ITM 1010

Computer and Communication Technologies

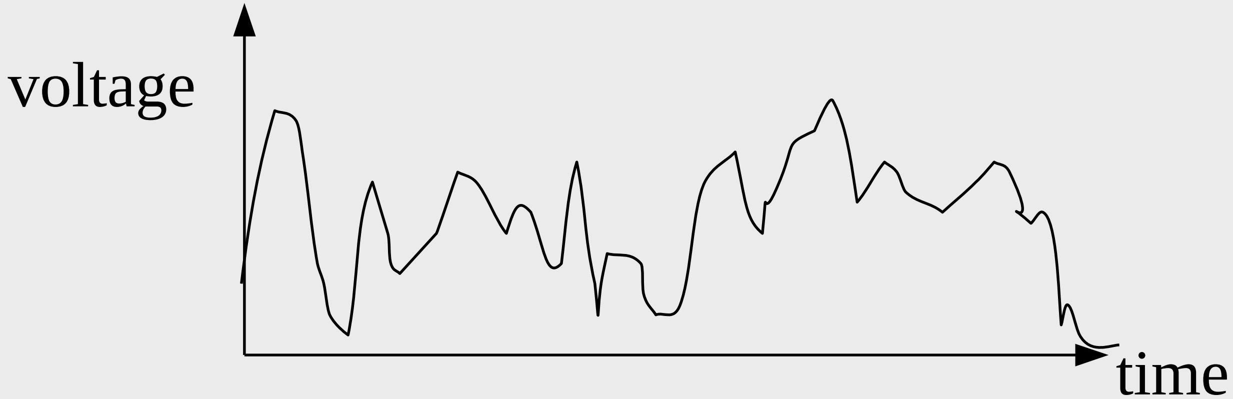
Lecture #11

Part II Introduction to Communication Technologies:

Signals

Signals

- Message to be transmitted in a communications system may by an analog signal e.g. for a television picture or human voice:



- What is the information content of such signals?
  - Need to introduce some signal analysis techniques to answer this question.

Waveforms

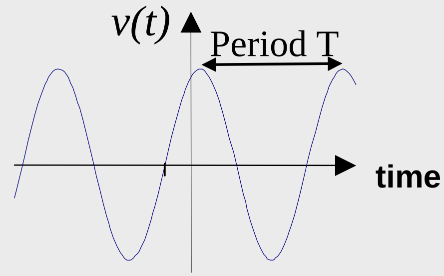
Waveforms may often be classified as
signals and noise,
digital or analog,
deterministic or nondeterministic,
physically realizable or nonphysically realizable,
periodic and nonperiodic,
and belonging to the power (mathematical models)
or energy (physical waveforms) type.

Physically Realizable Waveforms

- The waveform has significant nonzero values over a composite time interval that is finite.
- The spectrum of the waveform has significant values over a composite frequency interval that is finite.
- The waveform is continuous function of time.
- The waveform has a finite peak value.
- The waveform has only real values. That is, at any time, it cannot have a complex value $a + jb$, where b is nonzero.

Time and frequency domain

- Consider a signal which varies sinusoidally with time:



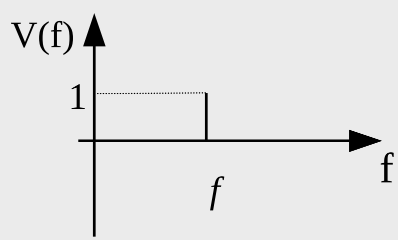
  - We can describe this signal in the time domain by:

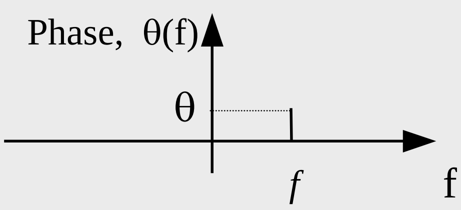
$$v(t) = \cos(2\pi f t + \theta)$$

where frequency f =1/T
θ is a constant phase angle

- Alternatively we can describe the signal in the frequency domain by specifying its frequency, amplitude and phase

$$V(f) = |V(f)| \angle \theta(f)$$

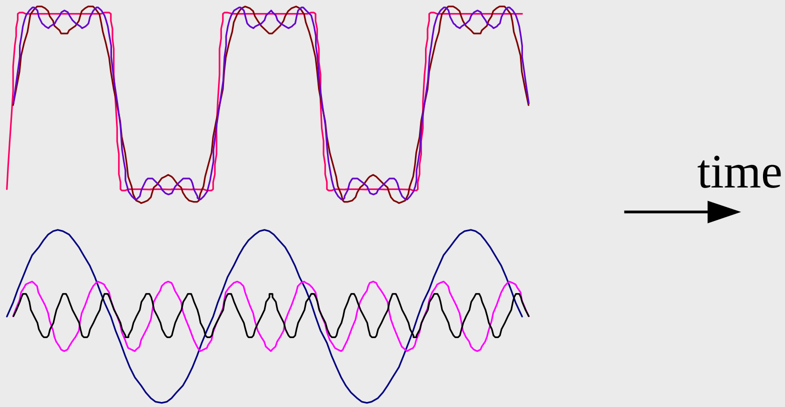




Fourier series

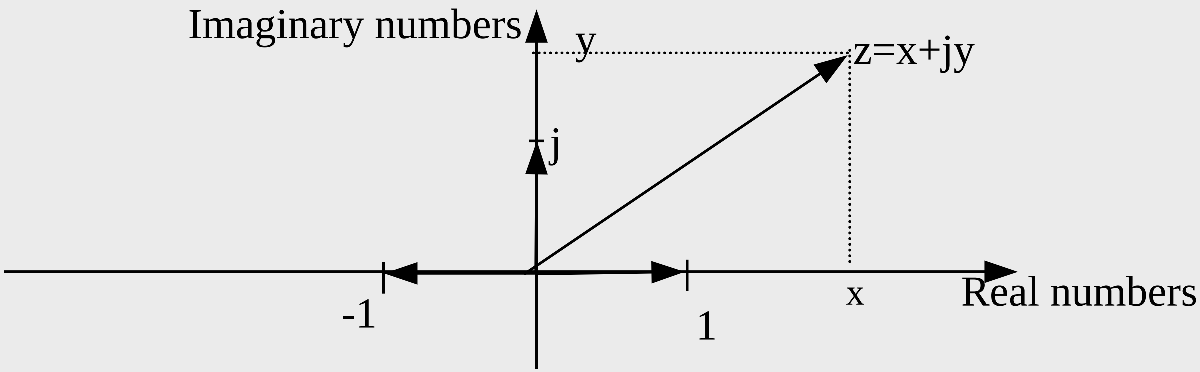
- A periodic function may be described by a sum of sinusoids; such a description is called a Fourier series expansion of the function. For example a "square" wave *v(t)* of period T=0.5π may be expanded as an infinite sum series:

$$v(t) = \sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \ldots$$



Complex Numbers: an introduction

- The square root of -1, which we denote by j, may be thought of as a 90 degree anticlockwise rotation in the complex plane (j^2 will give a 180 degree rotation so that $1 \times j^2 = -1$).



- A complex number z=x+jy is a point in the complex plane
- The complex exponential e^z is defined by: $e^z = e^x(\cos y + j \sin y)$
- If z is purely imaginary, $z = j\theta$, then Euler's formula is obtained:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Fourier series

