ELEG 2310B: Principles of Communication Systems **2021-22 First Term**

Handout 11: Digital Baseband Transmission

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Suggested Reading: Chapter 8 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wily & Sons Ltd; or Chapter 7 of B. P. Lathi and Z. Ding, *Modern Digital and Analog Communication Systems (4th Edition)*, Oxford University Press.

In this handout we study transmission of digital information over a baseband channel, e.g., a wireline communication channel. In particular, emphasis will be placed on digital PAM schemes.

1 Digital Baseband Transmission via PAM

1.1 Binary PAM

Let $\{b_n\}$ be an information bit stream, where each bit b_n takes a value of either 0 or 1. We may use the bit stream to represent analog signals digitally, for example, by means of PCM. Or, the bit stream can be other digital information such as a file to be uploaded or downloaded. In either case, we see $\{b_n\}$ as generic binary data wherein the content and the application served are not important. What is important here is that we wish to transmit the bit stream over a baseband channel. Here we consider a binary PAM scheme for transmitting $\{b_n\}$. In this scheme, the transmitted signal is formulated as

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT), \tag{1}$$

where a_n is called a *symbol* and is determined by

$$a_n = \begin{cases} -1, & \text{if } b_n = 0 \\ 1, & \text{if } b_n = 1 \end{cases}, \tag{2}$$

T is the *symbol interval*, and $g(t)$ is the *pulse shape.* To be more specific, the scheme described in (1)-(2) is often called the 2*-ary PAM scheme.* Figure 1 shows a system diagram of 2-ary PAM.

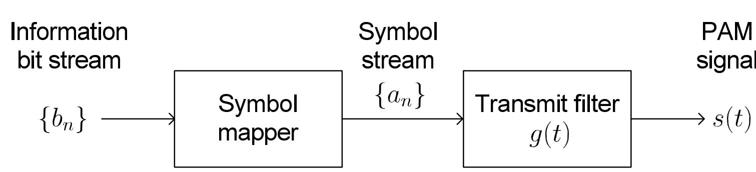


Figure 1: Digital PAM system diagram.

1.2 Pulse Shape

We should mention the choice of the pulse shape $g(t)$. A very commonly used pulse shape is the full-width rectangular pulse

$$g(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where A is the pulse amplitude. Figure 2(a) gives an illustration of the PAM waveform for the full-width rectangular pulse case. As can be seen, the resulting PAM signal is identical to that of the polar NRZ line code (see Handout 10). We can also adopt other pulse shapes. For example, consider the following pulse shape

$$g(t) = \begin{cases} A, & 0 \le t < \frac{T}{2} \\ -A, & \frac{T}{2} \le t < T \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Figure 2(b) illustrates the corresponding PAM waveform. It is observed that the PAM signal in this case is identical to that of the Manchester line code (again, see Handout 10).

