FRI SENSING: SAMPLING IMAGES ALONG UNKNOWN CURVES

Ruiming Guo and Thierry Blu

Department of Electronic Engineering, The Chinese University of Hong Kong
email: 1155100873@link.cuhk.edu.hk, thierry.blu@m4x.org

ABSTRACT

While sensors have been widely used in various applications, an essential current trend of research consists of collecting and fusing the information that comes from many sensors. In this paper, on the contrary, we would like to concentrate on a unique mobile sensor; our goal is to unveil the multidimensional information entangled within a stream of one-dimensional data, called FRI Sensing. Our key finding is that, even if we don't have any position knowledge of the moving sensors, it's still possible to reconstruct the sampling trajectory (up to a linear transformation and a shift), and then reconstruct an image that represents the physical sampling field under certain hypotheses. We further investigate the reconstruction hypotheses and propose novel algorithms that could make this 1D to 2D reconstruction feasible. Experiments show that the proposed approach retrieves the sampling image and trajectory accurately under the developed hypotheses. This method can be applied to geolocation localization applications, such as indoor localization and submarine navigation. Moreover, we show that the proposed algorithms have the potential to visualize the one-dimensional signal, which may not be sampled from a real 2D/3D physical field (e.g. speech and text signals), as a two- or three-dimensional image.

Index Terms— Mobile sensing, finite rate of innovation, sampling theory, image and trajectory reconstruction.

1. INTRODUCTION

The wide availability of cheap sensors of various kinds (inertia, magnetic field, light, temperature, pressure, chemicals etc.) makes it possible to render a series of technologies and applications [1], [2]. An important topic in sensor application research is combining data from various sensors, in particular fusing geolocation information with other sampled data [3], [4]. For example, in environmental monitoring [5], the positioning information is essential for pressure and temperature sensors to make its data meaningful.

Different from information fusion, conversely we aims to extract and reveal the multidimensional information hidden within a series of one-dimensional time samples. We propose to explore the possibility of reconstructing the physical field and moving trajectory from a sequence of one-dimensional signals obtained from a unique mobile sensor without any positioning device. More abstractly, the core problem is about reconstructing the two-dimensional image and the sampling trajectory from one-dimensional time samples without relying on extra location data. In this paper, we call this task as *FRI Sensing*. At first glance, this inverse recovery process may look like impossible since it suffers from the absence of multidimensional information (e.g. location, velocity, etc). However, in this work, we show that there is valuable and adequate spatial information (2D, 3D) hidden within the 1D sensor data.

Of course, for our program to be successful, both the trajectory of the sensor and the field sampled should satisfy some kind of conditions. In order to figure out these constraints, we thus abide a methodology where we start from the most simple case and generally relax the constraints step by step to fit the real applications more accurately. At first, we investigate the conditions when both the trajectory of the sensor and the sampling physical field satisfy some kind of sparsity. Namely, the sampling trajectory is made up of several straight line segments and the image is a finite sum of spatial sinusoids. Then, we relax the hypotheses on the sampling curve to make it curved and explore the feasible reconstruction constraints.

We should point out that being able to extract multidimensional information from as little as one stream of one-dimensional time samples may prove quite useful. A direct application to which the proposed methods could be applied is geolocation positioning. Usually, people choose to install a GPS (Global Positioning System) device to obtain the location information with the help of satellites. However, in many scenarios, positioning via GPS is infeasible and prohibitive due to energy limitations or environmental restrictions [6], [7]. For example, in wild animal tracking [8], localization is a very difficult, expensive process that require bulky tags that run out of energy quickly [9]. GPS signal can also be too weak to be detected under deep water [10], [11] or blocked by the barriers or obstacles [12]. Instead of relying on the GPS device, our proposed methods could reconstruct the moving trajectory only through the sampled data, which could potentially improve the positioning accuracy.

