SINGLE ANTENNA POWER MEASUREMENTS BASED DIRECTION FINDING WITH INCOMPLETE SPATIAL COVERAGE

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ABSTRACT

This paper consider the problem of relaxing the spatial coverage requirement on the mechanical rotation in the direction finding (DF) approach based on the received power measurements from single antenna pointing to different directions. Under incomplete spatial coverage, we show that the least square (LS) solution used to transform the problem into its spectral form is no longer accurate due to its ill-conditioned system matrix. To overcome this, we propose an approach based on spatial remodeling of the spatial power measurements such that its spatial periodicity can be adjusted according to the spatial coverage. The approach also incorporates the Tikhonov regularization in calculating the LS solution based on the new system matrix. Upon arriving at the new spectral form, the Cadzow-annihilating filter method can then be used to estimate the direction-of-arrival. Both simulation and experimental results are presented to show the efficacy of the proposed method.

Index Terms— direction-of-arrival, single antenna direction finding, annihilating filter, Cadzow denoising.

1. INTRODUCTION

Unlike antenna array based multiple channel direction finding (DF), single antenna DF requires less stringent receiver implementation requirements [1–3]. Besides using only one receiver [4], single antenna DF is free from the calibration issue as well as the trade-off between the inter-element spacing and resolution [5]. Traditional single antenna DF relies *solely* on the directivity of the receiving antenna to locate the direction-of-arrival (DOA) of the transmitting source. In single-source transmitting case, the DOA is estimated as the direction at which the received power is the strongest. When extended to multiple-source transmitting case, its ability to resolve two closely-separated sources is limited by the antenna beamwidth.

This resolution limitation can be overcome by formulating the problem as spatial sampling of multiple Diracs whose locations are the DOAs and the rotating antenna pattern serves as the sampling kernel [6–8]. This formulation allows the received power to be simplified in a similar Fourier series form and the DOAs can then be estimated from its Fourier series coefficients.

The Fourier series expansion exploits the fact that the spatial power profile is a 2π-periodic function of the rotating angle. This in itself imposes the requirement for the rotating antenna to cover the full range of 2π rotation. In practice, it may be difficult if not impossible to meet this requirement given the limited degree-of-freedom of the rotating antenna or its physical constraints. From another point-of-view, if there exists some pre-defined range of angles in which the transmitters are located, it becomes impractical to rotate the antenna out of this range. This motivates us to address the single antenna power measurements based DF problem with incomplete spatial coverage.

The approach taken to address the problem is based on spatial remodeling of the Fourier series expansion. This involves redefining the spatial periodicity based on the limited spatial coverage. As the new spatial periodicity is less than 2π, we show that the spatial power profile is now expressed as a function of the truncated antenna pattern. This truncation will introduce discrepancy between the spatial power measurements and its new model. This discrepancy causes over-fitting in the least squares (LS) solution for estimating the Fourier series coefficients, leading to instable and inaccurate DOA estimation. To prevent this, we incorporate the Tikhonov regularization to the LS solution and utilize the earlier proposed Cadzow denoising with annihilating filter method to estimate the DOA.

2. BACKGROUND AND SIGNAL MODEL

Consider a single directional antenna receiving the transmission from K stationary emitters while pointing to the direction $\tilde{\theta}$. In the multipath-free propagation, the received signal at the antenna is: $x(t,\tilde{\theta}) = \sum_{k=1}^{K} g(\tilde{\theta}-\theta_k)s_k(t)$, where $g(\tilde{\theta}-\theta_k)$ denotes the antenna attenuation for the signal impinging from θ_k while the antenna pointing to $\tilde{\theta}$. $s_k(t)$ is the received signal waveform from k-th emitters, which includes the antenna effect and the propagation path loss. Assuming that the K signals are uncorrelated, the received signal power over a duration T can be well approximated by

$$p(\tilde{\theta}) = \frac{1}{T}\int_T |x(t,\tilde{\theta})|^2 dt \approx \sum_{k=1}^{K} \underbrace{|g(\tilde{\theta}-\theta_k)|^2}_{a(\tilde{\theta}-\theta_k)} p_k \tag{1}$$

where $p_k = \frac{1}{T}\int_T |s_k(t)|^2 dt$. This approximation is valid under the assumption that the cross term is negligible.

