# A Full Wave Analysis of A Conductor Post Insert Reentrant Coaxial Resonator in Rectangular Waveguide Combline Filters

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## ABSTRACT

Due to its wide tunability region in performance and compactness in size, a conductor post insert reentrant coaxial resonators are widely used in combline filters for mobile communications. This paper, for the first time, presents a full wave electromagnetic model for the reentrant coaxial resonator in rectangular waveguide combline filters. The model is based on the orthogonal expansion method and provides a formally exact modal solution to the problem. The model has been extensively verified by experiments and can be used in calculating resonant frequency, coupling values, or in constructing a key building block in full electromagnetic design of waveguide combline filters.

## I. Introduction

Conductor post (reentrant) coaxial resonator waveguide combline filters have been widely used in mobile communications due to many of their unique merits, such as compactness, low-cost and high power handling capability. Figure 1(a) illustrates the conventional reentrant coaxial resonator. A conductor post is used in the resonator as a quarter-wave TEM line inductor and the resonator open-end gap serves as a lumped-capacitor. Since there is an optimal aspect ratio of post diameter and width of the waveguide cavity for the best unloaded Q, the open-end gap becomes very narrow in order to achieve the required capacitance. Such a narrow gap limits the tunability and power handling capability of the filters.

A solution which has been extensively used to this problem is to use the conductor post insert reentrant coaxial resonator structure shown in Figure 1(b). The insert conducting post increases the interfacing area of open-end gap, consequently it increases the capacitance of the gap. Obviously, the structure also provides a great improvement on the tunability.

Extensive research on modelling of a conventional reentrant coaxial resonator in rectangular waveguide has been done over the past few decades. Until very recently, a full wave electromagnetic model of a centred reentrant coaxial resonator in evanescent mode waveguides was proposed by Yao et al [1]. This model has been considered as the first attempt towards full EM designing of conducting post waveguide combline filters.

As an extension of Yao's work, we present in this paper a full wave analysis on a more general and practical case: a conductor post insert reentrant coaxial resonators in waveguide combline filters. The proposed model uses the orthogonal expansion method to characterize the generalized scattering matrix (GSM) of the post insert reentrant resonator in rectangular waveguide. The GSM will be the key building block not only in calculating coupling coefficient between two resonators, but also in full electromagnetic designing of waveguide combline filters.

## II. Formulation

Considering a centred post insert reentrant resonator in a rectangular waveguide as shown in Fig.1(c), we divide the whole structure into four cylindrical subregions: (1) the centre open-end gap region I ($r < R0$); (2) the side gap region II ($R0 < r < R1$); (3) the side open-end gap region III ($R1 < r < R2$); and (4) an artificial cylindrical post region IV ($R2 < r < A$). The transverse fields with respect to radial direction in each subregion can be expressed as:

$$\mathbf{E}^s_t = \sum_m \sum_k [E^{se}_{mk} J_m(k^{se}_{ck} r) + F^{se}_{mk} Y_m(k^{se}_{ck} r)] \, \mathbf{e}^{se}_{tmk} +$$
$$\sum_m \sum_k [E^{sh}_{mk} k^{sh}_{ck} J'_m(k^{sh}_{ck} r) + F^{sh}_{mk} k^{sh}_{ck} Y'_m(k^{sh}_{ck} r)] \, \mathbf{e}^{sh}_{tmk} \quad (1)$$

$$\mathbf{H}^s_t = \sum_m \sum_k [E^{se}_{mk} k^{se}_{ck} J'_m(k^{se}_{ck} r) + F^{se}_{mk} k^{se}_{ck} Y'_m(k^{se}_{ck} r)] \, \mathbf{h}^{se}_{tmk} +$$
$$\sum_m \sum_k [E^{sh}_{mk} J_m(k^{sh}_{ck} r) + F^{sh}_{mk} Y_m(k^{sh}_{ck} r)] \, \mathbf{h}^{sh}_{tmk} \quad (2)$$

where $J_n$ and $Y_n$ are Bessel functions of the first kind and the second kind, respectively; $s = I, II, III$ or $IV$. $F_{mk} = 0$ for region I. $e_{tmk}^{se,h}$ and $h_{tmk}^{se,h}$ are the transverse electric and magnetic eigen mode functions of TE and TM mode respectively for parallel planes bounded in y direction.