- The sum of sinusoids (Fourier series) used to describe a periodic function *v(t)* of period T=1/f_0 may be written in terms of complex exponentials:

$$v(t)=\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

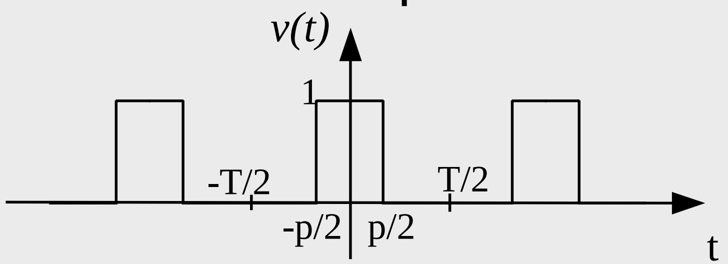
where $\omega_0 = 2\pi f_0$

- We can calculate the amplitudes of each sinusoid in the Fourier series by using the property that the integral of the product of two sinusoids over their period (or exact multiple of a period) is zero unless the two have the same frequency. The individual c_n may be calculated by integrating over any period:

$$c_n = \frac{1}{T}\int_0^T v(t)e^{-jn\omega_0 t}\,dt$$

Fourier series: an example

What is the Fourier series for the periodic function sketched below?



Answer $c_n = \frac{1}{T}\int_{-p/2}^{p/2} 1\cdot e^{-jn\omega_0 t}dt \quad = -\frac{1}{jn\omega_0 T}\left[e^{-jn\omega_0 t}\right]_{-p/2}^{p/2} \quad = \frac{\sin\left(\frac{n\omega_0 p}{2}\right)}{n\pi}$

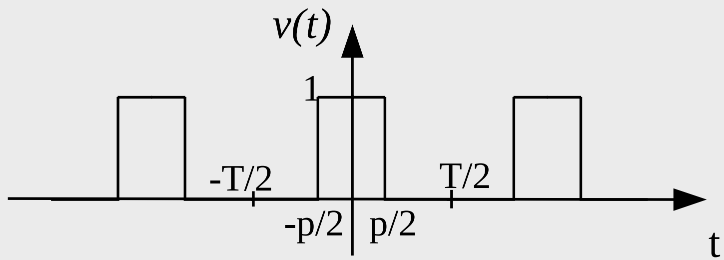
Hence

$$v(t) = \sum_{n=-\infty}^{\infty} \frac{\sin\left(\frac{n\omega_0 p}{2}\right)}{n\pi} e^{jn\omega_0 t}$$

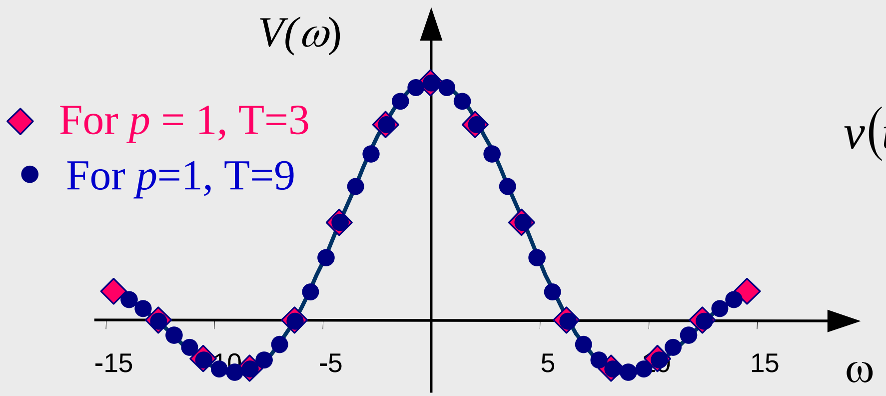
$$= \ldots + \frac{\sin\left(-\frac{\omega_0 p}{2}\right)}{-\pi}e^{-j\omega_0 t} + \frac{\omega_0 p}{2\pi} + \frac{\sin\left(\frac{\omega_0 p}{2}\right)}{\pi}e^{j\omega_0 t} + \frac{\sin\left(\frac{2\omega_0 p}{2}\right)}{2\pi}e^{j2\omega_0 t} + \ldots$$

$$v(t) = \frac{\omega_0 p}{2\pi} + 2\frac{\sin\left(\frac{\omega_0 p}{2}\right)}{\pi}\cos(\omega_0 t) + 2\frac{\sin\left(\frac{2\omega_0 p}{2}\right)}{2\pi}\cos(2\omega_0 t) + 2\frac{\sin\left(\frac{3\omega_0 p}{2}\right)}{3\pi}\cos(3\omega_0 t) + \ldots$$

