ELEG 2310B: Principles of Communication Systems **2021-22 First Term**

Handout 10: Pulse-Code Modulation

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Suggested Reading: Chapter 7 of Simon Haykin and Michael Moher, *Communication Systems (5th Edition)*, Wily & Sons Ltd; or Chapter 7 of B. P. Lathi and Z. Ding, *Modern Digital and Analog Communication Systems (4th Edition)*, Oxford University Press.

Pulse-code modulation (PCM) is a technique that aims at representing an analog message signal by a sequence of binary pulses. By doing so, we can perform digital transmission of analog signals. Note that PCM follows essentially the same principle in analog-to-digital and digital-to-analog conversions in digital signal processing. Also, PCM is a file format for storing voice and music.

1 System Description

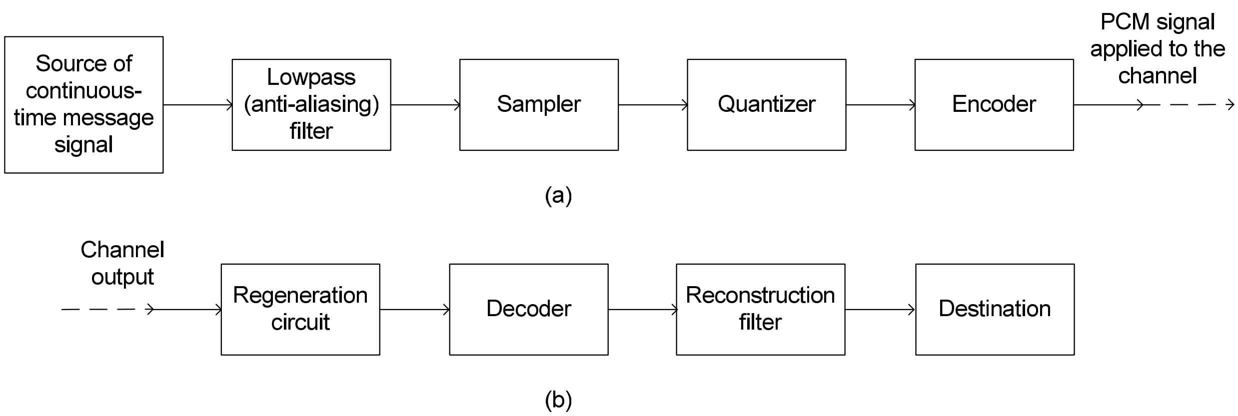


Figure 1: A PCM system diagram. (a) The transmitter. (b) The receiver.

Figure 1 shows a system block diagram of PCM. Some key components of PCM are described as follows.

- **Sampling:** The process is the same as the sampling process in PAM—transforming the incoming continuous-time signal into a discrete-time signal. The sampler obtains sample amplitudes $m(nT_s)$ from a message signal at times $t = nT_s$, with n taking integer values. In order to satisfy the perfect reconstruction requirement by the sampling theorem, the sampling rate $f_s = 1/T_s$ is assumed to be at least twice of the message bandwidth W. For a similar reason, a lowpass filter may be applied before the sampler to exclude frequency components greater than $f_s/2$.