The antenna pattern $a(\theta)$ is a 2π-periodic non-negative function: $a(\theta) = a(\theta + 2\pi)$, $0 < a(\theta) \leq 1$, $\forall\theta$; and its bandlimited characteristics allow us to express in the Fourier series form:

$$a(\theta) = \sum_{m=-M}^{M} a_m e^{jm\theta} \tag{2}$$

As a result, we can further simplify (1) and obtain the similar Fourier series expression form as

$$p(\tilde{\theta}) = \sum_{m=-M}^{M} a_m \underbrace{\sum\nolimits_{k=1}^{K} p_k e^{-jm\theta_k}}_{y_m(\boldsymbol{\theta})} e^{jm\tilde{\theta}} \tag{3}$$

where $a_m y_m(\boldsymbol{\theta})$ is the Fourier series coefficients.

If the rotating angle $\tilde{\theta}$ can be taken from a uniform grid, the spectral analysis form $y_m(\boldsymbol{\theta})$ can be calculated from the discrete Fourier transform (DFT) of $p(\tilde{\theta})$. In the practical scenario where $\tilde{\theta}$ is random, $y_m(\boldsymbol{\theta})$ can be solved in a more general form as the least square (LS) solution to the following linear system of equations $\mathbf{p} = \mathbf{A}\mathbf{y}(\boldsymbol{\theta}) + \mathbf{n}$, where $\mathbf{p} = [p(\tilde{\theta}_1), \cdots, p(\tilde{\theta}_L)]^T$, the matrix $\mathbf{A}$ is a $L \times (2M+1)$ matrix with its (l,m)-th element given by $\mathbf{A}_{l,m} \stackrel{\text{def}}{=} a_m e^{jm\tilde{\theta}_l}$, the vector $\mathbf{y}(\boldsymbol{\theta})$ is a $2M+1$ vector whose elements are $y_m(\boldsymbol{\theta})$ with $m = \{-M, -M+1, \cdots, M\}$, and $\mathbf{n}$ is the additive noise component. If the linear system is overdetermined ($L \geq (2M+1)$), we can use the LS solution to compute for $\mathbf{y}(\boldsymbol{\theta})$: $\hat{\mathbf{y}}(\boldsymbol{\theta}) = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{p}$, where the superscript H denotes the matrix conjugate transpose operation. It is worth mentioning that the LS solution converges to the DFT operation when $\tilde{\theta}_l$ is located at uniform grid as $\mathbf{A}^H\mathbf{A}$ will form a diagonal matrix, thus reducing the computational complexity.

In the case where the spatial coverage of the antenna is incomplete, the accuracy of the LS solution degrades because the linear system becomes ill-conditioned [9]. In this paper, we aim to overcome this problem so that the proposed single antenna power measurements based DF can be applied to a more general case. To be exact, we propose the extension of single antenna power measurements based DF for the case where the antenna spatial coverage is constrained within a presumed angular range $[\tilde{\theta}_a, \tilde{\theta}_b]$ in which the emitters are located. Briefly, given L measurements of the spatial power $p(\tilde{\theta}_l)$ at directions $\tilde{\theta}_l$, our goal is to estimate the DOA of the K emitters θ_k under the condition that $(\tilde{\theta}_l, \theta_k) \in \mathbb{S}_\theta$ where $\mathbb{S}_\theta$ denotes the angular range defined as

$$\mathbb{S}_\theta = \{\theta \in \mathbb{R} \mid \quad \tilde{\theta}_a \geq \theta \geq \tilde{\theta}_b, \tilde{\theta}_b - \tilde{\theta}_a < 2\pi\} \tag{4}$$