From Fourier series to Fourier Transform



- Knowing the Fourier series, we can now sketch the spectrum of the pulse train *v(t)*:



$$v(t)=\sum_{n=-\infty}^{\infty}\frac{\sin\left(\frac{n\omega_0 p}{2}\right)}{n\pi}e^{jn\omega_0 t}$$

$$\omega_0=\frac{2\pi}{\mathrm{T}}$$

- As the period T increases ω_0 decreases and the different frequency components come closer in the frequency domain
- If the period tends to infinity, the spectrum tends towards a continuous function of frequency and we arrive at the Fourier transform of an individual pulse.

Fourier Transform

- The Fourier transform gives the frequency spectrum, $V(\omega)$, of a time-varying signal $v(t)$. Unlike the Fourier series, the signal $v(t)$ need not be periodic:

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt$$

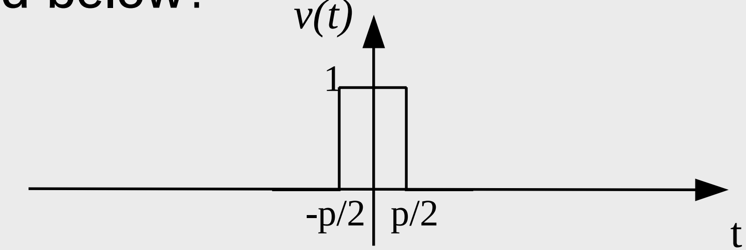
- In general $V(\omega)$ is a complex number (the magnitude is the amplitude of the sinusoidal component of frequency ω, and the phase of the complex number is the phase of the sinusoid).

- The inverse Fourier transform allows the time domain signal to be calculated from its frequency spectrum:

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega)\, e^{j\omega t}\, d\omega$$

Example: Fourier transform of a pulse

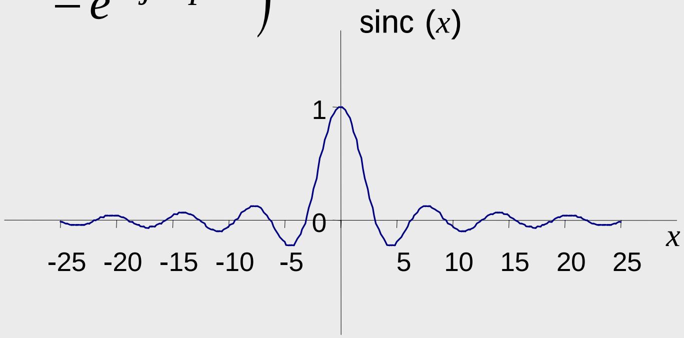
- What is the frequency spectrum associated with the rectangular pulse sketched below?



Solution

$$V(\omega)=\int_{-p/2}^{p/2} 1.\,e^{-j\omega t}dt=-\frac{1}{j\omega}\left[e^{-j\omega t}\right]_{-p/2}^{p/2}=\frac{1}{j\omega}\left(e^{j\omega p/2}-e^{-j\omega p/2}\right)$$

$$=\frac{1}{j\omega}\left(2j\sin\frac{\omega p}{2}\right)=p\frac{\sin\frac{\omega p}{2}}{\frac{\omega p}{2}}$$



- The function (sin x) /x is called sinc (x). $\frac{\sin x}{x}\equiv \operatorname{sinc} x$

Summary

- Any repetitive signal may be represented by a sum of sinusoids (Fourier series)

- Signals varying in the *time domain* may therefore be fully described simply by giving the amplitude, frequency and phase of its sinusoidal components (*frequency domain* picture)

- Fourier transform may be thought as an extension of Fourier series technique for non-periodic signals. Signals may be completely described either in time or frequency domains.
  - Fourier transform allows calculation of frequency domain function (spectrum) from time domain function and inverse Fourier transform performs the reverse operation.

