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An Order-16 Integer Cosine Transform

W. K. Cham and Y. T. Chan

Abstract—**It is possible to replace the real-numbered elements of a discrete cosine transform (DCT) matrix by integers and still maintain the structure, i.e., relative magnitudes and orthogonality, among the matrix elements. The result is an integer cosine transform (ICT). Thirteen ICT's have been found and some of them have performance comparable to the DCT.**

I. Introduction

In digital image processing, data compression is necessary to improve efficiency in storage and transmission. Transformation is one popular technique for data compression. By first transforming correlated pixels into weakly correlated ones, and after a ranking in their energy contents, for example, and retaining only the most significant components, high compression ratio is possible [1]. Since inverse transformation is needed to reproduce the original image from the compressed data, it is important that the transform process be simple and fast. The family of orthogonal transforms [2] is well suited for this application because the inverse of an orthogonal matrix is its transpose.

The discrete cosine transform (DCT) [2] is widely accepted as having a high efficiency (see Section III for a definition) for image data compression while the Walsh transform (WHT) and C-matrix transform (CMT) [3] are simpler to implement but possess lower efficiencies.

The DCT matrix elements are real numbers and [4] has shown that for a 16-order DCT 8 b are needed to represent these numbers in order to ensure perfectly negligible image reconstruction errors due to finite-length number representation. If the transform matrix elements are integers, then it may be possible to have a smaller number of bit representation and at the same time zero truncation errors.

Using the principle of dyadic symmetry, [5] has introduced the order-8 integer cosine transform (ICT) which has zero truncation errors, requires a small number of bit representation (as little as 2 b in one case) and comparable efficiency to the DCT. Briefly, an ICT matrix is in the form $I = KJ$ where I is the orthogonal ICT matrix, and K is a diagonal matrix whose elements take on values that serve to scale the rows of the matrix J so that the relative magnitudes of the ICT matrix I are similar to those in the DCT matrix. The matrix J is orthogonal with elements that are all integers. Many order-8 ICT's are given in [5] and, in particular, there is one ICT that performs nearly as well as the DCT and requires only 4 b for perfect representation of its elements.

In real-time applications, a transform is most likely implemented using a dedicated chip. Thus the shorter bit length ICT will lead to a simpler IC structure and shorter computation time. Furthermore, in applying the ICT to source coding, it is easy to eliminate the multiplication of K by absorbing it in the quantization process in the coder, and in the decoding process of converting bit streams into numbers in the decoder.

In [5], the order-8 ICT was derived using the principle of dyadic symmetry. This correspondence gives a different development that leads to the order-16 ICT. Equations relating the elements of the ICT matrix so as to satisfy the orthogonality conditions among the columns of the ICT matrix are first written. Then a search method is proposed to find integer solutions for these elements. Thirteen order-16 ICT's are given and two of them are perfectly representable by 6-b numbers, resulting in a reduction in computation time and a simpler IC structure. The development of the order-16 ICT is in Section II, and Section III compares its performance with other transforms. The conclusions are in Section IV.

II. The Order-16 ICT

The development of the order 16-ICT begins with the 16×16 DCT matrix:

$$D(i,j) = \begin{cases} \dfrac{1}{4}, & i = 1 \\ \dfrac{\sqrt{2}}{4} \cos\left[\dfrac{(i-1)(j-0.5)\pi}{16}\right], & 1 \le j \le 16. \end{cases}$$

Decomposing it into

$$D = K\Psi$$

Manuscript received August 28, 1989; revised November 7, 1990.
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IEEE Log Number 9042688.

where K = diagonal (k_i), and k_i, $1 \le i \le 16$ are the scaling factors, results in