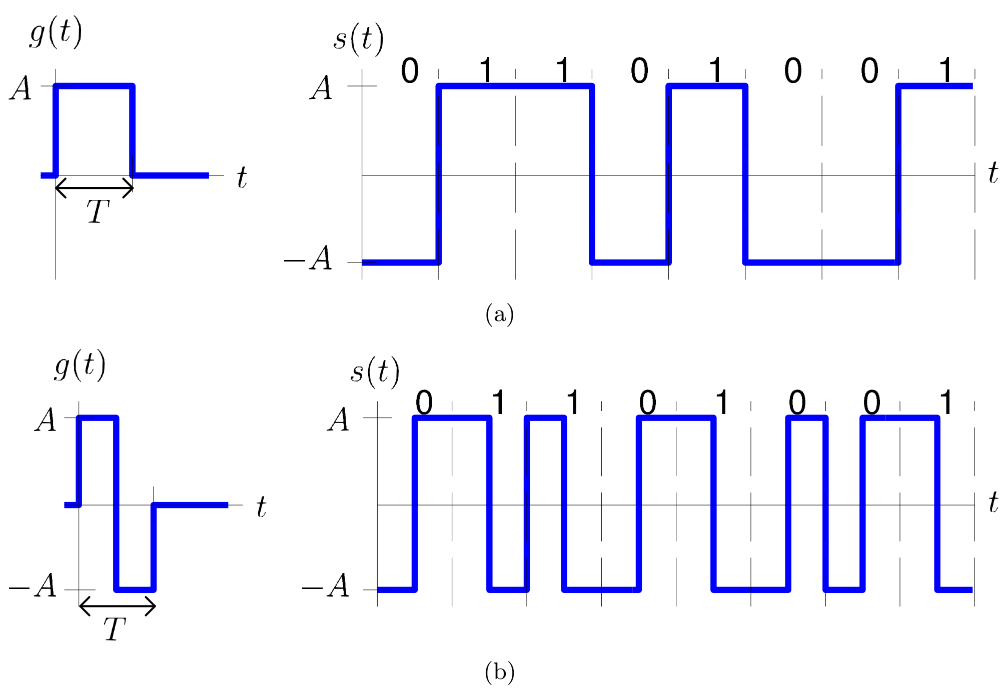


Figure 2: 2-ary PAM waveform illustration. (a) The full-width rectangular pulse shape. (b) The pulse shape used in the Manchester code.

From the above two examples, it seems logical to employ a pulse shape that has a support of $[0, T)$; that is to say, $g(t)$ is nonzero-valued for t lying in the time interval $[0, T)$, and must be zero-valued for t lying outside $[0, T)$. We will assume this being true at least in this handout. However, it will be studied later that pulse shapes are allowed to have their support going beyond $[0, T)$—and by doing so we can devise bandwidth-efficient PAM schemes.

1.3 Data Rate

We should also describe two key quantities for system performance specification, namely, *bit rate* and *symbol rate*. For convenience, let us denote R_b and R_s to be the bit rate and symbol rate, respectively. The quantity R_b is defined as the number of transmitted bits per second, while R_s the number of transmitted symbols per second. Apparently, one always desires to receive a high bit rate. The symbol rate for 2-ary PAM is

$$R_s = \frac{1}{T} \text{ symbols per second,} \tag{5}$$

since a symbol-carrying pulse is transmitted every T second. Also, the bit rate of 2-ary PAM is

$$R_b = R_s = \frac{1}{T} \text{ bits per second (bps),} \tag{6}$$

since one symbol represents one bit.

While 2-ary PAM has $R_s = R_b$, in a general context PAM can provide a bit rate higher than the symbol rate. Specifically, we can consider a more general PAM scheme, *M-ary PAM*, where the symbols a_n are allowed to take multi-amplitude values. Let us take 4-ary PAM as an example. In a standard 4-ary PAM scheme, the set of admissible symbol values is

$$a_n \in \{-3, -1, 1, 3\} \tag{7}$$

for all n. By mapping two bits to a symbol, e.g., using the mapping table in Table 1, we can transmit two bits in one symbol interval. Hence, the bit rate of 4-ary PAM is

$$R_b = 2R_s = \frac{2}{T} \text{ bps,} \tag{8}$$

which is twice of that of 2-ary PAM. Likewise, for a standard 8-ary PAM scheme, we have

$$a_n \in \{-7, -5, -3, -1, 1, 3, 5, 7\} \tag{9}$$

and the corresponding bit rate is $R_b = 3R_s = \frac{3}{T}$ bps.

a_n	b_{2n}, b_{2n+1}
3	10
1	11
-1	01
-3	00

Table 1: A bit-to-symbol mapping table for 4-ary PAM.

Unless specified, our focus will be on 2-ary PAM. The reason is to use a simple, yet representative enough, scheme to understand how digital transmission works.