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt$$

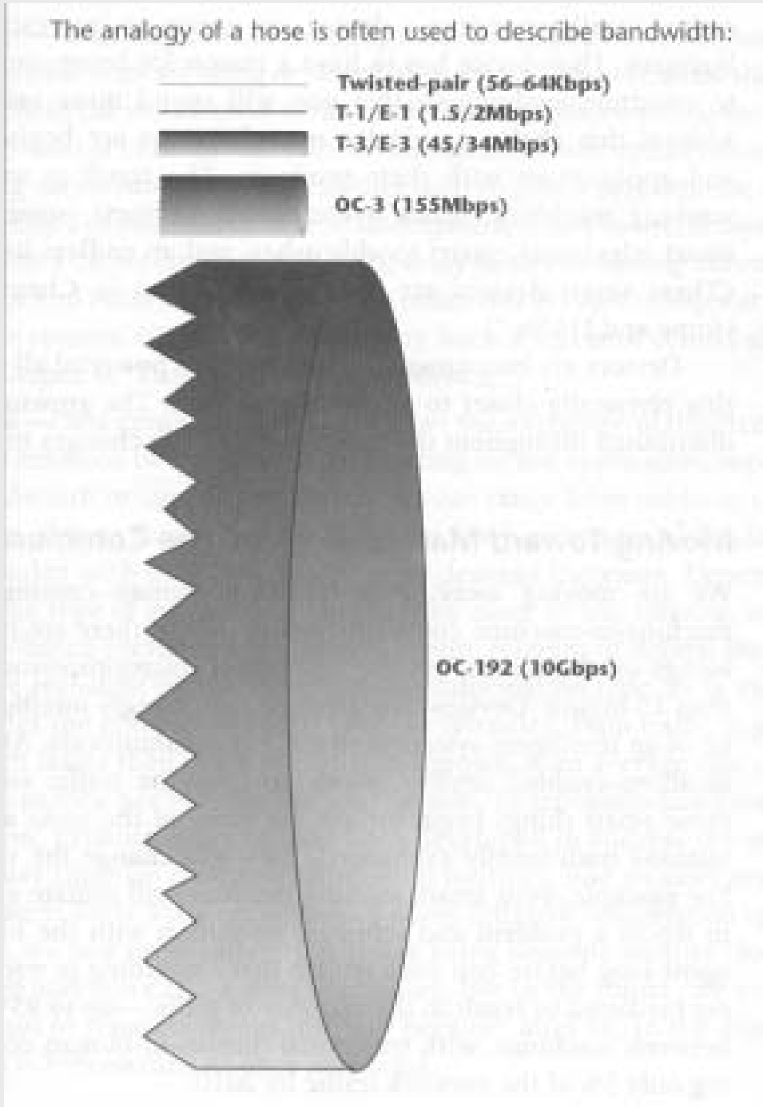
$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega)\, e^{j\omega t}\, d\omega$$

Bandwidth

What bandwidth is needed to transmit a signal?