$$\Psi = \left[\begin{array}{cccccccc|ccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & -A_8 & -A_7 & -A_1 \\
A_9 & A_{10} & A_{11} & A_{12} & -A_{12} & -A_{11} & -A_{10} & -A_9 & -A_9 & -A_{10} & A_9 \\
A_2 & A_5 & A_8 & -A_6 & -A_3 & -A_1 & -A_4 & -A_7 & A_7 & A_4 & -A_2 \\
A_{13} & A_{14} & -A_{14} & -A_{13} & -A_{13} & -A_{14} & A_{14} & A_{13} & A_{13} & A_{14} & A_{13} \\
A_3 & A_8 & -A_4 & -A_2 & -A_7 & A_5 & A_1 & A_6 & -A_6 & -A_1 & -A_3 \\
A_{10} & -A_{12} & -A_9 & -A_{11} & A_{11} & A_9 & A_{12} & -A_{10} & -A_{10} & A_{12} & A_{10} \\
A_4 & -A_6 & -A_2 & A_8 & A_1 & A_7 & -A_3 & -A_5 & A_5 & A_3 & -A_4 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\
A_5 & -A_3 & -A_7 & A_1 & -A_8 & -A_2 & A_6 & A_4 & -A_4 & -A_6 & -A_5 \\
A_{11} & -A_9 & A_{12} & A_{10} & -A_{10} & -A_{12} & A_9 & -A_{11} & -A_{11} & A_9 & A_{11} \\
A_6 & -A_1 & A_5 & A_7 & -A_2 & A_4 & A_8 & -A_3 & A_3 & -A_8 & -A_6 \\
A_{14} & -A_{13} & A_{13} & -A_{14} & -A_{14} & A_{13} & -A_{13} & A_{14} & A_{14} & -A_{13} & A_{14} \\
A_7 & -A_4 & A_1 & -A_3 & A_6 & A_8 & -A_5 & -A_2 & -A_2 & A_5 & -A_7 \\
A_{12} & -A_{11} & A_{10} & -A_9 & A_9 & -A_{10} & A_{11} & -A_{12} & -A_{12} & A_{11} & A_{12} \\
A_8 & -A_7 & A_6 & -A_5 & A_4 & -A_3 & A_2 & -A_1 & A_1 & -A_2 & -A_8
\end{array}\right].$$

It is readily seen that Ψ has symmetry with respect to the dotted line, with even and odd symmetry alternating. The basic idea of the ICT is to replace the elements A_i by integers and at the same time preserve the structure, i.e., the relative magnitudes of these elements and orthogonality, of the DCT matrix. For $\Psi\Psi^T$ to be diagonal, its elements must satisfy the conditions

$$A_1A_2 + A_2A_5 + A_3A_8 = A_4A_6 + A_3A_5 + A_1A_6 + A_4A_7 + A_7A_8 \quad (1)$$

$$A_1A_3 + A_2A_8 + A_5A_6 + A_1A_7 + A_6A_8 = A_3A_4 + A_2A_4 + A_5A_7 \quad (2)$$

$$A_1A_4 + A_1A_5 + A_4A_8 + A_6A_7 = A_2A_6 + A_2A_3 + A_3A_7 + A_5A_8 \quad (3)$$

$$A_9A_{10} = A_9A_{11} + A_{10}A_{12} + A_{11}A_{12}. \quad (4)$$

Additionally, if this matrix were to possess a DCT-like structure, its elements must also satisfy the constraints

$$A_1 > A_2 > A_3 > A_4 > A_5 > A_6 > A_7 > A_8 \quad (5)$$

$$A_9 > A_{10} > A_{11} > A_{12} \quad (6)$$

$$A_{13} > A_{14}. \quad (7)$$

Note that the elements A_{13} and A_{14} do not enter into the orthogonal conditions (1) to (4) and were assigned values of 3 and 1, respectively, in [5]. They were found to be the best integer values for giving high efficiency.

To find integers that satisfy (1) to (6), it is first noticed that (4) and (6) are independent of the other conditions, i.e., the elements A_9 to A_{12} can be found independently of the other elements to satisfy (4) and (6). Indeed, [5] has determined several sets of values that satisfy (4) and (6) and $A_9 = 55$, $A_{10} = 48$, $A_{11} = 32$, $A_{12} = 11$ is one such set, for example.

Next, let

$$\Delta_1 = (A_2 - A_6)(A_6 - A_7) - (A_3 + A_7)(A_2 - A_3) \quad (8)$$

$$\Delta_2 = (A_6 - A_7)(A_3 - A_7) - (A_2 - A_3)(A_2 + A_6) \quad (9)$$

$$\Delta_3 = A_6^2 - A_7^2 - A_2^2 + A_3^2 \quad (10)$$

$$\Delta_4 = (A_2 + A_3)(A_3 - A_7) - (A_2 + A_6)(A_6 + A_7) \quad (11)$$

$$\Delta_5 = A_2(A_3 + A_6) + A_7(A_3 - A_6) \quad (12)$$

which are all functions of the set $S_1 = \{A_2, A_3, A_6, A_7\}$. It follows from (1) to (3), after some manipulation, that