While a picture is worth a thousand words, as a completely different application, we plan to demonstrate the potential of this algorithm for the purpose of visualizing one-dimensional signals as a two- or three-dimensional image. Speech, text signals, but also many others could be considered eventually, allowing to identify visual clues and characteristics (geometry, texture, etc) in non-visual signals, which do not necessarily have the corresponding 2D/3D ground truth images.

The rest of the paper is organized as follows: We show the basic framework and analysis of FRI sensing in Sect.2. Then, the details of our experiments are presented in Sect.3. In Sect.4, we talk about the extensions and potential improvements of the proposed method. We conclude the paper in Sect. 5.

2. FRI SENSING

2.1. Problem Description

Sensor have been broadly used in various areas and render many applications possible, in particular by fusing data obtained from many sensors. In this paper however, we choose to concentrate on only one mobile sensor that provides one-dimensional time samples from a two-dimensional physical field (e.g. temperature, pressure, etc), and we aim to investigate how much two-dimensional information is hidden there, i.e. FRI sensing problems. Generally speaking, given the one-dimensional samples only, without any additional information, the problem we propose to investigate is twofold:

- reconstruct the two-dimensional sampling trajectory of the sensor (up to a linear geometrical transformation and a shift)
- reconstruct the two-dimensional image that represents the physical field

To be more specific, assume that a mobile sensor is sampling an image along a curve without any extra positioning information. From the sequence of sensor measurements, our goal is to retrieve the original image and sampling trajectory of the sensor. Fig. 1 sketches the above description in a more intuitive way.

Estimating the 2D image and curve from 1D curvilinear samples requires solving a number of problems. The first difficulty is in identifying the conditions under which it's possible to retrieve the target curve and image. Since the problem is ill-posed, achieving the reconstruction within given tolerance from as little as one-dimensional samples requires strong hypotheses on both the sampling curve and image. It's reasonable to require that the trajectory changes slowly with time and the image has some form of simplicity/sparsity. Thus, we need some form of sampling theorems to characterize the constraints between the image and curve. A second challenge is, given certain tolerance how to retrieve the curve information contained within the 1D samples. We need some method to extract the characterizations of the curve. A third problem is how to reconstruct the image as precise as possible. A possibility to use image interpolation algorithms to reconstruct