2 Channel

A channel, as a representation of a physical transmission medium, is rarely ideal. Received signals are often damaged and/or corrupted versions of the original transmitted signal. There are scenarios where the channel may be considered ideal or near-ideal, e.g., communication over very short distances, exceedingly large transmission power, and/or low-rate transmission. However, they are considered special cases. In general, the input-output relation of a channel may be modeled by the following formula

$$x(t) = \int_{-\infty}^{\infty} c(\tau)s(t-\tau)d\tau + \eta(t), \tag{10}$$

where $s(t)$ is the transmitted signal, $x(t)$ is the received signal, $c(t)$ is called the *channel impulse response*, and $\eta(t)$ is *noise*. The channel model in (10) means that the physical transmission medium is modeled as a linear time-invariant system (or a filter, if you will) and there is noise adding on the signal received. Figure 3 shows a system representation of the channel.

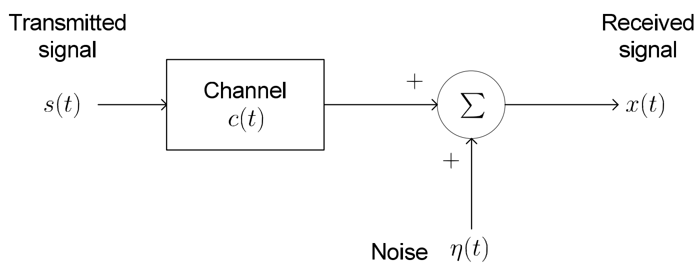


Figure 3: Channel model.

There are two main issues with the channel. The first is signal distortion, characterized by $c(t)$. Wireline transmission media generally behave like filters, typically lowpass, and the channel impulse response $c(t)$ is used to model such phenomena. An example is the plain old telephone line. Figure 4 shows a simulated telephone channel's amplitude spectrum where, by observation, the channel is arguably like a lowpass filter. It should be noted that real channels may exhibit non-ideal characteristics compared to an ideal lowpass system. For example, in the simulated telephone channel example in Figure 4, do you think that we can treat the channel as an ideal lowpass filter?

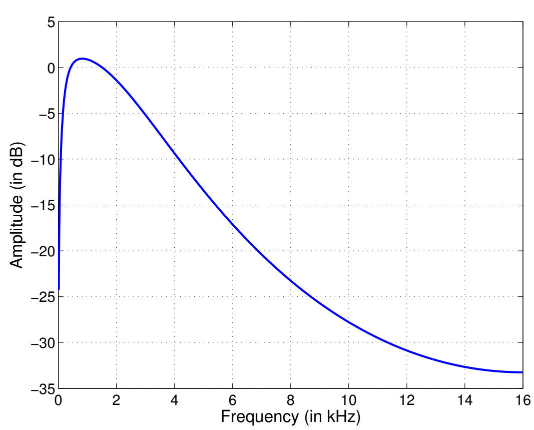
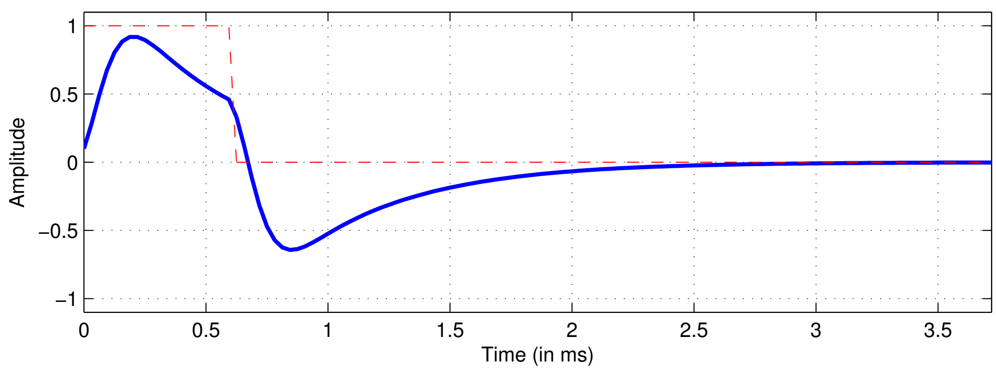
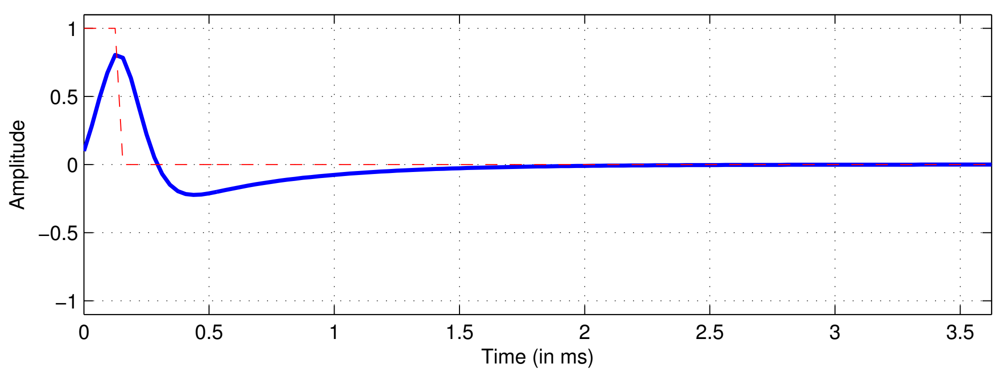