3. PROPOSED APPROACH

3.1. Redefining Spatial Periodicity

Let $\tau \stackrel{\text{def}}{=} \tilde{\theta}_b - \tilde{\theta}_a$ denote the new spatial periodicity defined based on the presumed angular range or the range between the left-most and right-most of the antenna pointing directions. Now the antenna pattern function $a(\theta)$ can be remodeled as a periodic function of τ, thus allows us to express it in its new Fourier series expansion

$$\breve{a}(\theta) = \sum_{m=-M_t}^{M_t} \breve{a}_m e^{jm(2\pi/\tau)\theta} \tag{5}$$

where $\theta \in [\tilde{\theta}_a, \tilde{\theta}_b]$. This approximation is made by truncating the initial 2π-periodic antenna pattern function $a(\theta)$ at $-\tau/2$ on the left and at $\tau/2$ on the right, as well as enforcing the periodicity at τ: $\breve{a}(\theta) = \breve{a}(\theta + \tau)$. This implies that the approximation requires that the truncation error is assumed to be negligible $\int_{-\pi}^{-\tau/2} a(\theta)e^{-jm\theta}d\theta = \int_{\tau/2}^{\pi} a(\theta)e^{-jm\theta}d\theta \approx 0$.

Observe that the new Fourier series expansion in (5) is a scaled version of the initial Fourier series expansion by a factor of $2\pi/\tau$. Therefore from the duality property of the Fourier series, the new antenna bandwidth $(2M_t + 1)$ is inversely proportional to the stretching factor $2\pi/\tau$ and the new antenna bandwidth can be calculated as $M_t \approx \lceil \tau M/2\pi \rceil$, where $\lceil\cdot\rceil$ denotes the ceiling operation.

Subsequently, we can deduce the new Fourier series expansion for the spatial power measurement $\breve{p}(\theta)$ by substituting (5) into the initial spatial power measurement in (1) as follows

$$\breve{p}(\tilde{\theta}) = \sum_{m=-M_t}^{M_t} \breve{a}_m \underbrace{\sum\nolimits_{k=1}^{K} p_k e^{-jm(2\pi/\tau)\theta_k}}_{\breve{y}_m(\boldsymbol{\theta})} e^{jm(2\pi/\tau)\tilde{\theta}} \tag{6}$$

where $\breve{a}_m \breve{y}_m(\boldsymbol{\theta})$ denote its Fourier series coefficients.

Notice that the new spatial power profile expression in (6) can be seen as the scale version of the original spatial power profile expression in (3). This means that we can conveniently calculate $\breve{y}_m(\boldsymbol{\theta})$ from the incomplete spatial power measurement $p(\tilde{\theta})$, provided that $p(\tilde{\theta})$ is a good approximation of $\breve{p}(\tilde{\theta})$: $\mathbf{p} \approx \breve{\mathbf{A}}\breve{\mathbf{y}}(\boldsymbol{\theta}) + \mathbf{n}$. The LS solution of $\breve{y}(\boldsymbol{\theta}) = [\breve{y}_{-M_t}(\boldsymbol{\theta}), \cdots, \breve{y}_{M_t}(\boldsymbol{\theta})]^T$ is

$$\widehat{\breve{\mathbf{y}}}(\boldsymbol{\theta}) = (\breve{\mathbf{A}}^H\breve{\mathbf{A}})^{-1}\breve{\mathbf{A}}^H\mathbf{p} \tag{7}$$

where $\breve{\mathbf{A}}$ is the new $L \times (2M_t + 1)$ system matrix with its (l,m)-th element given by $\breve{\mathbf{A}}_{l,m} \stackrel{\text{def}}{=} \breve{a}_m e^{jm(2\pi/\tau)\tilde{\theta}_l}$. Notice that the new system matrix is no longer ill-conditioned because of the scaling factor $2\pi/\tau$ and the vector $\breve{\mathbf{y}}(\boldsymbol{\theta})$ has a lower dimension as compared to its initial representation $\mathbf{y}(\boldsymbol{\theta})$ because $M_t < M$.