In waveguide regions W1 and W2, the total fields can be expressed in terms of regular $TE_{mi}$ and $TM_{mi}$ modes respect to z-direction. The field expressions for region W1 are

$$
\begin{aligned}
E_z^{Ip} &= \sum_m^M \sum_i^I [A_{mi}^{Ip} e^{-\gamma_{mi} z} - B_{mi}^{Ip} e^{\gamma_{mi} z}] \, e_{zmi}^p \\
H_z^{Ip} &= \sum_m^M \sum_i^I [A_{mi}^{Ip} e^{-\gamma_{mi} z} + B_{mi}^{Ip} e^{\gamma_{mi} z}] \, h_{zmi}^p \\
\boldsymbol{E}_t^{Ip} &= \sum_m^M \sum_i^I [A_{mi}^{Ip} e^{-\gamma_{mi} z} + B_{mi}^{Ip} e^{\gamma_{mi} z}] \, \boldsymbol{e}_{tmi}^p \\
\boldsymbol{H}_t^{Ip} &= \sum_m^M \sum_i^I [A_{mi}^{Ip} e^{-\gamma_{mi} z} - B_{mi}^{Ip} e^{\gamma_{mi} z}] \, \boldsymbol{h}_{tmi}^p
\end{aligned}
\tag{3}
$$

We define an inner product as $\langle \boldsymbol{e}, \boldsymbol{h} \rangle = \iint_s \boldsymbol{e} \times \boldsymbol{h} \cdot d\boldsymbol{s}$. Then, by matching the tangential fields on the artificial cylindrical boundary at $r = A$, we obtain the following matrix equation:

$$
\begin{bmatrix}
X_{11} & X_{12} & \cdots & X_{18} \\
X_{21} & X_{22} & \cdots & X_{28} \\
X_{31} & X_{32} & \cdots & X_{38} \\
X_{41} & X_{42} & \cdots & X_{48}
\end{bmatrix}
\begin{Bmatrix}
A^{Ih} \\
A^{IIh} \\
A^{Ie} \\
A^{IIe} \\
B^{Ih} \\
B^{IIh} \\
B^{Ie} \\
B^{IIe}
\end{Bmatrix}
=
\begin{Bmatrix}
E^{4e} \\
F^{4e} \\
E^{4h} \\
F^{4h}
\end{Bmatrix}
\tag{4}
$$

where A's and B's are the field coefficient vectors of the incident and reflected waves in the waveguide regions W1 and W2. These coefficients define the general scattering matrix of the key building block. To calculate the above matrix [X] analytically, the Bessel-Fourier series derived in [2] are used.

In order to solve the entire scattering problem, the boundary conditions at each subregion interface need to be satisfied. Enforcing the continuity of the tangential fields at boundary r = R0, R1 and R2, with the inner product defined above, gives

$$
[U1] \begin{Bmatrix} \boldsymbol{E}^2 \\ \boldsymbol{F}^2 \end{Bmatrix} = \{0\} \qquad for \quad r = R0 \tag{5}
$$

$$
[U2] \begin{Bmatrix} \boldsymbol{E}^3 \\ \boldsymbol{F}^3 \end{Bmatrix} = [V2] \begin{Bmatrix} \boldsymbol{E}^2 \\ \boldsymbol{F}^2 \end{Bmatrix} \qquad for \quad r = R1 \tag{6}
$$

and

$$
[U3] \begin{Bmatrix} \boldsymbol{E}^3 \\ \boldsymbol{F}^3 \end{Bmatrix} = [V3] \begin{Bmatrix} \boldsymbol{E}^4 \\ \boldsymbol{F}^4 \end{Bmatrix} \qquad for \quad r = R2 \tag{7}
$$

If the same number of modes are used in regions II, III and IV, by combining (5), (6) and (7), one can obtain

$$
[U1] \{ [U3] [U2]^{-1} [V2] \}^{-1} [V3] \begin{Bmatrix} E^{4e} \\ F^{4e} \\ E^{4h} \\ F^{4h} \end{Bmatrix} = \{0\} \tag{8}
$$

The generalized scattering matrix [ S ] of the conductor post insert reentrant coaxial resonator in rectangular waveguide can then be finally derived, from equations (4) and (8), as

$$
\begin{Bmatrix}
B^{Ih} \\
B^{IIh} \\
B^{Ie} \\
B^{IIe}
\end{Bmatrix}
=
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{Bmatrix}
A^{Ih} \\
A^{IIh} \\
A^{Ie} \\
A^{IIe}
\end{Bmatrix}
\tag{9}
$$

Similar to the analysis of single conducting post in a rectangular waveguide[1], the number of eigenmodes used in waveguide regions must be the same as those in regions II, III and IV with same y-variations. The number of modes in region I could be different from those in other regions. However, the mode index of $\Phi$-variations in region I must be the same as those of other cylindrical regions.