Figure 4: The amplitude spectrum $|C(f)|$ of a simulated telephone channel.

A well-known consequence of transmitting pulses over a non-ideal channel is pulse dispersion, which further leads to *intersymbol interference (ISI)* effects in PAM systems. Figure 5 gives an illustration where we use the simulated telephone channel in Figure 4 to examine pulse shapes after passing through the channel. We observe that the received pulse shapes are distorted in comparison to the transmitted pulse shapes. Also, the received pulses are dispersed in the sense that their pulse widths go beyond T. The presence of such dispersive effects, especially strong ones, means that received PAM pulses at adjacent symbol intervals overlap in time, and tend to interfere each other. The latter problem is called ISI. To illustrate, Figure 6 displays received PAM signals under the simulated telephone channel in Figure 4. ISI is more severe in the case in Figure 6(b), where a shorter T (and a higher symbol rate) is used.

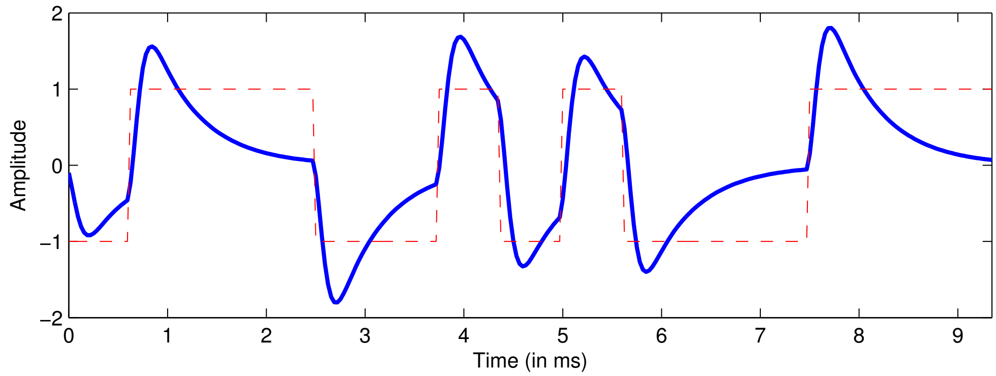


(a) $T = 0.625$ ms

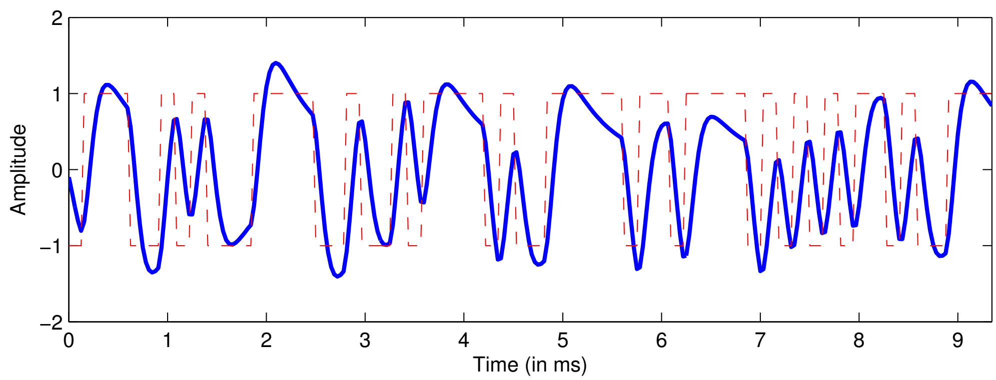


(b) $T = 0.15625$ ms

Figure 5: Dispersive effects of the channel. Red dashed line: transmitted pulse $g(t)$. Solid blue line: received pulse.



(a) $T = 0.625$ ms



(b) $T = 0.15625$ ms

Figure 6: Illustration of channel distortions to the PAM signals. Red dashed line: transmitted PAM signal $s(t)$. Solid blue line: received PAM signal $x(t)$.