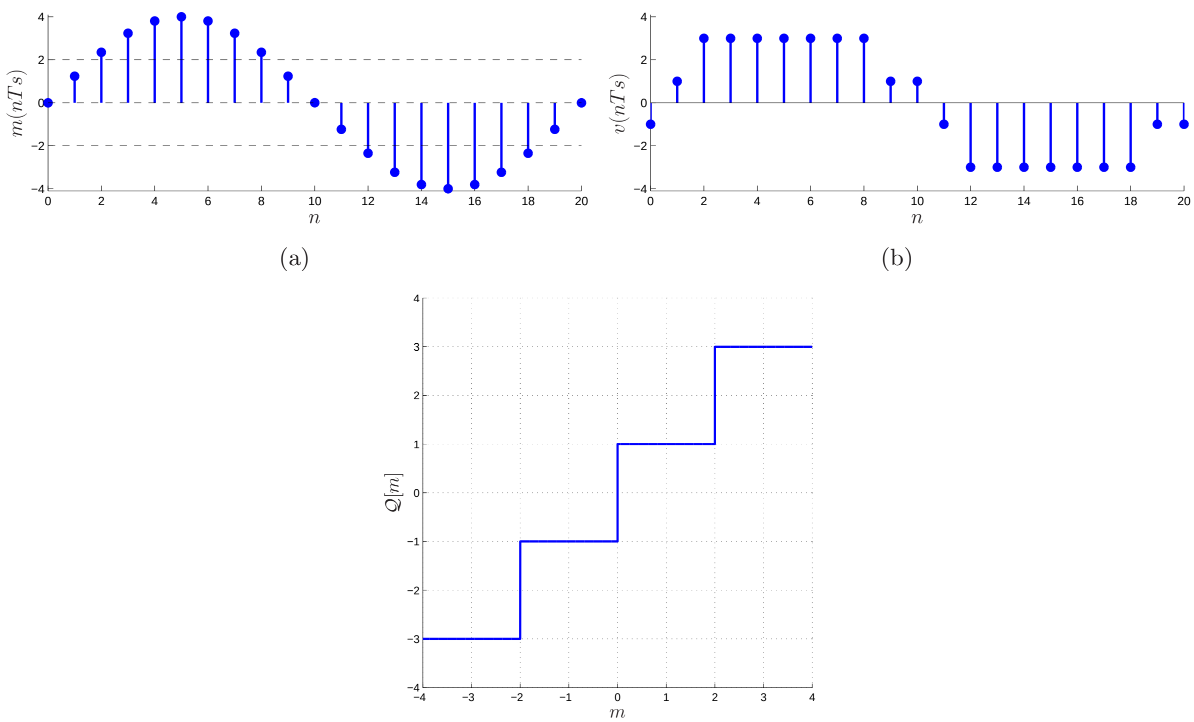
- **Quantization:** Quantization aims at discretizing the signal amplitudes. The quantization process may be described by the following formula

$$v(nT_s) = \mathcal{Q}[m(nT_s)], \tag{1}$$

where $v(nT_s)$ denotes the quantizer output, and $\mathcal{Q}[\cdot]$ is the quantizer function. The quantizer output $v(nT_s)$ has its values taken from a finite set of signal amplitudes or levels. Specifically, the values of $v(nT_s)$ are taken from a signal level set $\{v_1, v_2, \ldots, v_L\}$, where each v_i is a prespecified signal level and L is the number of levels. Figure 2 shows an example, where the number of levels is four and the levels are $v_1 = -3$, $v_2 = -1$, $v_3 = 1$, $v_4 = 3$. The quantizer function in this example is given by

$$\mathcal{Q}[m(nT_s)] = \begin{cases} -3, & m(nT_s) \leq -2 \\ -1, & -2 < m(nT_s) \leq 0 \\ 1, & 0 < m(nT_s) \leq 2 \\ 3, & 2 < m(nT_s) \end{cases}$$

More generally, the quantizer produces an output $\mathcal{Q}[m(nT_s)] = v_k$ if v_k is closest to $m(nT_s)$; that is, the distance $|m(nT_s) - v_k|$ is the smallest among all other distances $|m(nT_s) - v_i|$ for all $i = 1, \ldots, L$, $i \neq k$.



(c)

Figure 2: An example of the quantization process. (a) The sampled signal $m(nT_s)$. (b) The corresponding quantized signal. (c) The quantization function. The dashed lines in (a) represent the decision thresholds of the quantizer.

- **Encoding:** The purpose of encoding is to represent each quantized signal level v_i to a binary codeword. An example of the encoding process corresponding to the example in Figure 2 is shown in Table 1. For convenience, suppose that the number of levels L is a power of 2. Then we have

$$L = 2^R, \tag{2}$$

or equivalently,

$$R = \log_2(L), \tag{3}$$

where R is the *number of bits per sample*.