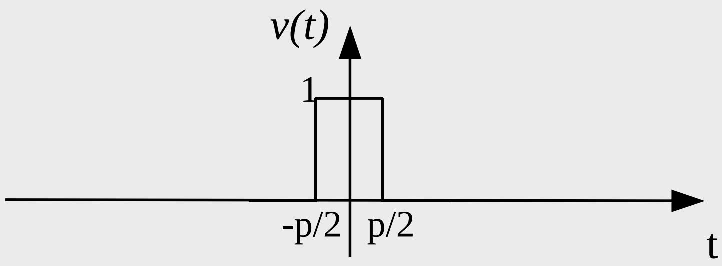
Bandwidth

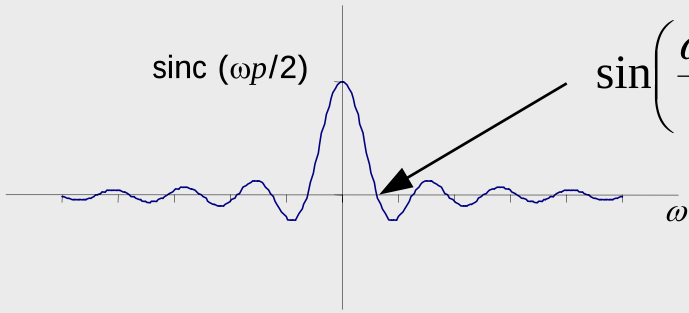
- Bandwidth is largely used today to refer to the capacity of a network a a telecom link, and it is generally measured in bits per second (bps). Bandwidth actually refers to the range of frequencies involved – that is, the difference between the lowest and the highest frequencies supported – and the greater the range of frequencies, the greater the bandwidth, and hence the greater the number of bits per second, or information carried.



Bandwidth of a pulse

- The frequency spectrum of a rectangular pulse which has a pulse-width of p seconds is:





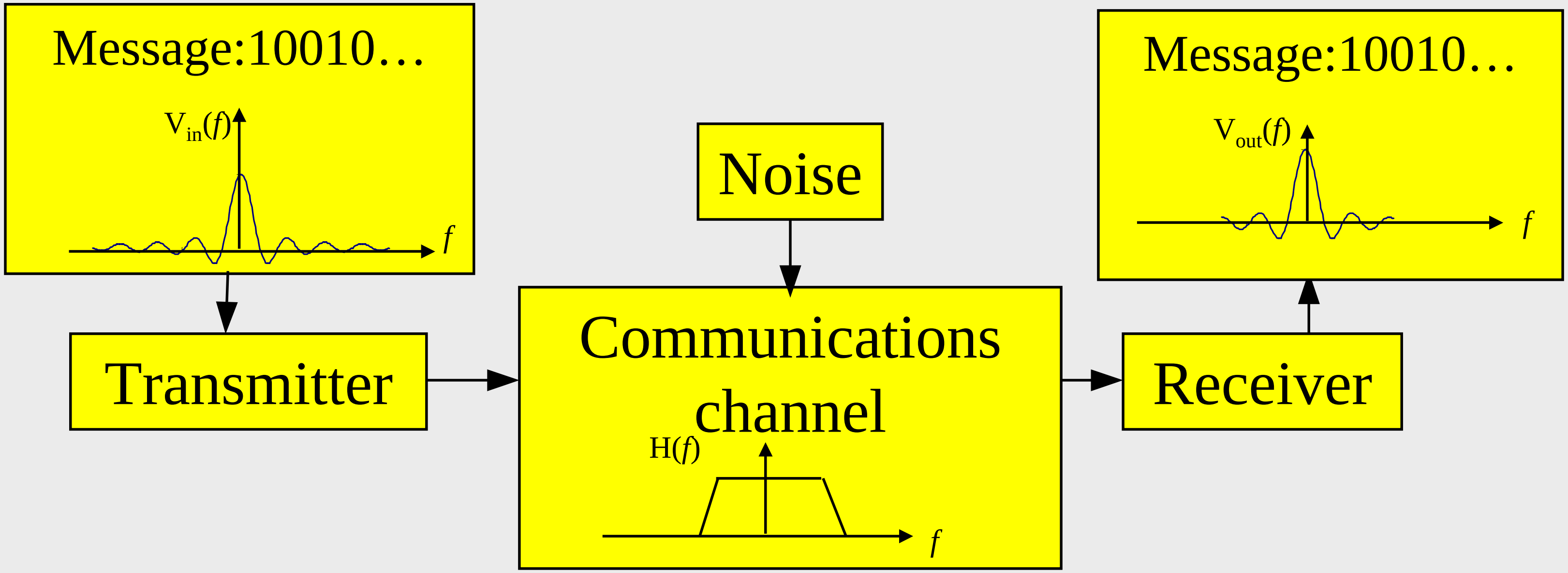
$$\sin\left(\frac{\omega p}{2}\right) = 0 \Rightarrow \omega = \frac{2\pi}{p}$$

$$f = \frac{1}{p}$$

- Most of the pulse energy lies within the frequency band $f<1/p$.