The second main issue in transmission over a channel is noise. Noise is commonly used to represent unwanted signals that are almost impossible (if not totally impossible) to characterize as deterministic signals; or, simply speaking, signals that behave like random and unpredictable. There are many possible sources of noise, and some examples are: thermal noise in electronic devices (e.g., in the receiver frontend), interference from other communication and electronic devices, to name a few. Figure 7 shows how noise affects the received signal; note that the demonstration above assumes no channel dispersion. We often resort to *probability theory* and *random processes* to model noise and find solutions to combat it. These two topics would somehow be beyond the scope of this course, although you should note that they are powerful and essential concepts in advanced communication studies. Also, basic knowledge of probability theory will still be required in one of the components we are going to study.

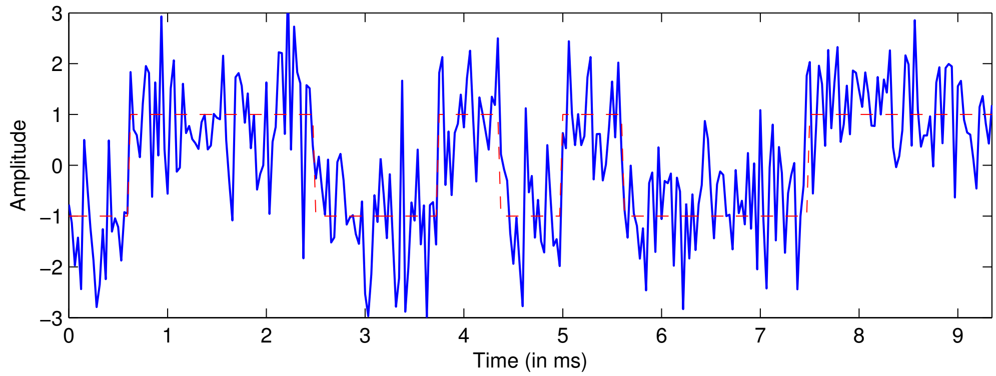


Figure 7: Illustration of noise effects on the received PAM signal. Red dashed line: transmitted PAM signal $s(t)$. Solid blue line: received PAM signal $x(t)$. No channel dispersion is assumed. $T = 0.625$ ms.

It should be noted that channel distortions and noise corruption exist simultaneously. In fact, seeking a best way to tackle both ISI and noise is far from being a trivial subject. In order to better understand how digital communication systems overcome these two problems, we will separate the two problems and study solutions for each individual problem.

3 Detection in Noise

This section concentrates on the problem of detecting the symbols a_n from the received signal, assuming presence of noise and no channel distortion.

