ITM1010
Computer and Communication Technologies

Lecture #2

Part I: Introduction to Computer Technologies

Introduction to Numbering Systems

Decimal Numbering System

- The decimal numbering system is a positional-weighted numbering system. This means that each digital position has a specific weight (value). For example, 0.1, 1, 10, 100 all contain a 1, but each 1 has a different place value.
- The decimal system uses ten different basic symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each of these symbols is called a digit. The base is 10.
- The value of a decimal number is determined by positional weight, the sum-of-weights method.

The Sum-of-Weights Method

- Example: the number 42,100.00

10^4	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}
4	**2**	**1**	**0**	**0**	**.**	**0**	**0**

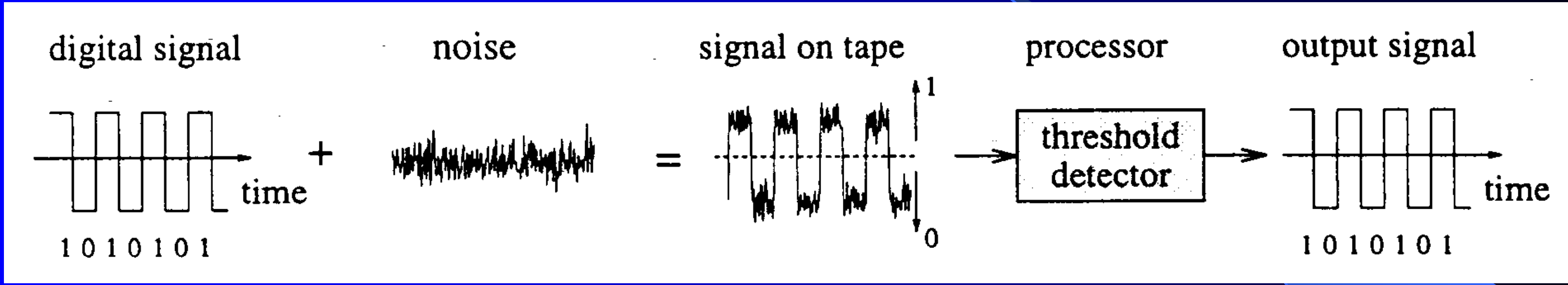
$(4x\ 10^4) + (2x10^3) + (1x10^2) + (0x10^1) + (0x10^0) + (0x10^{-1}) + (0x10^{-2})$

40,000+2000+100 + 0 + 0 + 0 + 0

= 42,100.00

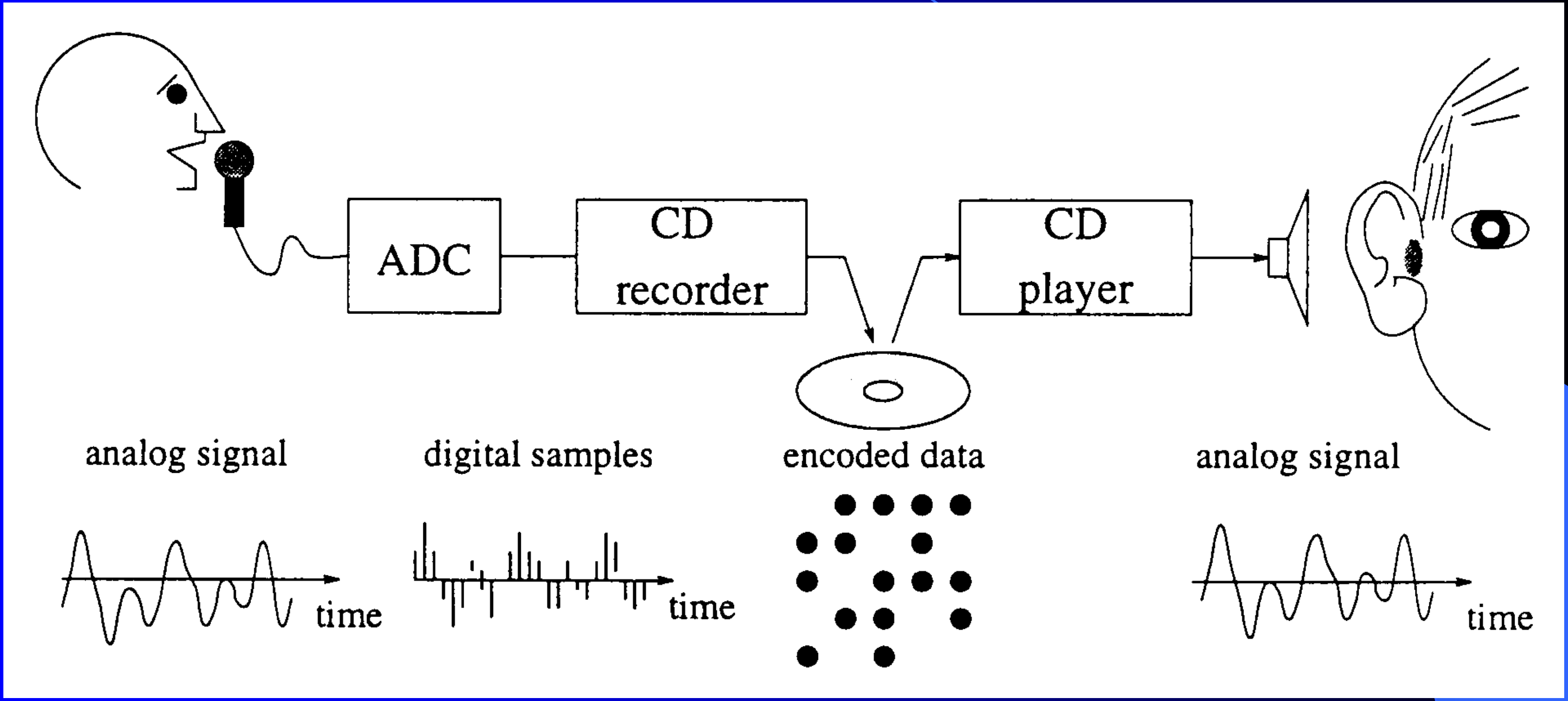
Why Binary Systems (Digital)