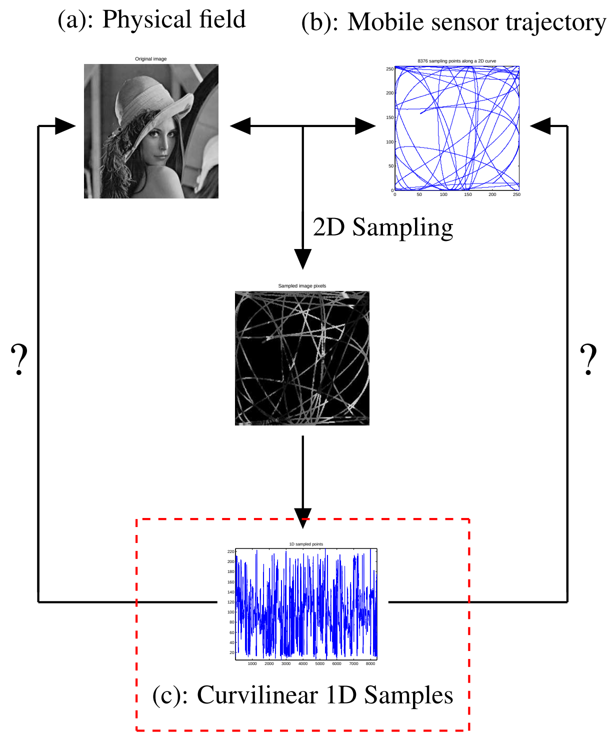


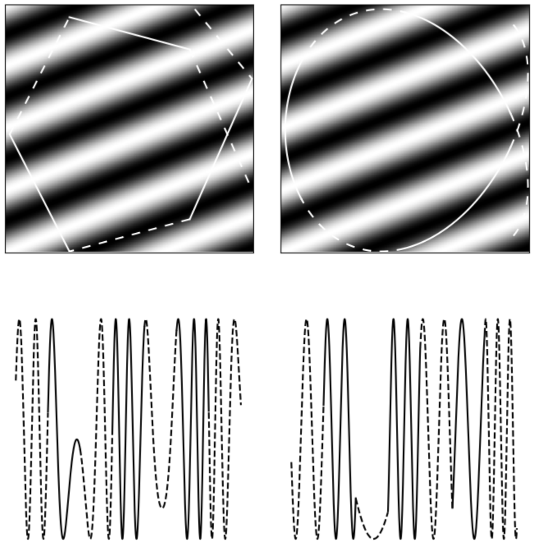
Fig. 1: Our goal ("FRI sensing") is to reconstruct the physical filed (a) and the sampling trajectory (b) of the mobile sensor from the measured curvilinear 1D time samples (c) (framed by a red box).

the two-dimensional physical field from the samples along the trajectory found earlier. But with the knowledge of the image frequency components obtained from the high-resolution frequency estimation algorithms, we may make it more accurate and robust.

2.2. Strict Hypotheses of FRI Sensing

At first sight, reconstructing both the curve and image from the given 1D samples seems quite ambitious. However, we will show that this is achievable and feasible if we control the hypotheses on the images and the sampling curve properly. Again, we aim to show that there is valuable spatial (2D, 3D) information entangled within the 1D uniform samples. For this reason, we intend to follow a methodology whereby we slowly relax the hypotheses needed for the algorithms to be successful, as depicted in Fig. 2.

First, we will investigate the conditions of feasible reconstruction when we know that the physical field is a finite sum of sinusoidal images and the trajectory is made up of several line segments. The idea that we will exploit here is that sampling a sum of two-dimensional sinusoids uniformly along a straight line results in a sum of one-dimensional sinusoids. If each segment is long enough, using a very robust high-resolution frequency estimation (Finite rate of innovation-FRI) [13], [14], [15] algorithm that we have developed and by pairing properly the results along the trajectory,



(a) (b)

Fig. 2: (a) Sampling a sum of spatial sinusoids along straight line segments results in an exact piecewise sinusoidal 1D signal. (b) Sampling the same image along a curved trajectory with small curvature results in an approximate piecewise sinusoidal 1D signal.

we will eventually obtain the full two-dimensional velocity of the mobile sensor as a function of time, and parameters which characterize completely the two-dimensional physical field. The parametric representation of the velocity can be converted into a tentative trajectory that can be refined based on the image found, the samples of which should match the one-dimensional data.

Then the framework will be extend to smooth curve if the curve is flat enough compared to the image spatial variations, which requires the curvature radius of the sampling curve satisfies some form of conditions. Next, we will then extend this algorithm so that it can be applied to fields that can be modelized locally as a sum of few sinusoids —namely, natural images. We will investigate the limitations on the trajectories and the images that can be reconstructed. The quantitative description of these constraints are beyond the scope of this paper and we will talk about this part of work in the following papers.

If the sampling is uniform along L segments of straight lines, it's easy to see that, in each segment, the samples obtained are sum of sinusoids. More specifically, assume that the image $I(\mathbf{r})$ (where $\mathbf{r} = (x, y)^T$ is a Cartesian location) can be expressed as

$$I(\mathbf{r}) = \sum_{k=1}^{K} C_k e^{j\mathbf{u}_k^T \mathbf{r}} \tag{1}$$

for some finite integer $K \geqslant 2$ and a sequence of K image frequency vectors $\mathbf{u_k}$. Assume, moreover that the sampling locations are uniform along L line segments:

$$r(t) = \mathbf{a}_l t + \mathbf{b}_l,\ l = 1, 2, \cdots L \tag{2}$$

Thus, the obtained samples $s_l(t),\ l = 1, 2, \cdots L$ along each of the L segments take the form of a sum of sinusoids

