kin Ma

Suggested Reading: Chapter 9 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wily & Sons Ltd.

We previously learnt digital transmission over a baseband channel. This handout studies digital transmission over a bandpass channel.

1 Binary Modulation Schemes

We introduce three basic digital carrier-modulation schemes, namely, amplitude-shift keying (ASK), phase-shift keying (PSK) and frequency-shift keying (FSK). Figure 1 illustrates how the three schemes work. It can be seen that

- ASK switches the carrier wave off and on to represent "0" and "1" of the transmitted bit stream, respectively;
- PSK uses 180° and 0° phase shifts of the carrier wave to represent "0" and "1" of the bit stream, respectively;
- FSK uses two different frequencies to represent "0" and "1" of the bit stream, respectively.

1.1 Signal Representations

The mathematical expressions of the three schemes are described as follows. Let $\{b_n\}$ denote the bit stream. In ASK, the transmitted signal can be expressed as

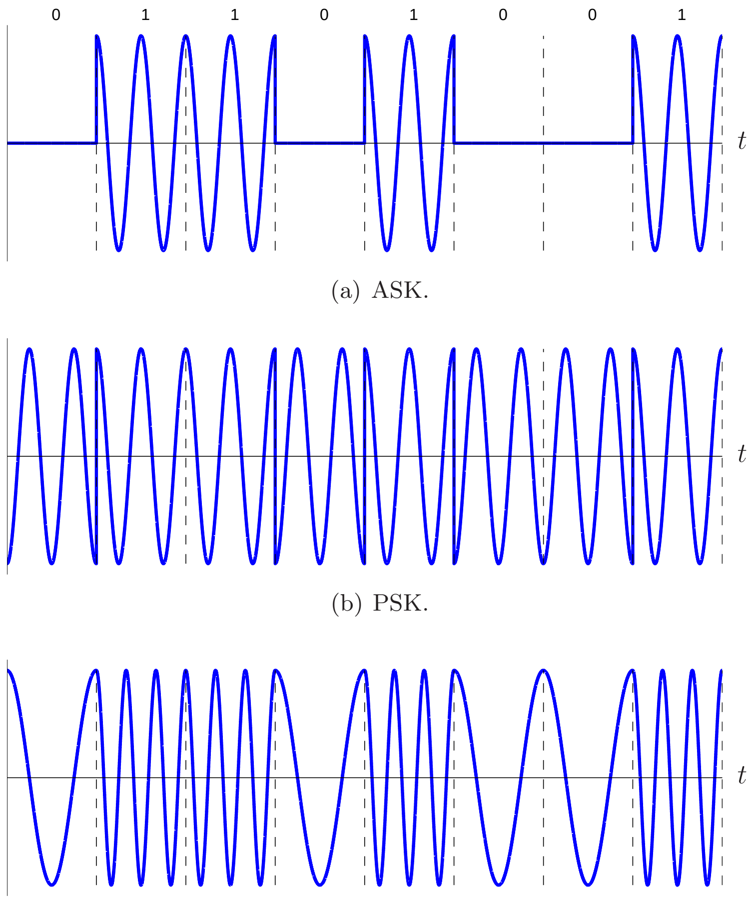
$$s(t) = \left[\sum_{k=-\infty}^{\infty} b_n g(t - kT) \right] \cdot A_c \cos(2\pi f_c t) \tag{1}$$

where T is the bit interval; $g(t)$ is a full-width rectangular pulse whose specific form is $g(t) = 1$ for $0 \leq t < T$ and $g(t) = 0$ otherwise; and f_c and A_c are the carrier frequency and amplitude, respectively. Following the same rationale as in our PAM study in Handout 11 (specifically, Section 3.1 there), let us focus only on the 0th bit interval for simplicity, i.e., $0 \leq t < T$. From (1), the transmitted ASK signal over $0 \leq t < T$ is given by

$$s(t) = \begin{cases} 0, & b_0 = 0 \\ A_c \cos(2\pi f_c t), & b_0 = 1 \end{cases} \tag{2}$$

Likewise, for PSK, the transmitted signal over 0th bit interval can be represented by

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi) = -A_c \cos(2\pi f_c t), & b_0 = 0 \\ A_c \cos(2\pi f_c t), & b_0 = 1 \end{cases} \tag{3}$$



(a) ASK.

(b) PSK.

(c) FSK.