To build the matrix $\breve{\mathbf{A}}$ for solving $\breve{y}_m(\boldsymbol{\theta})$ in (7), we need to find the unknown coefficients $\breve{a}_m$. To calculate this parameter, we can use the LS regression technique to fit the Fourier series expansion in (5) through the discretized antenna pattern $a(\theta_q)$ where $-\tau/2 \leq \theta_q \leq \tau/2$. That is, the coefficients $\breve{a}_m$ in the vector form $\mathbf{c} \stackrel{\text{def}}{=} [\breve{a}_{-M_t}, \breve{a}_{-M_t+1}, \cdots, \breve{a}_0, \cdots, \breve{a}_{M_t}]^T$ can be calculated using

$$\mathbf{c} = (\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H\mathbf{a} \tag{8}$$

where $\mathbf{a} = [a(\theta_1), \cdots, a(\theta_Q)]^T$ and the $Q \times (2M_t + 1)$ matrix $\mathbf{B}$ is composed of $\mathbf{B}_{q,m} = e^{jm(2\pi/\tau)\theta_q}$. It is important to note that the regression in (8) requires that $Q \geq (2M_t + 1)$.

3.2. Tikhonov Regularization

Recall that the LS solution of $\breve{\mathbf{y}}(\boldsymbol{\theta})$ requires that $\breve{p}(\tilde{\theta}_l)$ is a good approximation of $p(\tilde{\theta}_l)$. As $\breve{p}(\tilde{\theta}_l)$ is derived based on the truncated antenna pattern model in (5) while $p(\tilde{\theta}_l)$ is based on the initial 2π-periodic function, the mismatch between $\breve{p}(\tilde{\theta}_l)$ and $p(\tilde{\theta}_l)$ is due to the truncation error. Here, we introduce Tikhonov regularization term to the LS solution to prevent overfitting of the new spatial power profile to the received power measurements $p(\tilde{\theta}_l)$. Unlike the single antenna DF problem with complete spatial coverage, the need for the regularization term in the single antenna DF problem with incomplete spatial coverage arises due to this approximation error.

While the LS solution is obtained from minimizing the sum of square error $\|\mathbf{p} - \breve{\mathbf{A}}\breve{\mathbf{y}}(\boldsymbol{\theta})\|_2^2$ where $\|\cdot\|_2$ denote ℓ_2-norm operation, the Tikhonov regularization method introduces a regulating term to the minimization formulation

$$\min_{\breve{\mathbf{y}}(\boldsymbol{\theta})} \|\mathbf{p} - \breve{\mathbf{A}}\breve{\mathbf{y}}(\boldsymbol{\theta})\|_2^2 + \alpha^2\|\breve{\mathbf{y}}(\boldsymbol{\theta})\|_2^2 \tag{9}$$

where α is the regularization parameter. The explicit solution to the optimization in (9) is given by

$$\widehat{\breve{\mathbf{y}}}(\boldsymbol{\theta}) = (\breve{\mathbf{A}}^H\breve{\mathbf{A}} + \alpha^2\mathbf{I})^{-1}\breve{\mathbf{A}}^H\mathbf{p} \tag{10}$$

The choice of the regularization parameter α will determine how loosely fitted the spatial power measurements to its model. Large α will produce the solution that is smooth, thus de-emphasizing the high frequency components and decreasing the solution norm $\rho(\alpha)$ expressed as $\rho(\alpha) \stackrel{\text{def}}{=} \|\widehat{\breve{\mathbf{y}}}(\boldsymbol{\theta})\|_2 = \|(\breve{\mathbf{A}}^H\breve{\mathbf{A}} + \alpha^2\mathbf{I})^{-1}\breve{\mathbf{A}}^H\mathbf{p}\|_2$. On the other hand, when α is set near to zero, the solution will converge to the LS solution and the residual norm $\varepsilon(\alpha)$ will be minimum $\varepsilon(\alpha) \stackrel{\text{def}}{=} \|\mathbf{p} - \breve{\mathbf{A}}\widehat{\breve{\mathbf{y}}}(\boldsymbol{\theta})\|_2 = \|\mathbf{p} - \breve{\mathbf{A}}(\breve{\mathbf{A}}^H\breve{\mathbf{A}} + \alpha^2\mathbf{I})^{-1}\breve{\mathbf{A}}^H\|_2$.