- Information Integrity: Better noise immunity



- Information Manipulation: The binary numbering system offers a simple means of representing and processing information in both hardware and software.

Sources of Digital Information



Binary Numbering System

Powers of two	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Value (decimal)	256	128	64	32	16	8	4	2	1	0.5	0.25	0.125

Example:

$$1011.11_{(2)} = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$$

$$= 8 + 0 + 2 + 1 + 0.5 + 0.25$$

$$= 11.75_{(10)}$$

Binary Notation

Binary Digit is called BIT (**B**inary Dig**it**)

Possible representations:

	1	0
	high	low
	true	false
	0 volt	+5 volt

LSB – Least Significant Bit

Bit change with the least effect (on the right)

MSB – Most Significant Bit

Bit change with the most effect (on the left)

Binary & Decimal Conversion

- Binary-to-Decimal Conversion

  Sum-of-weights method. Example

  $1011.11_{(2)} = 1x2^3+0x2^2+1x2^1+1x2^0+1x2^{-1}+1x2^{-2} = 11.75_{(10)}$

- Decimal-to-Binary Conversion

  Radix division for integer & Radix multiplication for fractional:

  Example: 92.875

1	0	1	1	1	0	0
1	2	5	11	23	46	92
	2	2	2	2	2	2

0.875 x 2 = 1.75 => 1 MSB

0.75 x 2 = 1.5 => 1

0.5 x 2 = 1 => 1 LSB

$$92.875_{(10)} = 1011100.111_{(2)}$$

Class exercise

- Convert the following decimal number to its binary equivalent.

$$12.375_{(10)}$$

Binary Arithmetic

- Addition: Rules: 0 + 0 = 0, 1 + 0 = 1, 1 + 1 = 10

  1101 + 1001 = 10110, in decimal => $13_{(10)} + 9_{(10)} = 22_{(10)}$

- Subtraction
  - Rules: 0 – 0 = 0, 1 – 0 = 1, 1 – 1 = 0, & 10 – 1 = 1

  10110 - 1001 = 1101, in decimal => $22_{(10)} - 9_{(10)} = 13_{(10)}$

- Multiplication

  1101 x 101 = 1000001, in decimal => $13_{10} \times 5_{10} = 65_{10}$

- Division

  110111 ÷ 101 = 1011, in decimal => $55_{10} \div 5_{10} = 11_{10}$

Complement representations

- The process of subtraction can be accomplished by adding a negative number to a positive number.
- Complement number representations are designed for this purpose.
  - Ones complement
  - Twos complement
- Multiplication & division can be accomplished by repetitive addition and subtraction respectively.
- Many computers perform all basic mathematical operations almost entirely with adder circuits.

Ones Complement

- The 1s complement of a binary number of a binary number is derived by subtracting each bit in the number to be complemented from 1.
- e.g. 1s complement of 1100 is 0011

```
   1 1 1 1
 – 1 1 0 0
 ---------
   0 0 1 1  (1s comp)
```

Subtraction in 1s Complement

```
  1110
 -0010

Step 1:   -0010 = 1101 (1s comp)
Step 2:    1110
          +1101 (1s comp)
         (1) 1011
Step 3:   +   1 (End-around carry)
           1100 (Difference)
```

$14_{(10)}-2_{(10)}=12_{(10)}$

Carry = 1
→ positive result

$7_{(10)}-9_{(10)}= -2_{(10)}$

```
  0111
 -1001

Step 1:   -1001 = 0110 (1s comp)
Step 2:    0111
          +0110
         (0) 1101
Step 3:   +   0
           1101 (Difference in 1s complement form)
           1101 = -0010
```