Figure 1: Illustration of ASK, PSK and FSK waveforms.

where $0 \leq t < T$. Also, for FSK, we can write

$$s(t) = \begin{cases} A_c \cos(2\pi f_0 t), & b_0 = 0 \\ A_c \cos(2\pi f_1 t), & b_0 = 1 \end{cases} \tag{4}$$

for $0 \leq t < T$, where f_0 and f_1 are the two frequencies used to represent "0" and "1", respectively. Note that a common choice of f_0 and f_1 is to have $f_0 = f_c - \Delta f$, $f_1 = f_c + \Delta f$, for some $\Delta f > 0$.

1.2 Detection

The above three binary carrier-modulation schemes can be coherently detected by a two-path correlation receiver, which is similar to the (one-path) correlation receiver considered in Handout 11. To describe the method, it would be convenient to represent the three schemes under a unifying signal model

$$s(t) = \begin{cases} s_0(t), & b_0 = 0 \\ s_1(t), & b_0 = 1 \end{cases} \tag{5}$$

for $0 \leq t < T$, where $s_0(t)$ and $s_1(t)$ denote two waveforms for representing "0" and "1". For example, for ASK, we have $s_0(t) = 0$ and $s_1(t) = A_c \cos(2\pi f_c t)$, $0 \leq t < T$.

Figure 2 shows the two-path correlation receiver. We assume that the received signal is just a noisy version of the transmitted signal $s(t)$:

$$x(t) = s(t) + \eta(t) \tag{6}$$

where $\eta(t)$ is noise. The idea of the two-path correlation receiver is to consider the correlations of the two binary-representing waveforms

$$r_0 = \int_0^T x(t)s_0(t)dt, \tag{7}$$

$$r_1 = \int_0^T x(t)s_1(t)dt; \tag{8}$$

and then to make decision by comparing the values of r_0 and r_1, seeing which one is higher. Specifically, the decision rule is

$$\hat{b}_0 = \begin{cases} 0, & r_1 - r_0 < \lambda \\ 1, & r_1 - r_0 \geq \lambda \end{cases} \tag{9}$$

where λ is a threshold value. An example for the choice of λ is as follows.

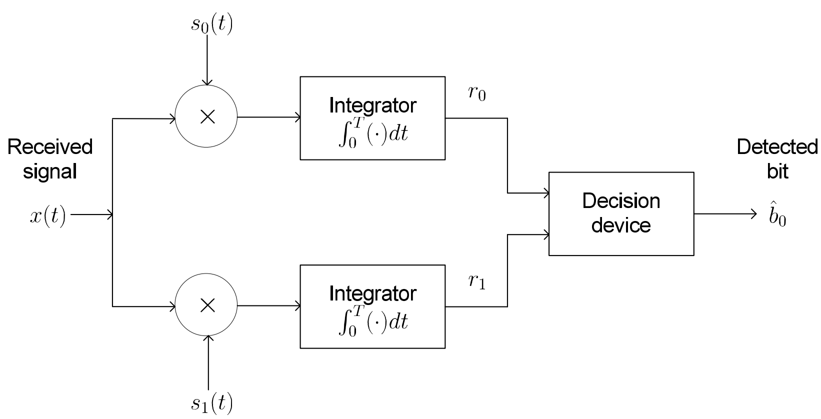


Figure 2: The two-path correlation receiver.

Example 1 Consider PSK where $s_0(t) = -A_c \cos(2\pi f_c t)$, $s_1(t) = A_c \cos(2\pi f_c t)$, $0 \leq t < T$. Also, for simplicity, assume that noise is absent. We evaluate the difference $r_1 - r_0$ under the two hypotheses of the bit b_0, and then use them to determine the value of the threshold λ.

Let us start with hypothesis $b_0 = 0$. Under a mild assumption that $f_c = m/T$ for some integer m, we can easily show that

$$r_0 = \frac{A_c^2 T}{2}, \tag{10}$$

$$r_1 = -\frac{A_c^2 T}{2}. \tag{11}$$

The difference of r_1 and r_0 is $r_1 - r_0 = -A_c^2 T$. We denote $\ell = -A_c^2 T$ to be the value of $r_1 - r_0$ under hypothesis $b_0 = 0$.

Similarly, under hypothesis $b_0 = 1$, it can be shown that

$$r_0 = -\frac{A_c^2 T}{2}, \tag{12}$$

$$r_1 = \frac{A_c^2 T}{2}, \tag{13}$$

under the assumption $f_c = m/T$ for some integer m. We denote $u = A_c^2 T$ to be the value of $r_1 - r_0$ under hypothesis $b_0 = 1$.