Hence, the good setting of α should be balanced between minimizing the solution and residual norms

$$\alpha_{opt} = \arg\min_{\alpha} \quad \rho(\alpha) + \varepsilon(\alpha) \tag{11}$$

Although there is no closed-form expression to α_{opt}, the numerical solution can be obtained using the L-curve method as described in [10]. Thus, the final estimate of $\widehat{\breve{\mathbf{y}}}(\boldsymbol{\theta})$ is given by (10) with the value α calculated in (11).

Having obtained the good estimate of a typical spectral analysis formulation $\breve{y}_m(\boldsymbol{\theta})$, we can now apply the Cadzow-annihilating filter method for extracting the DOA information from the exponent terms of $\widehat{\breve{y}}_m(\boldsymbol{\theta})$. It is worth mentioning that since the exponent terms in $\breve{y}_m(\boldsymbol{\theta})$ is defined as $(2\pi/\tau)\theta_k$, the relationship between the polynomial roots z_k of the annihilating filter coefficients and the DOA is now given by $z_k = e^{-j(2\pi/\tau)\theta_k}$. Therefore, the DOA estimates is calculated using

$$\hat{\theta}_k = j(\tau/2\pi)\log z_k \tag{12}$$

In summary, the proposed algorithm for single antenna power measurements based DF with the spatial coverage τ can be listed as follows

1. Compute the new antenna pattern coefficients $\breve{a}_m$ using (8).
2. Estimate $\widehat{\breve{\mathbf{y}}}(\alpha)$ using (10) with the regularization parameter α computed using (11).
3. Form a Toeplitz matrix from $\widehat{\breve{\mathbf{y}}}$ and apply Cadzow denoising.
4. Compute the annihilating filter coefficients from the $K+1$ eigenvectors of the denoised Toeplitz matrix.
5. Find the K polynomial roots z_k from the $K+1$ annihilating filter coefficients.
6. The DOA estimates can be calculated using (12).

3.3. Cramér Rao Bound

Here, we extend the Cramér Rao Bound (CRB) derivation from [8] and obtain a general expression for both complete and incomplete spatial coverage. Assume that the spatial power measurements generally follow the signal model in (6) and the approximation error is included in the additive noise, we have

$$\mathrm{CRB}^{-1}(\boldsymbol{\theta}) = 2\,\mathrm{Re}\left\{\mathrm{diag}[\mathbf{r}]^H \breve{\mathbf{D}}^H \breve{\boldsymbol{\Sigma}}_y^{-1} \mathbf{P}_{\breve{\mathbf{Q}}}^{\perp} \breve{\mathbf{D}}\,\mathrm{diag}[r]\right\} \tag{13}$$

with the following matrices and vectors definitions: $\mathbf{r} = [p_1, \cdots, p_K]^T$, $\mathbf{P}_{\breve{\mathbf{Q}}}^{\perp} = \mathbf{I} - \breve{\mathbf{Q}}\left(\breve{\mathbf{Q}}^H\breve{\mathbf{Q}}\right)^{-1}\breve{\mathbf{Q}}^H$, $\breve{\mathbf{Q}} = [\breve{\mathbf{q}}_1, \cdots, \breve{\mathbf{q}}_K]^T$, $\breve{\mathbf{q}}_k = [e^{-jN(2\pi/\tau)\theta_k}, \cdots, 1{+}2\sigma^2, \cdots, e^{jN(2\pi/\tau)\theta_k}]^T$, $\breve{\mathbf{D}} = \frac{\partial}{\partial(2\pi/\tau)\theta_k}\breve{\mathbf{Q}}$, and $\breve{\boldsymbol{\Sigma}}_y = \left(\breve{\mathbf{A}}^{\dagger}\right)\mathrm{diag}\{[\sigma_{p1}^2, \cdots, \sigma_{pL}^2]^T\}\left(\breve{\mathbf{A}}^{\dagger}\right)^H \cdot \left(\breve{\mathbf{A}}^H\breve{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}^H$. Notice that, when the spatial coverage is complete or the periodicity τ approaches 2π, the term $\breve{\mathbf{D}}^H\breve{\boldsymbol{\Sigma}}_y^{-1}\mathbf{P}_{\breve{\mathbf{Q}}}^{\perp}\breve{\mathbf{D}}$ will converge to the term $\mathbf{D}^H\boldsymbol{\Sigma}_y^{-1}\mathbf{P}_{\mathbf{Q}}^{\perp}\mathbf{D}$ defined in the CRB derivation in [8].

