# ITM 1010
## Computer and Communication Technologies

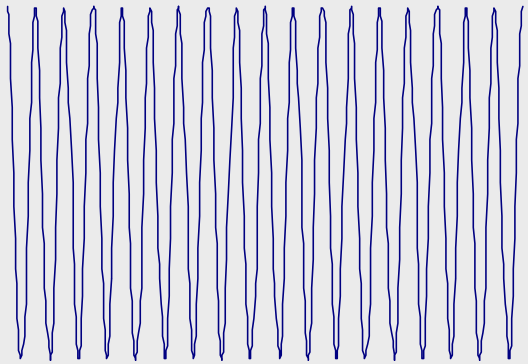
### Lecture #12

**Part II Introduction to Communication Technologies:**

# Modulation

# Modulation

- Modulation is the term used to describe how an information carrying signal m(t) is placed on a continuous “carrier” wave (c.w.) such as a radio wave or a light wave.



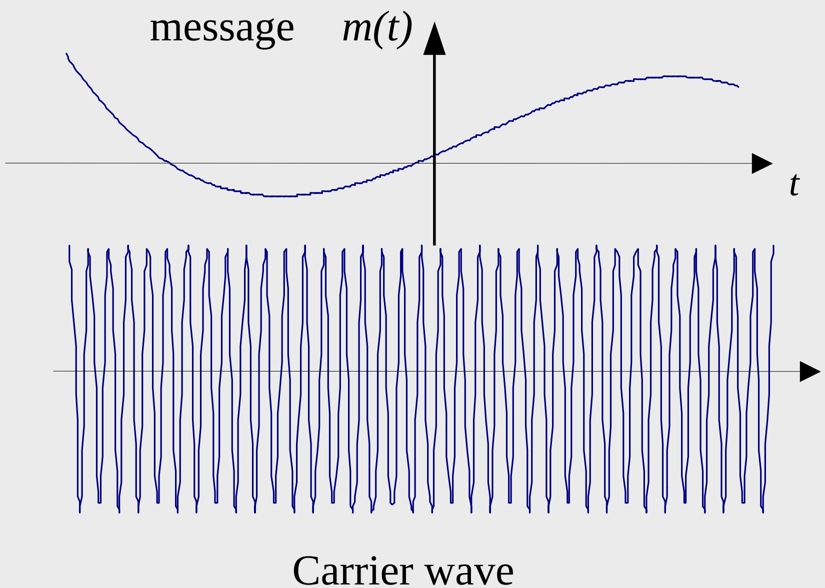
Carrier wave

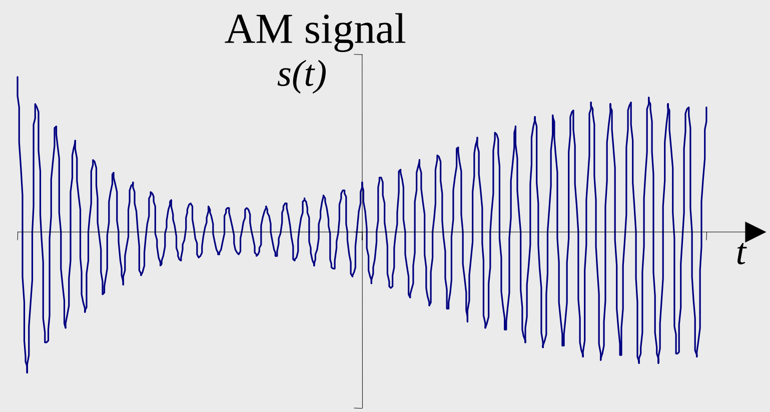
Message signal m(t)

- Many different types of modulation have been invented: we shall introduce some of the basic concepts for
  - Amplitude Modulation (AM)
  - Frequency Modulation (FM)

# Amplitude Modulation

- Amplitude modulation of a sinusoidal carrier wave, $A_c\cos(2\pi f_c t)$, by a base band signal *m(t)* will produce a signal *s(t)* which may be depicted as follows:



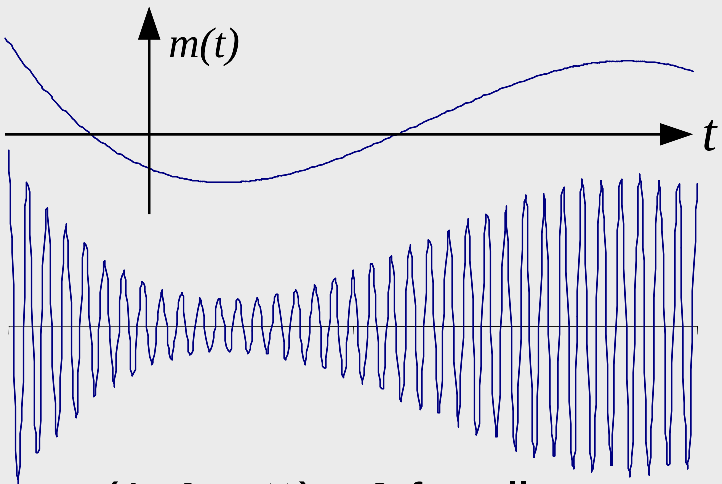

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

$k_a$ is called the modulation sensitivity

- Original message is simply the <u>envelope</u> of *s(t)* if :
  - $|k_a m(t)| < 1$ so that $(1 + k_a m(t))$ is positive;
  - frequency of carrier is greater than base band bandwidth.

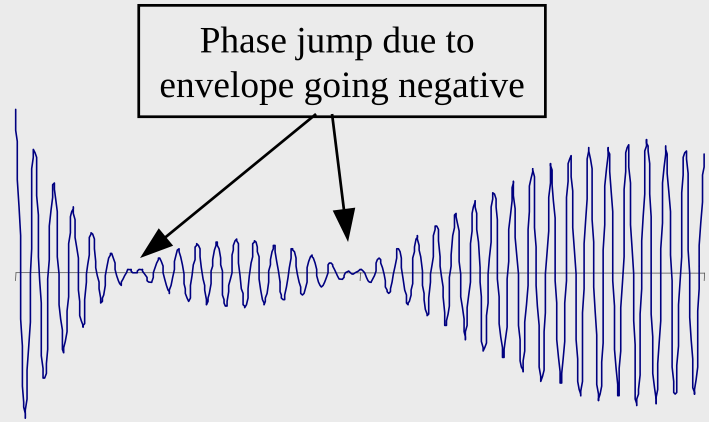
# Over-modulated AM signal

- If the modulation sensitivity $k_a$ is too large $(1+k_a m(t))$ may become negative leading to an over-modulated AM signal.
- Envelope of over-modulated signal is distorted by amplitude sign change which occurs whenever $(1+k_a m(t)) < 0$.




$(1+k_a m(t)) > 0$ for all t

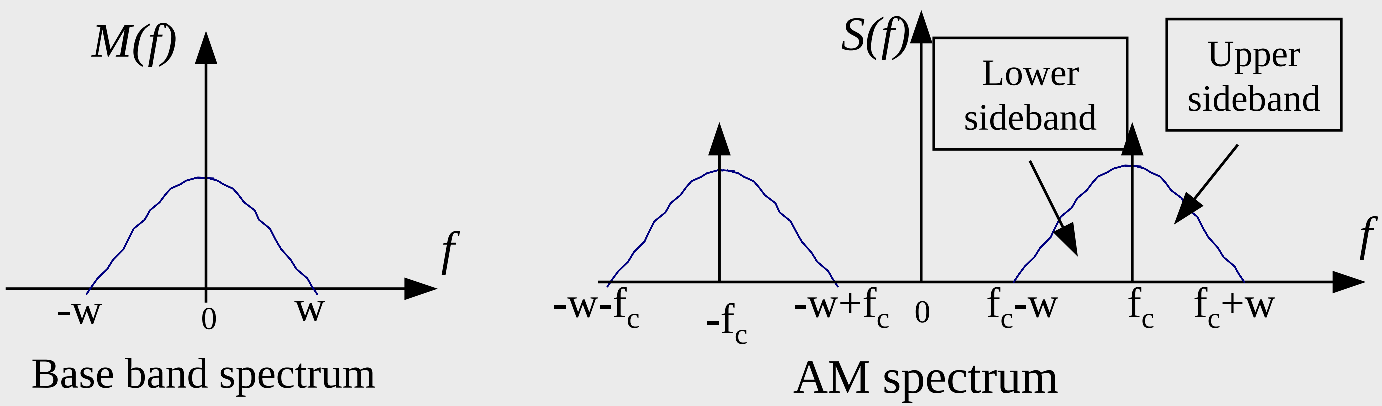
normal AM signal




$(1+k_a m(t)) < 0$ between 2 arrows

Over-modulated AM signal

# Spectrum of AM signal




- AM is wasteful of bandwidth because:
  - Carrier frequency contains power which must be sent by the transmitter but there is no information in this signal
  - Identical information is contained in the upper and lower sideband
  - If base band bandwidth is w, the AM signal occupies a bandwidth of 2w (from $f_c$-w to $f_c$+w)
- AM is widely used for broadcast radio because of simplicity in receiver design (only an envelope detector is needed)

# Double sideband and single sideband

- More efficient usage of bandwidth and transmitter power can be achieved by filtering out the un-needed frequency components from the transmitted spectrum but these need more complicated receivers.