$$A_1\Delta_1 + A_8\Delta_2 = A_4\Delta_3 \quad (13)$$

$$A_1\Delta_3 + A_5\Delta_2 = A_4\Delta_4 \quad (14)$$

$$A_1(A_4 + A_5) + A_8(A_4 - A_5) = \Delta_5. \quad (15)$$

Clearly, (8) to (12) decouple S_1 from $S_2 = \{A_1, A_4, A_5, A_8\}$ and (13) to (15) is the relationship between the elements of S_1 and S_2 and they come from (1) to (3). Now a computer search can be systematically carried out by first choosing integers from S_1 that satisfy (5) and then finding solutions from (13) to (15) for S_2, retaining only integer solutions that satisfy the constraint (5). It turns out that there are 13 solutions and they are listed in Table I.

III. Performance Comparison

The efficiency of a transform is generally defined as its ability to decorrelate a vector or random elements. Let the n-vector f contain elements of samples from a one-dimensional, zero mean, unit variance first-order Markov process with correlation coefficient ρ and covariance matrix C whose (i, j)th element is $\rho^{|i-j|}$. Let the transformation matrix be Φ and the transformed vector F. Then

$$F = \Phi f \quad (16)$$

and

$$E\{FF^T\} = \Phi E\{ff^T\}\Phi^T = \Phi C \Phi^T = B = \{b(i, j)\}. \quad (17)$$

The efficiency is

$$\eta = \frac{\sum_{i=1}^{n} |b(i, i)|}{\sum_{i=1}^{n}\sum_{j=1}^{n} |b(i, j)|}. \quad (18)$$

Fig. 1 is a plot of the transform efficiency of DCT, WHT, CMT, and ICT against the correlation coefficient ρ. The ICT used is the one in the second row of Table I, whose elements require only 6-b representations. The ICT efficiency is slightly below that of the DCT but is considerably higher than that of the WHT and CMT.

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TABLE I
13 INTEGER COSINE TRANSFORMS

A1	A2	A3	A4	A5	A6	A7	A8
120	114	103	94	68	57	34	14
42	38	37	32	22	19	10	4
62	61	49	47	37	31	21	5
120	108	104	85	69	52	32	2
128	124	119	100	88	67	22	12
121	119	107	97	79	68	19	15
87	80	70	65	43	40	25	7
81	76	64	61	41	38	25	7
94	93	73	70	58	51	26	6
117	106	90	82	59	50	42	1
121	111	105	89	69	63	15	8
108	107	81	76	70	61	29	1
134	119	118	98	70	69	11	10

$A_9 = 55,\ A_{10} = 48,\ A_{11} = 32,\ A_{12} = 11,\ A_{13} = 3,\ A_{14} = 1$

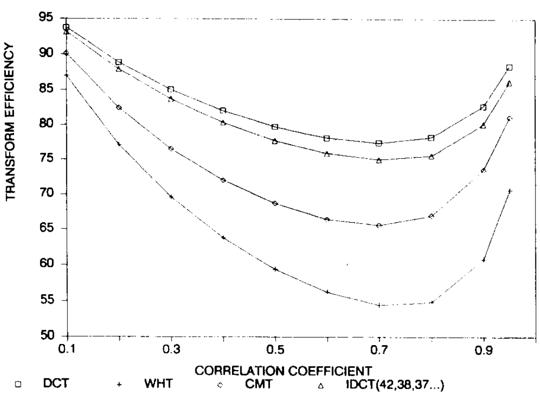

Fig. 1. Transform efficiency.

Two other transform performance measures are the maximum reducible bits (MRB) [6] defined by

$$\mathrm{MRB} = -\frac{1}{2n}\sum_{i=1}^{n}\log_2 b(i, i) \tag{19}$$

and, for a 2-D Markov model, the basis restriction error [7]

$$e(k) = 1 - \frac{\sum\sum_{u,v \in \omega} \sigma^2(u, v)}{\sum_{u=1}^{n}\sum_{v=1}^{n} \sigma^2(u, v)} \qquad 1 \le k \le 15. \tag{20}$$

They both serve to quantify the compression ability of a transform. In (20), $\sigma^2(u, v)$ denotes the variance of the 2-D transformed elements. Let ω be the set containing k index pairs (u, v) corresponding to the largest k $\sigma^2(u, v)$, where $1 \le k \le 15$ is the number of coefficients retained. Then the basis restriction mean-square error is as given in (20). Figs. 2 and 3 contain the plots of MRB and $e(k)$, where it is seen that the ICT and DCT have equal compression ability and they are both superior to the WHT and CMT.