3.1 Correlation Receiver

We make the following two assumptions:

1. There is no channel distortion or dispersion.
2. The support of the pulse shape $g(t)$ is $[0, T)$.

Under the above two assumptions, the received signal over the time interval $[0, T)$ is simply

$$x(t) = a_0 g(t) + \eta(t), \quad 0 \leq t < T, \tag{11}$$

where the signal above depends on the 0th symbol a_0 and does not depend on the other symbols $\{a_k\}_{k \neq 0}$. More generally, for each symbol time interval $[nT, (n+1)T)$, the received signal is

$$x(t) = a_n g(t - nT) + \eta(t), \quad nT \leq t < (n+1)T. \tag{12}$$

We see that (11) and (12) look exactly the same in terms of formulation. For convenience, and without loss of generality, we will focus on (11).

The objective here is to detect the symbol a_0 from the received signal $x(t)$ over the time window $[0, T)$. This can be accomplished by the *correlation receiver* shown in Figure 8. The correlation receiver first performs the following operation

$$y_0 = \int_0^T x(t)g(t)dt. \tag{13}$$

The above equation is called the *correlation* of $x(t)$ and $g(t)$, carried out in the time interval $[0, T)$. In a general context, correlation is a measure of similarity of two signals. Very roughly speaking, for two arbitrary signals $a(t)$ and $b(t)$ with support $[0, T)$, the correlation $\int_0^T a(t)b(t)dt$ is

i) positive and large in amplitude if $a(t)$ and $b(t)$ are similar;

ii) small in amplitude if $a(t)$ and $b(t)$ are not too related; and

iii) negative and large in amplitude if $a(t)$ and $b(t)$ are opposing each other.

Hence, the correlation operation in (13) is to access how similar $x(t)$ and $g(t)$ is. In fact, in the noiseless case where $\eta(t) = 0$, it is easy to verify from (11) and (13) that the correlation y_0 equals

$$y_0 = a_0 \int_0^T g^2(t)dt = a_0 \cdot E_g, \tag{14}$$

where $E_g = \int_0^T g^2(t)dt$ denotes the pulse energy. Since we have

$$y_0 = \begin{cases} -E_g, & \text{if } a_0 = -1 \\ E_g, & \text{if } a_0 = 1 \end{cases} \tag{15}$$

we can detect a_0 by

$$\hat{a}_0 = \begin{cases} -1, & \text{if } y_0 \le 0 \\ 1, & \text{if } y_0 > 0 \end{cases} \tag{16}$$

where the notation $\hat{a}_0$ stands for the detected symbol of a_0. Eq. (16) is the second operation of the correlation receiver, i.e., that of the decision device.

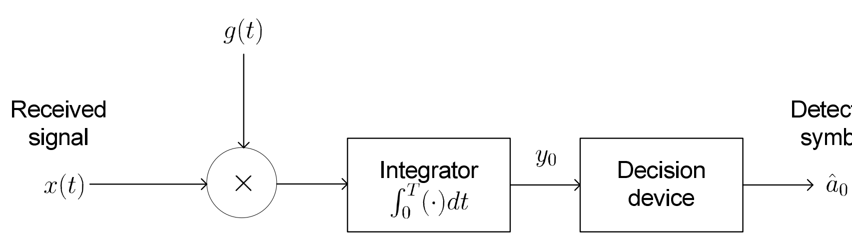


Figure 8: The correlation receiver.

While we describe the operations of the correlation receiver using the noiseless case, the correlation receiver is designed to deal with the noisy case. In the noisy case, y_0 should be rewritten as

$$y_0 = a_0 \cdot E_g + v_0, \tag{17}$$

where

$$v_0 = \int_0^T g(t)\eta(t)dt \tag{18}$$

is a noise term. Applying the above model to the decision rule in (16), we observe the following fact: If the noise term v_0 is bounded such that $|v_0| < E_g$, then the threshold decision operation in (16) *always* makes the right decision—and exact recovery of a_0 is guaranteed. However, the above condition is not always true, especially when the signal-to-noise ratio (SNR) is low. What we may hope for is that there is a high chance, or a large probability, that $\hat{a}_0$ is the correct decision of the true symbol a_0. To understand how large the probability of correct detection is, and how that probability scales with the SNR, we will need to go through probabilistic analysis; this will be considered in later handouts.