$$s_l(t) = \sum_{k=1}^{K} C_{l,k} e^{j\omega_{l,k} t},\ l = 1, 2, \cdots L \tag{3}$$

where $C_{l,k} = C_k e^{j\mathbf{u}_k^T \mathbf{b}_l}$ and $\omega_{l,k} = \mathbf{u}_k^T \mathbf{a}_l$, for $l = 1, 2, \cdots L$. Since the values of t are uniformly sampled within each segment by assumption, it's possible to retrieve $C_{l,k}$ and $\omega_{l,k}$ using a high-resolution frequency estimation algorithm. Here, we utilize our recent very accurate and robust FRI algorithm which is able to approximate arbitrary signals as a sum of sinusoids—up to the noise level.

Then, we observe that the following $K \times L$ matrix by appropriate arrangement

$$\mathbf{\Omega} = \begin{bmatrix} \omega_{1,1} & \omega_{2,1} & \cdots & \omega_{L,1} \\ \omega_{1,2} & \omega_{2,2} & \cdots & \omega_{L,2} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{1,K} & \omega_{2,K} & \cdots & \omega_{L,K} \end{bmatrix} \tag{4}$$

$$= \underbrace{\left[\mathbf{u}_1, \mathbf{u}_2 \cdots \mathbf{u}_K\right]^T}_{K \times 2 \text{ matrix } \mathbf{U}} \cdot \underbrace{\left[\mathbf{a}_1, \mathbf{a}_2 \cdots \mathbf{a}_L\right]}_{2 \times L \text{ matrix } \mathbf{A}} \tag{5}$$

is actually a matrix of rank at most equal to 2. This means that, we need an image made of at least $K = 2$ different sinusoids, and a curve with at least $L = 2$ segments to retrieve the matrix of spatial frequencies $\mathbf{U}$ and the matrix of curve directions $\mathbf{A}$ up to an arbitrary 2×2 linear geometric transformation $\mathbf{Q}$: $\mathbf{U}^{'} = \mathbf{Q}^{-T}\mathbf{U}$ and $\mathbf{A}^{'} = \mathbf{QA}$. Of course, an image with richer frequency components ($K \geqslant 3$) and a curve with more line segments ($L \geqslant 3$) will lead to more interesting results and also more robust reconstruction (Utilizing SVD to exploit the rank-2 property of the matrix $\mathbf{\Omega}$).

2.3. Frequency pairing

Although we did not mention it earlier, pairing the frequencies found in two different segments could be an issue. However, there are several clues that help identify which frequency in segment l corresponds to which frequency in segment $l^{'}$. One available choice is the amplitude of $C_{l,k}$ of the sinusoid attached to that frequency, whose absolute value should be invariant across segments. Another clue is, of course, that when the frequencies have been paired accurately, the matrix $\mathbf{\Omega}$ should be of rank-2—or in the situation of inaccuracies, can be approximated accurately by a rank-2 matrix. They both provide effective criterion to evaluate the paired results.

2.4. Relaxing the hypotheses on the curve

Instead of assuming that the sampling curve is made of straight line segments, we suppose that the curvature radius—i.e. the inverse of the curvature—of the curve is small enough compared to the spatial frequencies of the sinusoidal images, then we can still apply the line segment strategy, as illustrated in Fig. 2. The idea we exploit here is that using several line segments to approximate the original sampling curve.

2.5. Scheme Overview

Based on the above process, we propose a novel algorithm to reconstruct the physical field and sampling trajectory from one-dimensional samples as shown in Algorithm 1.

Algorithm 1: Reconstructing image and curve from 1D samples

Input: 1D uniform samples $s(t)$

1: Divide the samples into several sub-signals $s_l(t),\ l = 1, 2, \cdots L$

2: Estimate local frequency components $C_{l,k}$ and $\omega_{l,k}$

3: Obtain the frequency matrix $\mathbf{\Omega}$ through pairing process

4: Estimate the matrix of spatial frequencies $\mathbf{U}$ and curve directions $\mathbf{A}$

Output: The reconstructed physical field $I(\mathbf{r})$ and the sampling curve $\mathbf{r}(t)$

3. EXPERIMENTAL RESULTS

In order to estimate the performance of our proposed method, we sample an image made up of 5 spatial sinusoids ($K = 5$) along a trajectory. Here, the samples sensed are corrupted by 10dB PSNR noise during acquisition. The sampling process and the reconstruction results are shown in Fig.3.