IV. CONCLUSIONS

In this correspondence, we have developed a new order-16 ICT. It retains the important properties of the DCT such as orthogonality and proper ordering of the values of the matrix elements. Its main advantage lies in having only integer values which in two cases can be represented perfectly by 6-b numbers, thus providing a potential reduction in the computation complexity. Additionally, it has comparable performance to that of the DCT.

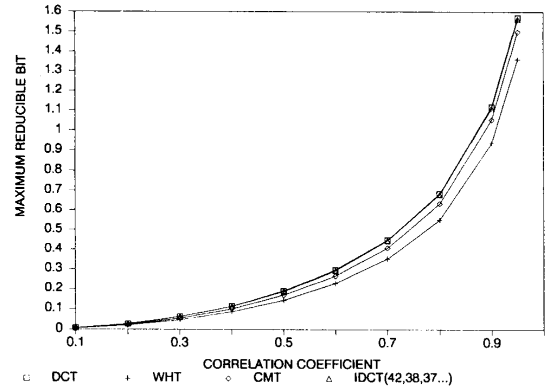

Fig. 2. Maximum reducible bit.

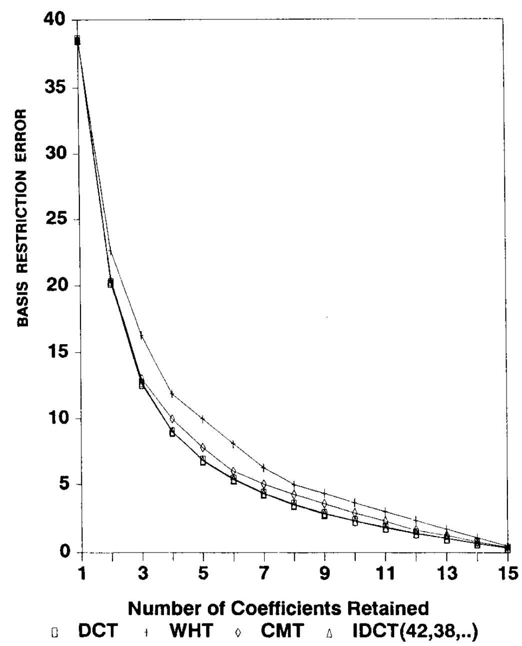

Fig. 3. Basis restriction error.

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Improved Spatial Smoothing Techniques for DOA Estimation of Coherent Signals

Weixiu Du and R. L. Kirlin

***Abstract*—In the context of coherent signal classification, spatial smoothing is necessary for the application of the eigen-based direction-of-arrival (DOA) estimation methods. However, the currently known spatial smoothing algorithms not only reduce the effective aperture of the array, but also do not consider the cross correlations of the subarray outputs. In this correspondence, an improved spatial smoothing algorithm is presented, which can fully utilize the correlations of the array outputs and produce a more stable estimate of the covariance matrix. Simulation results are provided to verify the theoretical prediction.**

I. Introduction

The eigen-based method has proven to be an effective means of obtaining direction-of-arrival (DOA) estimates of multiple signals from the outputs of a sensor array. However, the performance of algorithms based on this method will severely degrade when some of the signals are coherent or highly correlated. The spatial smoothing method for the coherent signals was first proposed by Evans *et al.* [1] and later developed by Shan *et al.* [2], Williams *et al.* [3], and Pillai and Kwon [4]. The solution is based on a preprocessing scheme that partitions the total array of sensors into subarrays and then generates the average of the subarray output covariance matrices.

In the resulting covariance matrix, the source covariance will possess full rank, and the eigen-based method can be applied effectively. However, this result is obtained at the cost of losing effective spatial aperture. In order to achieve a larger effective aperture, the method of modified spatial smoothing or forward-backward spatial smoothing, has been proposed by Evans *et al.* [1] and was extensively studied by Williams *et al.* [3] and Pillai and Kwon [4]. Using the improved spatial smoothing method, a uniform linear array can resolve as many as $2M/3$ coherent signals (with M representing the number of sensor elements), which coincides with the bounds of the number of coherent signals resolvable by such an array [5].