As an exercise, try the following: follow the above expressions and derive the correlation receiver for (12); i.e., detection of a_n.

3.2 Matched Filter

The *matched filter* is an alternative approach to implementing the correlation receiver. A system diagram for the matched filter is shown in Figure 9. The matched filter first carries out a filtering operation

$$y(t) = \int_{-\infty}^{\infty} \varphi(\tau)x(t-\tau)d\tau, \tag{19}$$

where $\varphi(t)$ is the impulse response of the filter, and $y(t)$ denotes the filter output. Then, a sampling operation is applied every $(n+1)T$ second on $y(t)$, resulting in a sequence of samples

$$y_n = y((n+1)T). \tag{20}$$

The samples y_n then pass through the decision device to detect a_n; the decision rule is the same as (16).

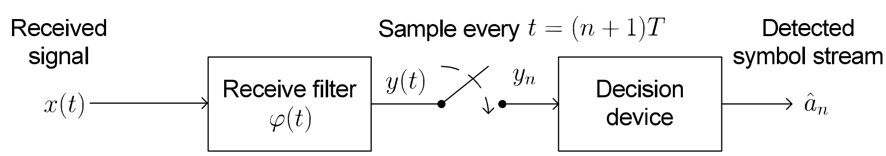


Figure 9: The matched filter.

By an appropriate choice of the filter $\varphi(t)$, the matched filter can perform exactly the same operation as in the correlation receiver. Let us consider $n = 0$. We get

$$\begin{aligned} y_0 = y(T) &= \int_{-\infty}^{\infty} \varphi(\tau)x(T-\tau)d\tau \\ &= \int_{-\infty}^{\infty} \varphi(T-\tau)x(\tau)d\tau \qquad \text{(by commutativity of convolution)} \end{aligned} \tag{21}$$

Suppose that we choose $\varphi(T-t) = g(t)$, or, equivalently,

$$\varphi(t) = g(T-t). \tag{22}$$

Eq. (21) can be expressed as

$$
\begin{aligned}
y_0 &= \int_{-\infty}^{\infty} g(\tau)x(\tau)d\tau \\
&= \int_{0}^{T} g(\tau)x(\tau)d\tau \qquad (23)
\end{aligned}
$$

which is exactly the same as the correlation operation in (13). Figure 10 pictorially shows the relationship of $g(t)$ and $\varphi(t)$.

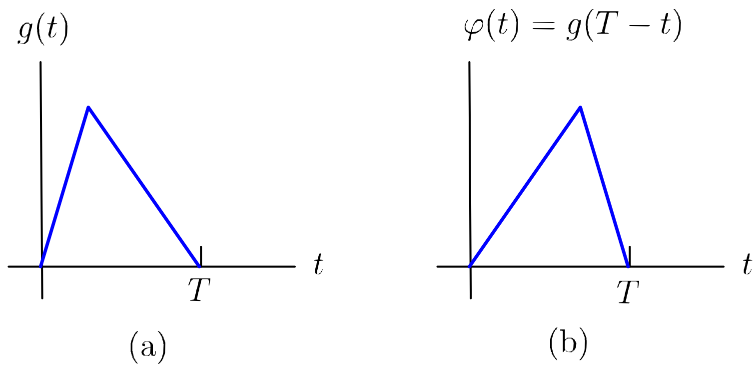


Figure 10: Relationship of the pulse shape and the matched filter impulse response.

Again, as an exercise, try the following: show that for general n each matched filter output y_n provides the same correlation receiver operation for the symbol a_n.