We need to define some measures to evaluate the error between the final reconstruction and the ground truth. Let C_1 and C_2 denote the ground truth and the reconstructed curve, respectively. Then, we define:

$$\mathrm{D} = \max_{z_1 \in C_1} \min_{z_2 \in C_2} ||z_1 - z_2|| \tag{6}$$

which characterizes the largest distance between the reconstruction trajectory C_2 and the ground truth C_1. As for the image, we use the PSNR value to characterize the image reconstruction accuracy.

Thanks to the the robust high-resolution frequency estimation methods, we can extract the frequency contents with expected accuracy. The experimental results show that the proposed method can reconstruct the sampling curve (D = 2.14 pixels) and the image accurately (PSNR = 20.56 dB). Note that, compared to the frequency estimates, amplitudes are less accurate and robust, which reminds us that it is not necessary to reconstruct the sampled image only through the estimated parameters of the image contents. Since we have retrieved the samples position, a possibility is to use radial basis functions to perform the high quality image interpolation algorithm to reconstruct the two-dimensional physical field for scattered data.

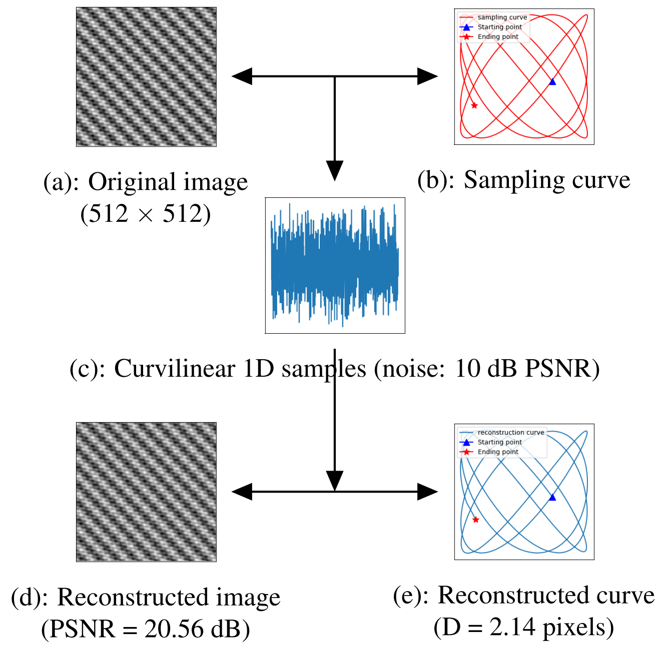


Fig. 3: Reconstruction of an image made up of 5 spatial sinusoids from noisy curvilinear samples.

4. EXTENSION AND FUTURE WORK

In the future, we plan to further relax the hypotheses on the physical field. Relaxing the hypotheses on the 2D physical field means making the global sum of sinusoids assumption to hold only locally. Hence, we only expect to be able to reconstruct the curve locally, which implies that the global reconstruction may suffer from some form of drift. We plan to develop a sampling theorem that guarantees reconstruction under hypotheses on both the physical field and on the curve. We also plan to extend our method to three-dimensional physical field sampled along 3D curves. We hope that this work will ultimately provide new tools for processing one-dimensional data as images.

5. CONCLUSIONS

In this paper we show it is possible to retrieve the multidimensional information that is hidden within a stream of one-dimensional time samples. A novel method (FRI sensing) based on high-resolution frequency estimation techniques is proposed to obtain the 2D image and curve reconstruction from the one-dimensional uniform samples. Experimental results verify our theory, showing that the proposed method is capable of reconstructing the physical field and the sampling trajectory accurately.