Level v_i	Binary codeword
-3	01
-1	00
1	10
3	11

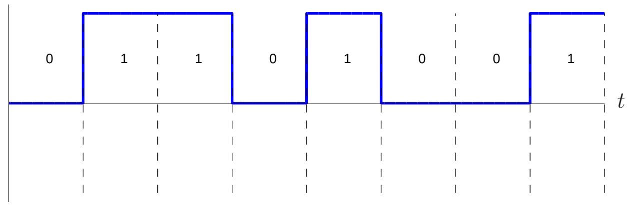
Table 1: An example of the encoding table for the four-level case.

- **Line codes:** The encoding process results in a binary data stream (like, in form of 011010...). A line code describes a scheme of converting the binary data stream to electrical (or physical) signals for transmission over the channel. There are many line codes, and some examples are shown in Figure 3. The most natural line codes are unipolar nonreturn-to-zero (NRZ) signaling and polar NRZ signaling. Unipolar NRZ signaling is simply an on-off signaling scheme, using zero and a positive amplitude to represent "0" and "1" respectively. Polar NRZ signaling uses a negative amplitude, say, $-A$, and a positive amplitude A to represent "0" and "1" respectively. Moreover, unipolar return-to-zero (RZ) signaling is a variant of unipolar NRZ signaling where a half-width pulse is used in place of the full-width pulse in unipolar NRZ signaling. The reason for doing so is to allow the receiver to extract timing from the received signals. See the textbooks for more details on various line codes, including the bipolar RZ signaling scheme and the Manchester code shown in Figure 3.

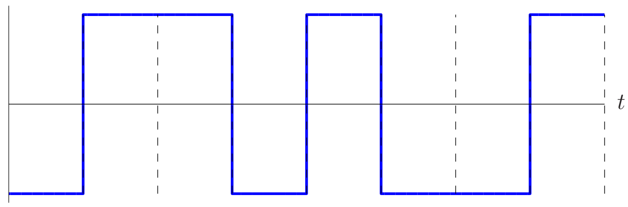
- **Regeneration:** A channel is rarely ideal in practice. The signal at the receiver is often subjected to noise corruptions and signal distortions introduced by the channel. It should be noted that in the previous handouts, we have assumed ideal channels so as to facilitate our understanding of modulation and demodulation; in real world, this is not true. The task of the regeneration circuit is to recover the data stream from the received signal. It is important to note at this point that digital transmission allows us to perform signal recovery quite accurately. Simply speaking, the idea is to apply a decision-making device, where for each bit interval the device would observe the received signal and decide whether the received bit is a "0" or a "1".

  As an additional note, the receiver also has to have timing synchronization with the transmitter in order to recover the data stream. Timing synchronization can be accomplished by a variety of ways; the problem will not be considered here, but we should notice that the timing synchronization task is also done in the regeneration circuit.

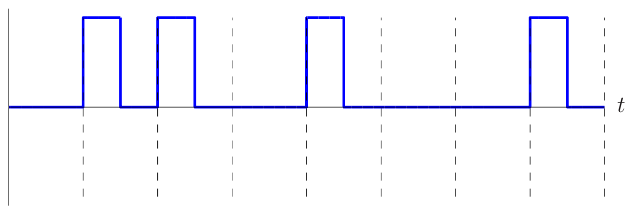
We should also mention that i) decoding at the receiver is the inverse of encoding, converting received codewords back into signal levels; and that ii) the reconstruction filter at the receiver is to implement the interpolation or perfect reconstruction formula in the sampling theorem.



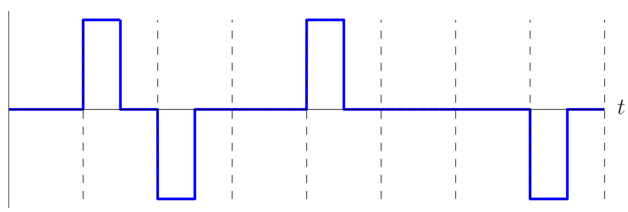
(a) Unipolar nonreturn-to-zero signaling.



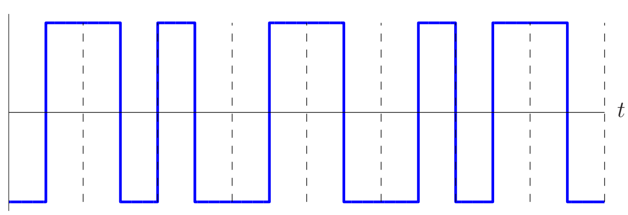
(b) Polar nonreturn-to-zero signaling.