- Examples of such techniques include:
  - Double sideband suppressed carrier (DSB-SC) which removes the carrier frequency from the transmitted signal
  - Single sideband (SSB) transmission which filters out both the carrier and either the lower sideband or upper sideband

- SSB and DSB receivers must multiply the received signal with a local oscillator to recover the base band signal.

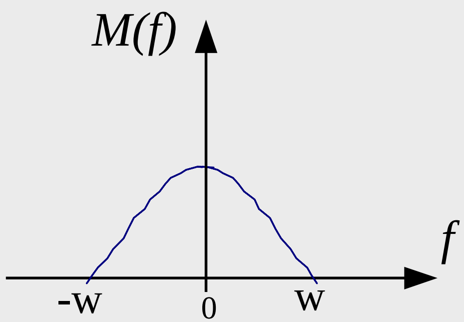
# Double sideband modulation

- Double sideband suppressed carrier modulation may be represented by

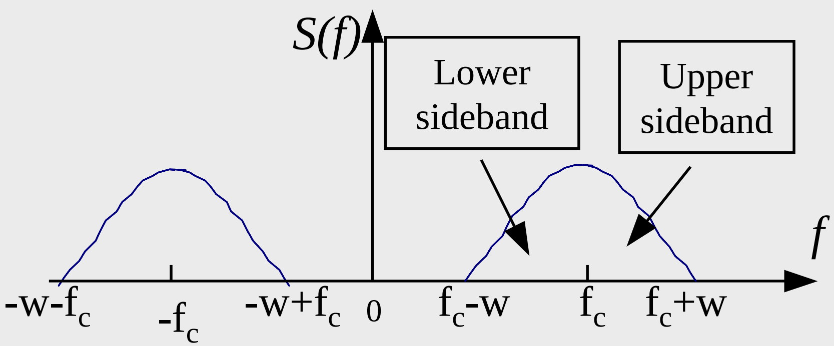
$$s(t) = m(t) A_c \cos(2\pi f_c t)$$

- *s(t)* undergoes a phase jump whenever the sign of *m(t)* changes: envelope of DSB-SC is NOT the same as *m(t)*

- In DSB-SC modulation the base band signal m(t) is simply translated in frequency and no carrier frequency is present.



Base band spectrum

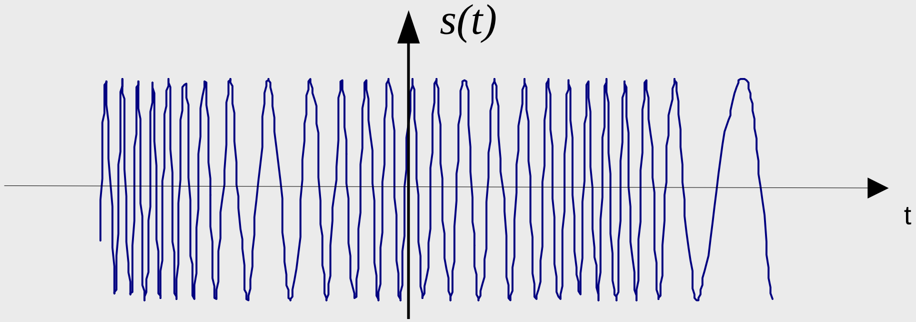


DSB-SC spectrum

# Frequency Modulation (FM)

- FM involves varying the instantaneous frequency of the carrier wave, $f_i$ , in proportion to amplitude of base band signal *m(t)* :

$$f_i = f_c + k_f \ m(t)$$




- $k_f$ is the modulation sensitivity (if *m(t)* is in volts, $k_f$ has units of Hz/V).

- FM signal has constant amplitude and thus is less prone to noise than AM ( noise usually only appears in the amplitude of a received radio signal).