6. REFERENCES

[1] P. M. Wadke, M. T. Burrows, D. Meldrum, and A. J. Davies, "Using magneto-resistive sensors to monitor animal behaviour: a case study using limpets," in *O-CEANS 2007*, Sept 2007, pp. 1–6.

[2] D. Niculescu and Badri Nath, "Ad hoc positioning system (aps) using aoa," in *IEEE INFOCOM 2003. Twenty-second Annual Joint Conference of the IEEE Computer and Communications Societies (IEEE Cat. No.03CH37428)*, March 2003, vol. 3, pp. 1734–1743 vol.3.

[3] J. P. Dominguez-Morales, A. Rios-Navarro, M. Dominguez-Morales, R. Tapiador-Morales, D. Gutierrez-Galan, D. Cascado-Caballero, A. Jimenez-Fernandez, and A. Linares-Barranco, "Wireless sensor network for wildlife tracking and behavior classification of animals in doana," *IEEE Communications Letters*, vol. 20, no. 12, pp. 2534–2537, Dec 2016.

[4] I. F. Akyildiz, Weilian Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Communications Magazine*, vol. 40, no. 8, pp. 102–114, Aug 2002.

[5] Alan Mainwaring, David Culler, Joseph Polastre, Robert Szewczyk, and John Anderson, "Wireless sensor networks for habitat monitoring," in *Proceedings of the 1st ACM International Workshop on Wireless Sensor Networks and Applications*, New York, NY, USA, 2002, WSNA '02, pp. 88–97, ACM.

[6] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, R. L. Moses, and N. S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54–69, July 2005.

[7] Xiang Ji and Hongyuan Zha, "Sensor positioning in wireless ad-hoc sensor networks using multidimensional scaling," in *IEEE INFOCOM 2004*, March 2004, vol. 4, pp. 2652–2661 vol.4.

[8] Rebecca N. Handcock, Dave L. Swain, Greg J. Bishop-Hurley, Kym P. Patison, Tim Wark, Philip Valencia, Peter Corke, and Christopher J. ONeill, "Monitoring animal behaviour and environmental interactions using wireless sensor networks, gps collars and satellite remote sensing," *Sensors*, vol. 9, no. 5, pp. 3586–3603, 2009.

[9] S. Ehsan, K. Bradford, M. Brugger, B. Hamdaoui, Y. Kovchegov, D. Johnson, and M. Louhaichi, "Design and analysis of delay-tolerant sensor networks for monitoring and tracking free-roaming animals," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1220–1227, March 2012.

[10] H. P. Tan, Z. A. Eu, and W. K. G. Seah, "An enhanced underwater positioning system to support deepwater installations," in *OCEANS 2009*, Oct 2009, pp. 1–8.

[11] T. Shimura, Y. Watanabe, H. Ochi, and H. C. Song, "Long-range time reversal communication in deep water: Experimental results," *The Journal of the Acoustical Society of America*, vol. 132, no. 1, pp. EL49–EL53, 2012.

[12] M. L. Yin and R. Arellano, "A risk analysis framework on gps user range accuracy," in *2012 Proceedings Annual Reliability and Maintainability Symposium*, Jan 2012, pp. 1–6.

[13] C. Gilliam and T. Blu, "Fitting instead of annihilation: Improved recovery of noisy fri signals," in *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2014, pp. 51–55.

[14] Z. Doan, C. Gilliam, T. Blu, and D. Van De Ville, "Reconstruction of finite rate of innovation signals with model-fitting approach," *IEEE Transactions on Signal Processing*, vol. 63, no. 22, pp. 6024–6036, Nov 2015.

[15] H. Pan, T. Blu, and M. Vetterli, "Towards generalized fri sampling with an application to source resolution in radioastronomy," *IEEE Transactions on Signal Processing*, vol. 65, no. 4, pp. 821–835, Feb 2017.