- We introduce the concept of bandwidth of a signal here by (loosely) defining it as the range of positive frequencies needed to contain "most" of the energy of the signal.

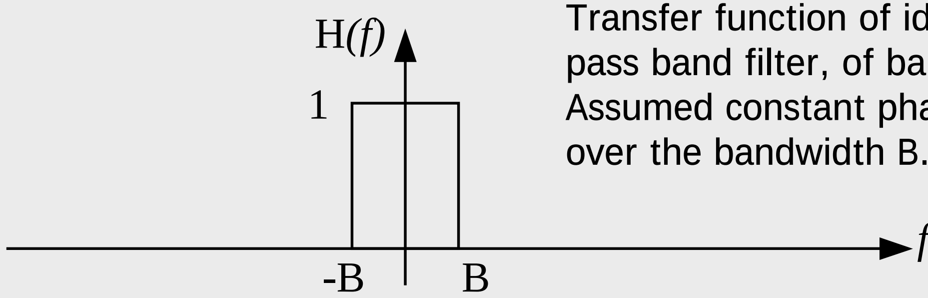
Bandwidth of a communications channel



- The channel must be capable of carrying the different frequency components of the message to allow it to arrive reasonably undistorted at the receiver.
- Transfer function H(*f*) may be used to describe the channel (normally used for describing frequency filters) and is defined by the ratio of output to input (in the frequency domain) :

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

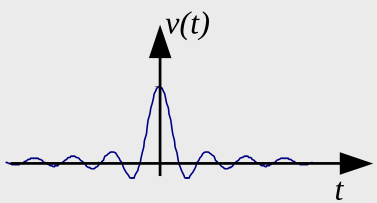
Idealized Filter Response



Transfer function of ideal pass band filter, of bandwidth B. Assumed constant phase response over the bandwidth B.

- A signal which has a flat (assumed to be of unit amplitude for simplicity) spectrum wider than B will, after passing through the filter have a spectrum H(f).
- The filtered signal in the time domain can be calculated by taking the inverse Fourier Transform

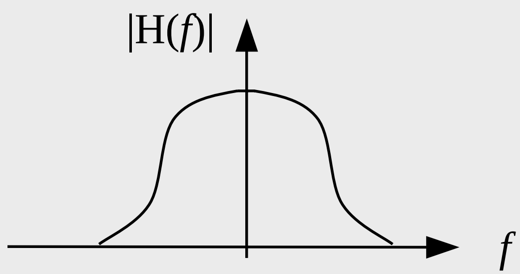
$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} e^{j\omega t} d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-2\pi B}^{2\pi B}$$



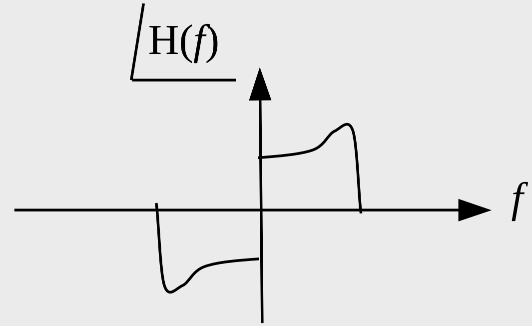
$$= \frac{1}{2\pi jt}\left(\cos(2\pi Bt) + j\sin(2\pi Bt) - \cos(2\pi Bt) + j\sin(2\pi Bt)\right) = \frac{1}{\pi t}\sin(2\pi Bt) = 2B\,\mathrm{sinc}(2\pi Bt)$$

Limitations on transfer functions from causality

- It is not possible to realize the ideal filter response since the ideal filter's inverse Fourier transform would lead to signals appearing BEFORE they were generated.
- In practice filters and communication channels must satisfy causality: it is impossible for the signal to appear at the communication channel output before the signal has even been sent!
- Physically realizable band pass filters typically have a less abrupt transition than the ideal filter and an odd phase response e.g.:



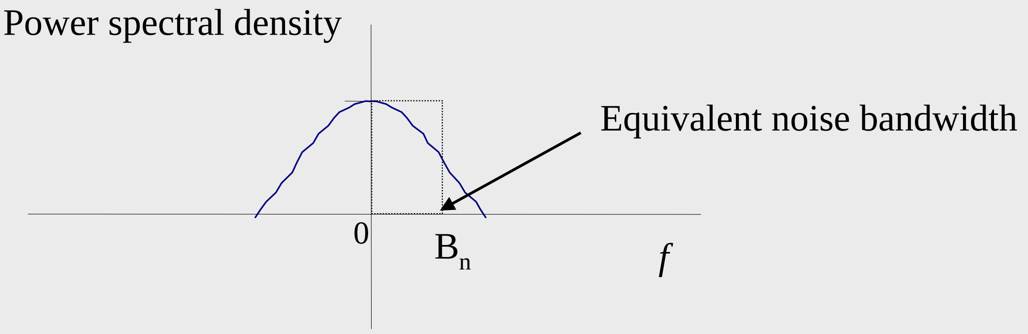
Magnitude response



Phase Response

Definitions of bandwidth

- Many different definitions of bandwidth exist. Some commonly used definitions include:
  1. 3 dB bandwidth: the range of positive frequencies beyond which the signal power drops by over 3 dB.
  2. Absolute bandwidth: the range of positive frequencies which contain all the signal power (the spectrum is zero outside this range)
  3. Equivalent Noise Bandwidth, B_n: the width of a fictitious rectangular spectrum of amplitude equal to the peak signal amplitude, such that the power in the rectangular band is equal to the power associated with the spectrum over positive frequencies.



Signals and their bandwidths

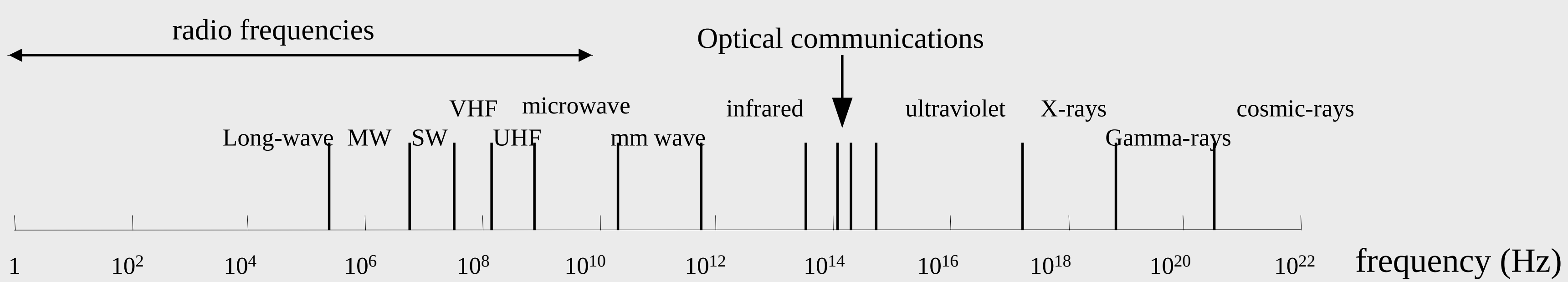
- The original source of the signal is called the base band signal.
- Usually the base band signal is modulated onto a carrier (e.g.. a radio wave)

Table 1. Bandwidths of common signals

Signal	Base band bandwidth
Telephone	4KHz
AM Radio	4.5KHz
FM radio	15KHz
Television (PAL)	6.5MHz
Satellite TV (analog NTSC))	5.5MHz

The Electromagnetic Spectrum

- Communications systems often make use of electromagnetic (EM) waves to carry the signal to be transmitted
- EM waves consist of sinusoidally oscillating electric and magnetic fields which travel at the speed of light. Light, Radio waves and X-rays are prominent examples of EM waves.



Electromagnetic spectrum plotted on a log scale.

The Electromagnetic Spectrum