The GSM obtained in (9) can then be incorporated with other rigorous junction modules to calculate the resonant frequencies and coupling coefficients.

## III. Numerical Results

To implement the formulation discussed above, a general computer program has been developed and integrated with the mode matching module for calculating a rectangular iris in rectangular waveguide. A number of rectangular cavities with conductor post insert resonator of different sizes have been built and tested to verify the analysis. Fig.2 shows the comparison of measured and calculated magnitude of the scattering coefficients of the propagating dominant TE10 mode for a conductor post insert reentrant resonator in a rectangular waveguide. The calculated results agree well with the measured data.

As the second example, the resonant frequency for an isolated resonator cavity is calculated and measured for different dimensions, where the higher order modes must be accurately taken into account. Figure 3 shows the comparisons of the calculated and the measured resonant frequencies versus the

depth of penetration of the tuning post for different cases with dimensions listed in Table 1. The calculated results are within 1-2% relative error region of the measured data. Considering the mechanical tolerance in alignment of the tuning post and the high sensitivity of the resonant frequency to the tolerance, the achieveable accuracy is quite acceptable.

Table 1  Dimensions in example II

| A=0.5, L1=L2=0.601, B=0.85  All dimension in Inches | | | | | |
|---|---|---|---|---|---|
| | 2 X R0 | 2 X R1 | 2 X R2 | Lb | Lc |
| Case 1 | 0.200 | 0.304 | 0.400 | 0.598 | 0.203 |
| Case 2 | 0.200 | 0.304 | 0.400 | 0.610 | 0.150 |
| Case 3 | 0.1585 | 0.304 | 0.400 | 0.594 | 0.149 |

## IV.  Conclusion

A new full wave electromagnetic analysis has been presented for a conductor post insert reentrant coaxial resonator in rectangular waveguide combline filters. The rigorous analysis can be used in conjunction with other full EM module to calculated the resonant frequency and the coupling coefficient of practical combline filter structures. This feature is very important in combline filter designs. The analysis has been verified by measurements. Excellent agreements are obtained.

## Achnowledgement

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## References

[1]    H.-W. Yao, K.A. Zaki, A.E. Atia and R. Hershtig,"Full wave modeling of conducting posts in rectangular waveguides and its applications to slot coupled combline filters," IEEE Trans. Microwave Theory Tech., Vol.43, No.12, pp.2824-2830, Dec. 1995.

[2]    X.-P. Liang and K.A. Zaki,"Modeling of cylindrical dielectric resonators in rectangular waveguides and cavities," IEEE Trans. Microwave Theory Tech., Vol.41, No. 12, pp.2174-2181, Dec. 1993.
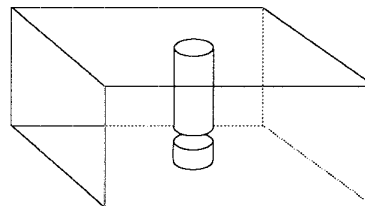
Fig.1(a) A conventional post reentrant coaxial resonator in a rectangular waveguide.
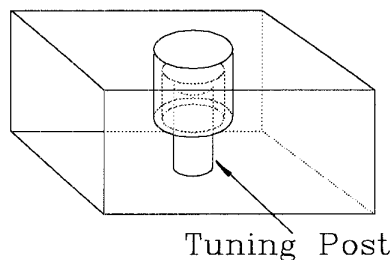