4. SIMULATION AND EXPERIMENTAL RESULTS

Consider a directional antenna with the antenna pattern, mathematically modeled as (2) with $M = 11$ and its Fourier coefficients a_m completely known, as shown in Fig. 1. It is used to receive the propagating uncorrelated signals impinging from $K = 2$ sources at $\theta_k = \{71.3588^\circ, 49.3241^\circ\}$. As many as $L = 26$ received power values are calculated when the antenna is pointing at various directions from 0° to 130°. The signal-to-noise ratio (SNR) for each observations is assumed to be -5dB.

Fig. 2 shows the continuous spatial power profile and its discrete measurements recorded from the received power at random pointing directions. The DOA estimates are obtained by applying different variants of the proposed algorithms as compared to the actual DOAs. For brefity, we use the label 'P1' and 'P2' as the proposed method where $\widehat{\breve{\mathbf{y}}}$ is calculated using (10) and (7), respectively. And the label '[8]' refers to the previously proposed method in [8] where $\hat{\mathbf{y}}$ (not $\widehat{\breve{\mathbf{y}}}$) is computed using the LS solution. As shown in Fig. 2, we can see that the Tikhonov regularization plays an important role in significantly increasing the accuracy of the estimation when the operating SNR is relatively low.

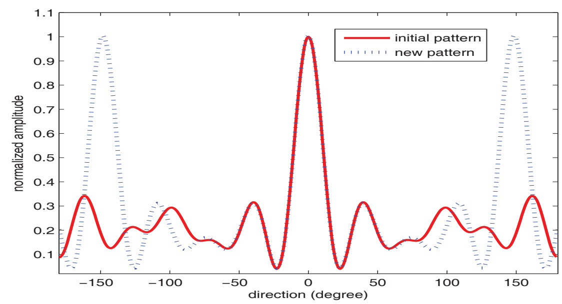

Fig. 1. Comparison between a typical realization of antenna pattern $a(\theta)$ simulated according to (2) with $M = 11$ and its truncated and remodeled as $\breve{a}(\theta)$ expressed in (5) with $\tau = 130^\circ$.

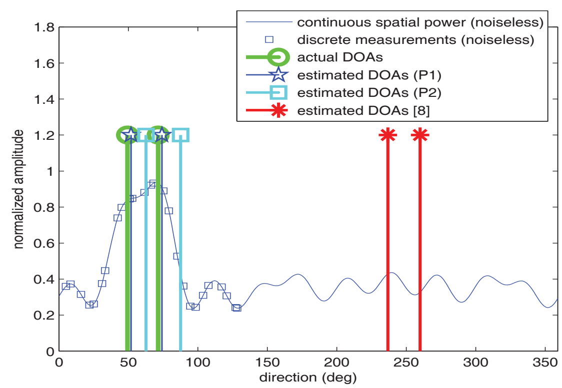

Fig. 2. Illustration of the continuous spatial power profile and its discrete measurements in the noiseless case, with the estimated DOAs.

In the following, we consider Monte Carlo simulations with 1000 realizations with the same parameters as the previous simulation over a wide range of SNR value. From the results of these realizations, we then calculate the root-mean-square error (RMSE) for every SNR value evaluated. Fig. 3 shows the RMSE plots as a function of SNR. In general, the RMSE reduces as the SNR increases. In comparison with the results obtained from applying the method without Tikhonov (P2), the method with Tikhonov (P1) achieves better robustness in low SNR environment. Although the performance of both P1 and P2 converges to that of the lower bound as SNR increases, the method P1 first converges at 0 dB SNR while the method P2 achieves that after 5 dB SNR.