While many efforts have been made to increase the effective spatial aperture for the spatial smoothing scheme, any information in the cross correlations of the outputs of the subarrays has been ignored until a method was proposed [6], which improves the estimate of subset of parameters based on their covariance with the other parameters [7]. In the conventional spatial smoothing scheme, only the autocorrelations of the outputs of the individual subarrays are utilized to obtain the final estimate of the covariance matrix. In this correspondence, a new spatial smoothing method is presented to overcome the disadvantage of losing cross correlation in the conventional algorithms.

Manuscript received December 7, 1989; revised July 11, 1990.

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IEEE Log Number 9042693.

II. Spatial Smoothing for Eigen-Based Method

Consider a uniform linear array composed of M omnidirectional sensors receiving d $(d < M)$ narrow-band plane-wave signals from directions $\theta_1, \theta_2, \cdots, \theta_d$, centered at frequency ω_0. It is assumed that the signals and noises are stationary, zero mean uncorrelated random processes and further, the noises are temporarily and spatially white with variance σ^2. Using complex signal representation, the $M \times 1$ vector of the array outputs can be expressed as

$$\boldsymbol{r}(t) = \sum_{i=1}^{d} \boldsymbol{a}(\theta_i) s_i(t) + \boldsymbol{n}(t) \tag{1}$$

where $s_i(t)$ is the ith signal, $\boldsymbol{a}(\theta_i)$ is the $M \times 1$ steering or direction vector in the direction of θ_i:

$$\boldsymbol{a}(\theta_i) = [1, e^{-j\omega_0\tau_i}, \cdots, e^{-j(M-1)\omega_0\tau_i}]^T$$

where $\tau_i = (\Delta/c)\sin\theta_i$, with Δ representing the sensor spacing and c the propagation speed. And

$$\boldsymbol{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$$

represents the $M \times 1$ noise vector. Rewriting (1) in the alternative form, we have

$$\boldsymbol{r}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{2}$$

where

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_d)]$$

is the $M \times d$ steering matrix and

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_d(t)]^T$$

is the $d \times 1$ signal vector. We can now form the $M \times M$ covariance matrix of the array outputs

$$\begin{aligned} \boldsymbol{R} &= E[\boldsymbol{A}\boldsymbol{s}(t)\boldsymbol{s}^H(t)\boldsymbol{A}^H] + E[\boldsymbol{n}(t)\boldsymbol{n}^H(t)] \\ &= \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \sigma^2\boldsymbol{I} \end{aligned} \tag{3}$$

where $\boldsymbol{S} = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$ is the $d \times d$ signal covariance matrix and H denotes Hermitian transpose. The nonsingularity of the signal covariance matrix is the key to a successful application of the eigen-based method. Spatial smoothing is a preprocessing scheme that can guarantee this property even when the signals are coherent [1].

Assume that d coherent signals are received by a uniform linear array of M sensors. Let us partition this linear array into L overlapping subarrays of size $m \geq d + 1$, with sensors $\{1, 2, \cdots, m\}$ forming the first subarray, sensors $\{2, 3, \cdots, m + 1\}$ forming the second subarray, etc. It is easy to verify that the $m \times m$ cross-covariance matrix of the ith and the jth subarrays is given by

$$\boldsymbol{R}_m^{ij} = \boldsymbol{A}_m\boldsymbol{D}^{i-1}\boldsymbol{S}(\boldsymbol{D}^{j-1})^H\boldsymbol{A}_m^H + \sigma^2\boldsymbol{I}\delta_{ij} \tag{4}$$

where $\boldsymbol{A}_m$ is the $m \times d$ steering matrix of the reference subarray (usually the first subarray) and

$$\boldsymbol{D} = \mathrm{diag}\,[e^{-j\omega_0\tau_1}, e^{-j\omega_0\tau_2}, \cdots, e^{-j\omega_0\tau_d}] \tag{5}$$

is a $d \times d$ diagonal matrix. Note that in the above we assume noises from different subarrays are uncorrelated. However, when subarrays are overlapped, this assumption does not hold. In this case we can either approximately remove the noise covariance using its estimate or simply ignore its impact on our algorithm. For $M \gg m$ the latter choice is reasonable; effects in our simulations have shown to be negligible. To facilitate expressions for the backward smoothing, we form the $m \times m$ cross-covariance matrix

$$\overline{\boldsymbol{R}}_m^{ij} = \boldsymbol{J}(\boldsymbol{R}_m^{ij})^*\boldsymbol{J} \tag{6}$$

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