# Mathematical description of FM

- An easy mistake is to describe an FM signal $s(t)$ by

$$s(t) = A_c \cos(2\pi f_i t) \qquad \text{WRONG}$$

- Need to consider the effect of previous $f_i$ which will contribute to the phase $\theta_i$ of the carrier:

$$s(t) = A_c \cos(\theta_i)$$

The phase change in a short time interval $dt$ is:

$$d\theta_i = 2\pi f_i dt = \omega_i \, dt$$

$$\text{Hence} \quad \theta_i = \int_0^t \omega_i \, dt = \omega_c t + 2\pi k_f \int_0^t m(t) dt$$

- The general expression for FM is thus:

$$s(t) = A_c \cos\left( \omega_c t + 2\pi k_f \int_0^t m(t) dt \right)$$

# Analysis of FM

- Suppose the base band signal *m(t)* is represented by a sinusoid with a frequency equal to its highest frequency Fourier component $f_m$ and an amplitude $V_m$ which equals the maximum voltage in *m(t)* :

$$m(t) = V_m \cos \omega_m t$$

- The FM signal may then be written as:

$$s(t) = A_c \cos\left( \omega_c t + 2\pi k_f \int_0^t V_m \cos(\omega_m t) dt \right)$$

$$s(t) = A_c \cos\left( \omega_c t + \frac{2\pi k_f V_m}{\omega_m} \sin(\omega_m t) \right) = A_c \cos\left( \omega_c t + \frac{k_f V_m}{f_m} \sin(\omega_m t) \right)$$

- We define the modulation index, β , as the maximum FM frequency deviation divided by the base band bandwidth

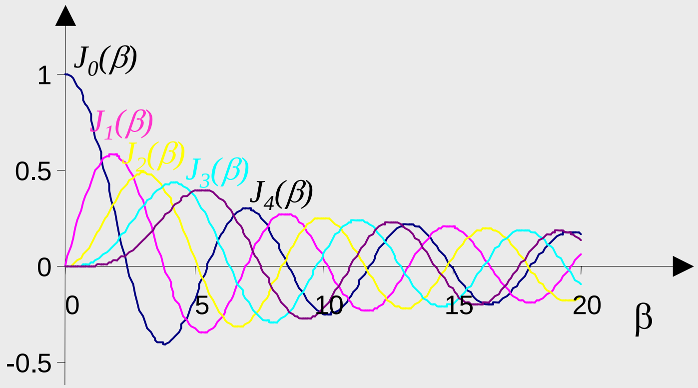
$$\beta = \frac{k_f V_m}{f_m}$$

Hence $s(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))$

# Bessel function expansion of FM

- It can be shown that the FM signal may be expressed as a sum of Bessel functions of the first kind, $J_n(\beta)$:

$$s(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))$$
$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c t + n\omega_m t)$$




Bessel functions $J_n(\beta)$ of different order n

- Since for negative n, $(-1)^n J_{-n}(\beta) = J_n(\beta)$, the spectrum of of the FM signal *s(t)* contains an infinite set of side frequencies located symmetrically about the carrier frequency.

# Carson's rule for bandwidth of FM signal

- Bandwidth depends on the modulation index
- Carson defined the bandwidth as that containing 98% of the signal power and arrived at the following approximation for the bandwidth $B$ of an FM signal as a function of modulation index $\beta$ and bandwidth $f_m$ of base band message

$$B = 2(1+\beta)f_m$$

- Carson's rule gives a slight underestimate of the actual bandwidth needed for an FM signal
- If modulation index $\beta$ is small (much less than 1) the bandwidth needed for narrowband FM is $2f_m$ , the same as for AM
- In practice broadcast FM radio employs a maximum frequency deviation of 75KHz, and requires a bandwidth of 200KHz (225KHz in UK). Broadcast FM use a carrier frequency between 88MHz to 108MHz, and provides a base band bandwidth of 15KHz.

# Summary

- A signal's bandwidth is the range of positive frequencies which contain most of the signal's power.
- Signals may be transmitted by modulating a carrier wave, such as a continuous EM wave (radio wave or light signal)
- AM uses amplitude of carrier wave to carry the base band signal.
  - Required bandwidth is 2w where w is base band bandwidth
  - Variants on AM include DSB-SC (to avoid wasting transmitter power) and SSB which reduces bandwidth needed from 2w to w.
- FM uses the frequency of the carrier wave to carry the base-band signal
  - Required bandwidth of FM is NOT just the maximum frequency deviation of the modulation
  - FM theoretically generates an infinitely wide spectrum (but the power in higher order Bessel components falls rapidly).
  - Bandwidth needed by FM is at least that of AM and depends on the modulation index (defined as maximum frequency deviation divided by the bandwidth of the base band signal).