2s Complement

- Twos (2s) Complement = 1s complement + 1.

```
   1011
  −0101
  -----
        Step 1:   −0101  =  1010(1s comp)
                          +    1
                          --------
                            1011(2s comp)
        Step 2:    1011
                  +1011
                  -----
        Step 3: (1) 0110 (Difference)
        Carry ←─┘
```

$11_{(10)} - 5_{(10)} = 6_{(10)}$

Carry = 1
→ positive result

```
   1000
  −1100
  -----
                  −1100  =  0011(1s comp)
                          +    1
                          --------
                            0100(2s comp)
                   1000
                  +0100
                  -----
                (0) 1100
        Carry ←─┘
```

$8_{(10)} - 12_{(10)} = -4_{(10)}$

Class Exercise

- Subtract the following binary number using 2s complement.

```
    0 1 0 1          0 0 1 1
  - 0 1 0 0        - 0 1 0 1
  ---------        ---------
```

Sign Bit

- A single bit, usually the leftmost bit, may be used to distinguish positive and negative numbers. The meaning of the sign bit can be fixed arbitrarily. But normally,

  sign bit

0	-	positive number
1	-	negative number

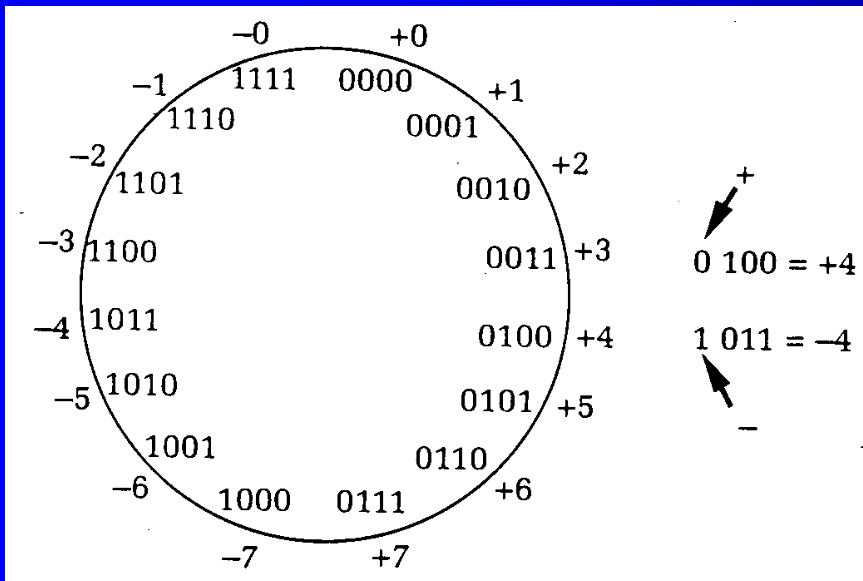
  e.g. $-5_{(10)} = 1101$

  $+5_{(10)} = 0101$

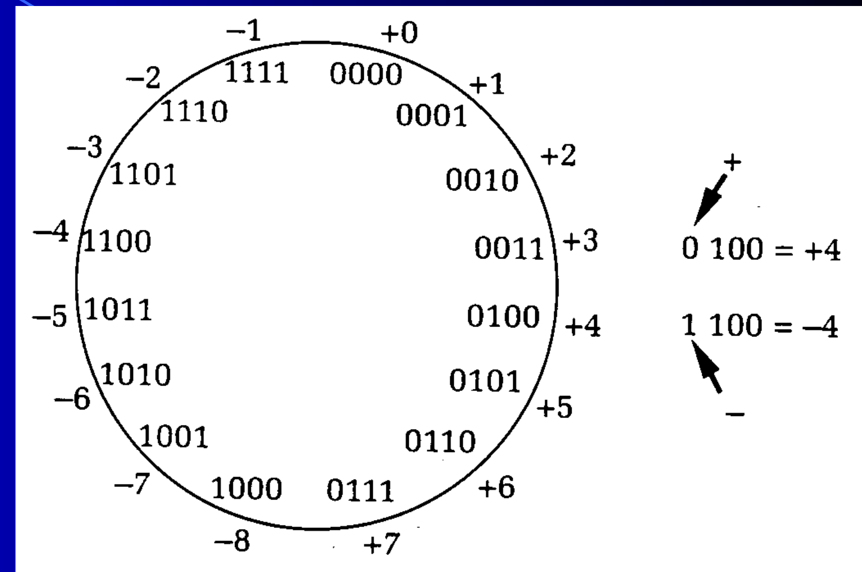
  – Note: the magnitude of a number is represented by the lower three bits

Sign Bit

1s Complement



2s Complement



The leftmost bit still indicates sign.

- In two's complement representation, two numbers can be added simply as two positive numbers e.g. 6 + (-2)= 4
- Overflow occurs whenever the sum of two positive numbers yields a negative result or when two negative numbers are summed and the result is positive.