In the following, we present the experimental results using the hardware realization of the single antenna DF system described in [8]. The received power measurements data is recorded from the experiments conducted at the foyer of Research Techno Plaza (RTP), where two sources are setup to be transmitting from 242^o and 286^o with respect to true north as depicted in Fig. 4.

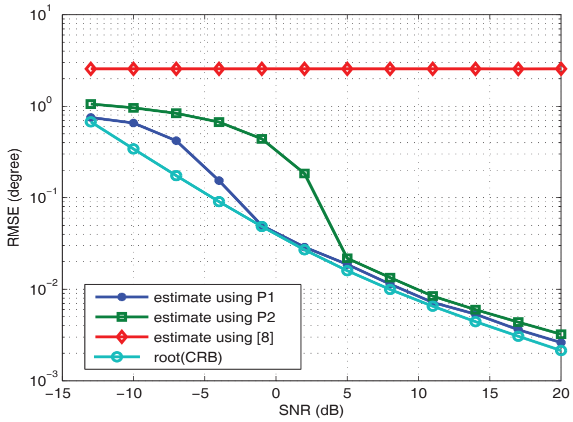

Fig. 3. RMSE performance plot as a function of SNR, in comparison with the square root of CRB in (13).

As many as $L = 14$ received power measurements are recorded while the antenna is rotated pointing to random directions from 120° to 360°. The normalized received power measurements as a function of the pointing directions are shown in Fig. 5. We first remodeled the antenna pattern with the periodicity defined as $\tau = 240^\circ$ and the new Fourier coefficients are calculated using (8). Note that the initial antenna pattern is modeled using the Fourier series expansion with $M = 8$ while the new model uses only $M_t = 6$ coefficients. The reduction in the antenna pattern bandwidth from $(2M + 1)$ to $(2M_t + 1)$ implies that lesser power measurements L is required to satisfy the requirement of the LS solution. In this case, the new antenna pattern requires only 13 or more measurements.

The estimation result using the method P1 is compared with the actual DOAs and the results obtained from applying the earlier method in [8]. We can conclude from these results that the proposed method helps to recover the estimation from the break down in the case of incomplete spatial coverage measurements.

5. CONCLUSSION

In this paper, we address multiple signal DOA estimation problem based on the received power measurements from single antenna pointing to different directions with limited spatial coverage. Our approach is based on redefining the periodicity of the spatial power measurements according to its spatial coverage, so that we can approximate the vector of power measurements as a new linear matrix equation with the system matrix that is no longer ill-conditioned. This motivates us to propose the LS solution with regularization that transforms the problem into a spectral analysis problem so that the DOAs can be estimated using the Cadzow-annihilating filter method. We demonstrated using simulations as well as experimental results the efficacy of the proposed approach. Furthermore, we also extended the CRB derivation and show that the performance converges to the CRB.

6. REFERENCES

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Fig. 4. Experimental setup for RTP Foyer experiment.

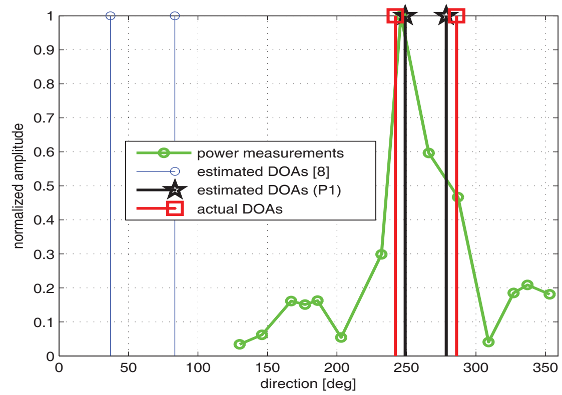

Fig. 5. Experimental results for RTP Foyer experiment.