Intuitively, it seems logical to choose the decision threshold λ to be the midpoint of ℓ and u. By this belief, we have

$$\lambda = \frac{\ell + u}{2} = 0. \tag{14}$$

2 M-ary Modulation Schemes

Just like M-ary PAM for digital baseband transmission, we can also consider M-ary data transmission for passband transmission. Here we study two such schemes, namely the M-ary QAM scheme and the M-ary PSK scheme.

2.1 M-ary QAM

M-ary PAM can also be used for digital passband transmission. Essentially, we simply carrier-modulate a baseband M-ary PAM signal; that is,

$$s(t) = \left[\sum_{k=-\infty}^{\infty} a_n g(t - kT) \right] \cdot A_c \cos(2\pi f_c t), \tag{15}$$

where $\{a_n\}$ denotes the symbol stream. For ease of explanation later, we let $\mathcal{A}$ be the set of all possible values of a_n. The set $\mathcal{A}$ is commonly called the *constellation*. For example,

- the constellation in 2-ary PAM is $\mathcal{A} = \{\pm 1\}$; and
- the constellation in 4-ary PAM is $\mathcal{A} = \{\pm 1, \pm 3\}$.

Also, assuming that the standard full-width rectangular pulse is employed, the transmitted signal over the 0th symbol interval reduces to

$$s(t) = a_0 \cdot A_c \cos(2\pi f_c t), \quad 0 \le t < T. \tag{16}$$

Roughly speaking, M-ary QAM may be regarded as a complex-valued version of M-ary PAM. Following the above setup for M-ary PAM, the M-ary QAM scheme is described as follows. The M-ary QAM transmitted signal over the 0th symbol interval is given by

$$s(t) = \mathrm{Re}[a_0 \cdot A_c e^{2\pi f_c t}], \quad 0 \le t < T, \tag{17}$$

where a_0 (as well as the other symbols a_n) is a *complex-valued* symbol. Figure 3 shows two examples of QAM constellations, namely, the 4-ary and 16-ary QAM constellations. Mathematically,

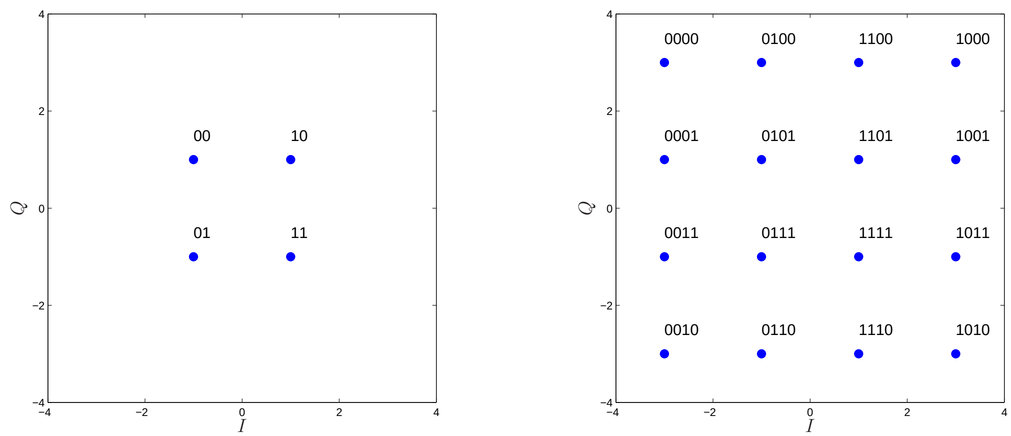
- the 4-ary QAM constellation is $\mathcal{A} = \{\pm 1 \pm j\}$; and
- the 16-ary QAM constellation $\mathcal{A} = \{\pm 1 \pm j, \pm 3 \pm j, \pm 1 \pm 3j, \pm 3 \pm 3j\}$.

M-ary QAM can be seen as using analog QAM to multiplex two digital PAM signals. It can be shown that (17) is equivalent to

$$s(t) = \mathrm{Re}[a_0] \cdot A_c \cos(2\pi f_c t) - \mathrm{Im}[a_0] \cdot A_c \sin(2\pi f_c t), \quad 0 \leq t < T. \tag{18}$$

We observe from (18) that M-ary QAM uses the carrier wave's in-phase and quadrature-phase components to send the symbol's real and imagery parts, respectively. Also, for the example of 4-ary QAM, we see that $\mathrm{Re}[a_0], \mathrm{Im}[a_0] \in \{\pm 1\}$—which means that the real and imagery parts of a_0 lie in the 2-ary PAM constellation. For the same reason, you should be able to see that a 16-ary QAM symbol has its real and imagery parts lying in the 4-ary PAM constellation.