Tuning Post

Fig.1(b) A conductor post insert reentrant coaxial resonator in a rectangular waveguide.
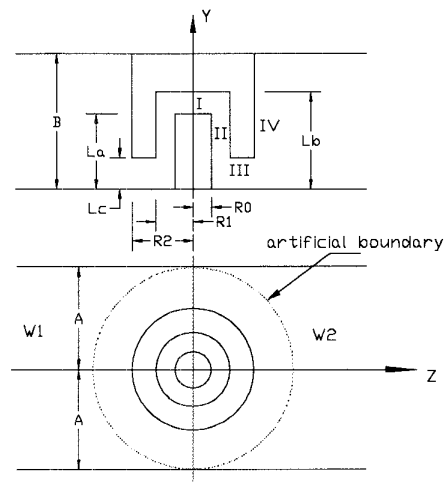


Fig.1(c) Computation regions in the analysis of post insert reentrant coaxial resonator.
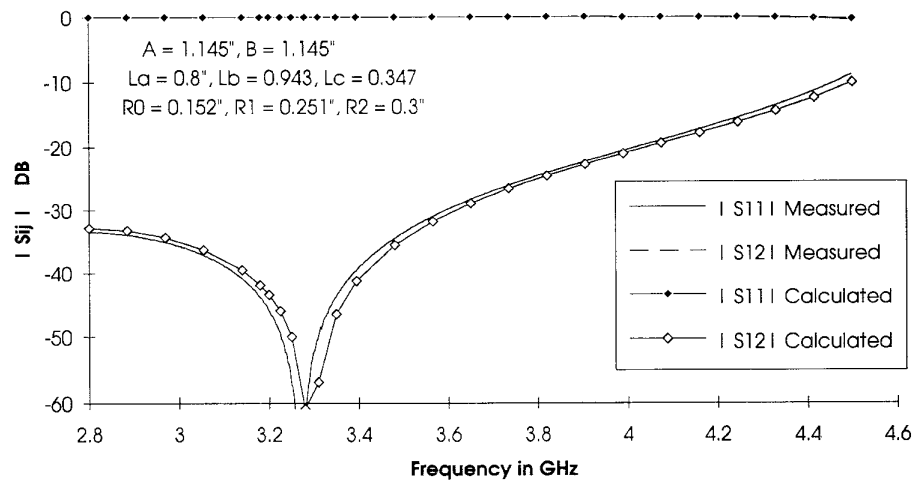
Fig.2 The calculated and the measured magnitude of scattering coefficient of a conductor post insert reentrant coaxial resonator in a rectangular waveguide WR229.
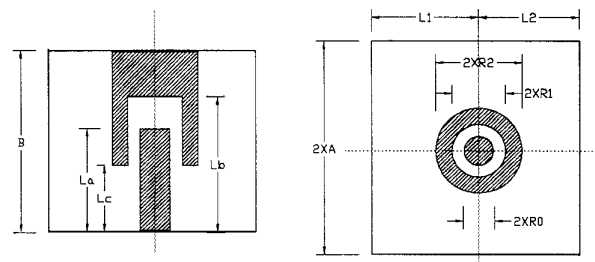


Fig.3(a) An isolated post insert reentrant coaxial resonator.
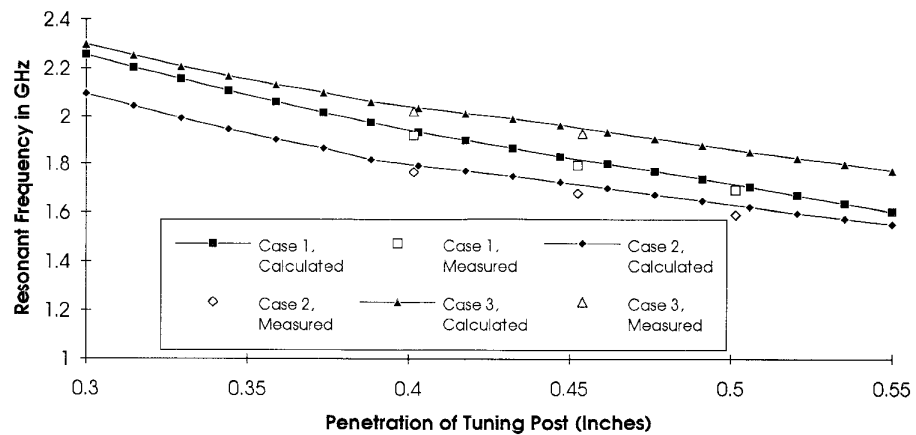


Fig.3(b) Resonant frequencies of an isolated post insert reentrant coaxial resonator versus the penetration of the tuning post.