(c) Unipolar return-to-zero signaling.



(d) Bipolar return-to-zero signaling.



(e) Manchester code.

Figure 3: Illustration of some line codes.

2 Quantization Noise

While quantization enables binary representation of analog signals, it also introduces errors. To describe the problem, let

$$q(nT_s) = m(nT_s) - v(nT_s) \tag{4}$$

define the *quantization noise*, the error caused by quantization. Figure 4 provides an illustration of the quantization noise in the eight-level case.

We are interested in quantifying the quantization noise effects. To do so, we assume uniform quantization where

$$|v_i - v_{i+1}| = \Delta, \quad \text{for every } i = 1, \ldots, L-1, \tag{5}$$

where Δ is called the *step-size*. In this uniform case, the quantizer output $\mathcal{Q}[m(nT_s)]$ produces an output v_k if v_k is such that $v_k - \Delta/2 < m(nT_s) \leq v_k + \Delta/2$. Also, the step-size is chosen as

$$\Delta = \frac{2|m|_{\max}}{L}, \tag{6}$$

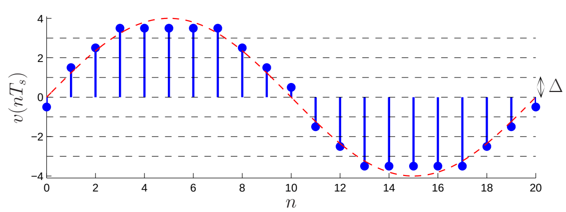
where $|m|_{\max} = \max_t |m(t)|$ is the maximum amplitude of $m(t)$. It follows that

$$|q(nT_s)| \leq \frac{\Delta}{2} = \frac{|m|_{\max}}{L}. \tag{7}$$

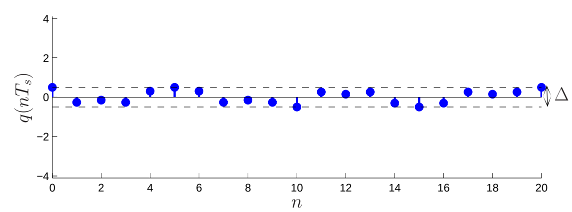
Also, by recalling that $L = 2^R$, the above equation can be further expressed as

$$|q(nT_s)| \leq \frac{\Delta}{2} = |m|_{\max} 2^{-R}. \tag{8}$$

Eq. (8) provides us with the following implication: *fixing the signal dynamic range specified by $|m|_{\max}$, the quantization noise amplitude reduces exponentially with the number of bits per sample R.* In other words, every time we add one bit, the quantization noise amplitude is reduced by a factor of 2.



(a)



(b)

Figure 4: Illustration of quantization noise. The red dashed line in (a) is the original message signal $m(t)$.

3 Further Discussion

Digital transmission allows us to recover or detect transmitted data from a received signal that has been subjected to impairments by the channel, and this is accomplished via exploitation of the discrete nature of the transmitted signal (of course, the recovery accuracy is up to a certain limit which depends on how worse the channel impairments are). In fact, this is one of the key reasons why digital communication is popular. For the same reason, we can also recover the transmitted data from the incoming (and impaired) digitally modulated signal and then regenerate a clean version of the modulated signal for further transmission—a device that performs the above mentioned task is generally called a *regenerative repeater*. An illustration for the application of regenerative repeaters is given in Figure 5. The scenario is that of a long transmission path, where regenerative repeaters are placed at intermediate points to detect and re-transmit the signal. By doing so, coverage can be extended or performance may be enhanced.

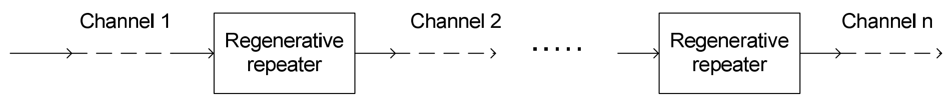


Figure 5: Illustration of using regenerative repeaters to extend transmission distances.