The bit rate of M-ary QAM, with M being a power of 2, is $R_b = \log_2(M)/T$ bps.



(a) (b)

Figure 3: QAM Constellations. (a) 4-ary QAM. (b) 16-ary QAM. The notations "I" and "Q" represent the real and imagery parts of the constellation points, respectively.

2.2 M-ary PSK

The M-ary QAM scheme uses the amplitudes of the in-phase and quadrature-phase carrier components to perform M-ary signaling. M-ary PSK does the latter by using only the carrier phase. Specifically, the M-ary PSK transmitted signal over the 0th symbol interval can also be expressed as

$$s(t) = \mathrm{Re}[a_0 \cdot A_c e^{2\pi f_c t}], \quad 0 \leq t < T, \tag{19}$$

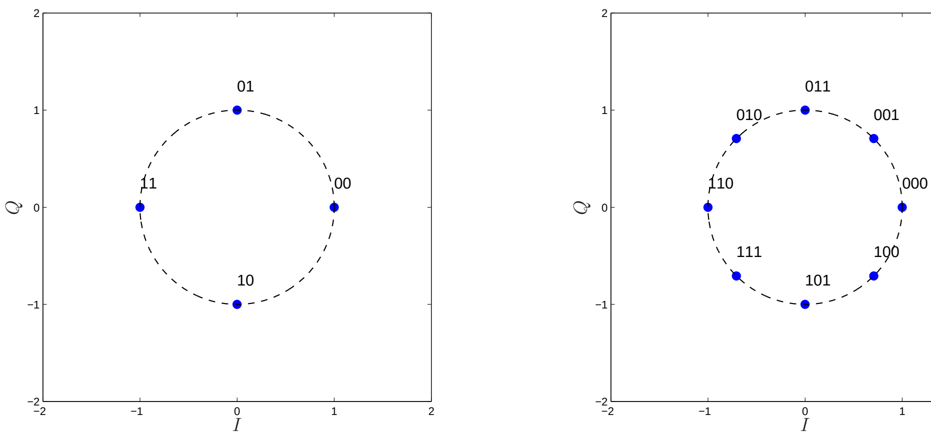
but with the constellation given by

$$\mathcal{A} = \left\{ e^{j\theta} \;\middle|\; \theta = \frac{2\pi m}{M}, m = 0, 1, \ldots, M-1 \right\}. \tag{20}$$

Figure 4 illustrates two PSK constellations. Again, the bit rate of M-ary PSK is $R_b = \log_2(M)/T$ bps, assuming that M is a power of 2. It can be shown from (19)–(20) that the transmitted signal can be written as

$$s(t) = A_c \cos(2\pi f_c t + \theta), \quad 0 \le t < T, \tag{21}$$

where $\theta = \frac{2\pi m}{M}$, $m = 0, 1, \ldots, M-1$. It is clear from the above equation that M-ary PSK uses phase to carry information digitally.



(a) (b)

Figure 4: PSK constellations. (a) 4-ary PSK. (b) 8-ary PSK.

2.3 Detection

We have seen in (18) that M-ary QAM uses the in-phase and quadrature-phase carrier components to transmit the real and imagery parts of the symbol, respectively. In fact, the same concept applies to M-ary PSK. Hence, we can use a one-path correlation receiver (studied in Handout 11) to extract the in-phase component of the received M-ary QAM or PSK signal, and, at the same time, use another correlation receiver to extract the quadrature-phase component. Then, decision can be made based on the extracted in-phase and quadrature-phase components. The idea is arguably a straight extension of what we previously learnt in binary detection, and please see the textbooks if you wish to see the details.

3 Further Discussion

We studied only coherent detection, specifically, the correlation receiver. As previously studied in amplitude modulation-based schemes, phase synchronism at the receiver is the key assumption and requirement in coherent detection. One can also consider noncoherent detection—and naturally ASK and FSK are able to support noncoherent detection (how about PSK and QAM? Do you think they can be noncoherently detected?).

To simplify our study in this handout, we assume full-width rectangular pulse. In fact, pulse shaping can also be applied in digital carrier modulation. It should be noted that ISI also appears in bandpass channels, especially when one wishes to transmit at a higher data rate. For example, in wireless channels, ISI is caused by multipath propagation characteristics of RF signals. Read the textbooks (or other digital communication textbooks) to get more inspiration.