A New Stable and Very Dispersive Boundary Condition for the FD-TD Method

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ABSTRACT

In this paper, we present a new stable and very dispersive boundary condition for the finite difference time domain (FD-TD) method. Compared with existing absorbing boundary conditions (ABC's), the new boundary condition has a similar computational complexity but much better absorbing performance. As well, the new boundary condition is more stable than presently existing ABC's.

INTRODUCTION

The finite-difference time-domain (FD-TD) method for solving Maxwell's equations has been widely used and its popularity continues to increase because of its flexibility and accuracy. Because computers can handle only a limited computational domain and the applications are typically open region problems, absorbing boundary conditions (ABC's) have to be employed to terminate the computational space. A better absorbing boundary condition will result in more accurate numerical solutions. On the other hand, a better absorbing boundary condition can also bring the boundaries closer to the modeled structure and thus save considerably computer memory space and computation time. Various absorbing boundary conditions have been proposed during the past years. The authors believe that most commonly used absorbing boundary conditions in FD-TD analysis are Mur's first order ABC [1] and the dispersive boundary condition (DBC) [2]. However, it has been found that neither Mur's first order ABC nor the DBC are always numerically stable [3], [4]. In addition, the absorbing performance of these existing boundary conditions is not good enough for many dispersive applications, such as waveguide applications. The purpose of this paper is to present a new stable boundary condition which has much better absorbing performance and a similar computational complexity, compared with the DBC and Mur's first order ABC.

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NEW BOUNDARY CONDITION

Consider the boundary condition of the form:

$$\prod_{i=1}^{N}\left(\frac{\partial}{\partial x}+\frac{1}{v_i}\frac{\partial}{\partial t}\right)E = 0. \qquad (1)$$

We can use a difference scheme, i.e.,

$$\frac{E^n(M)-E^n(M-1)}{\Delta x}+\frac{1}{v_i}\frac{E^n(M)-E^{n-1}(M)}{\Delta t} \qquad (2)$$

to replace the differential factor:

$$\left(\frac{\partial}{\partial x}+\frac{1}{v_i}\frac{\partial}{\partial t}\right)E \qquad (3)$$

The above difference scheme is different from that used in [1] and [2].

We can prove that (2) contains the pole

$$p_i = \frac{1}{1+\rho_i(1-e^{\sqrt{-1}k_x\Delta x})} \quad (4)$$

if

$$E = E_0 e^{\sqrt{-1}(\omega n\Delta t - k_x i_x \Delta x - k_y i_y \Delta y - k_z i_z \Delta z)}, \quad (5)$$

where

$$\rho_i = \frac{v_i \Delta t}{\Delta x}. \quad (6)$$

The magnitude of the pole is less than 1 except at $k_x \Delta x = 0$ as shown in Figure 1. This means that the boundary conditions obtained by replacing the differential factor (3) with the difference scheme (2) will always be stable as long as $k_x \neq 0$ [5].

Now we give the exact form of the new stable boundary condition. Let

$$a_i = \frac{\rho_i}{1+\rho_i} \quad (7)$$

$$b_i = \frac{1}{1+\rho_i}. \quad (8)$$

The difference scheme (2) can be expressed by the operator

$$k_i^d D_i E^n(M) \quad (9)$$

where k_i^d is a non-zero constant and

$$D_i = I - a_i Z_M^{-1} - bi Z_n^{-1}. \quad (10)$$

I, Z_M^{-1} and Z_n^{-1} are the shift operators such that

$$IE^n(M) = E^n(M) \quad (11)$$

$$Z_M^{-1} E^n(M) = E^n(M-1) \quad (12)$$

$$Z_n^{-1} E^n(M) = E^{n-1}(M). \quad (13)$$

The new boundary condition can then accordingly be expressed by the operator

$$DE^n(M) = 0. \quad (14)$$

For $N = 3$,

$$\begin{aligned} D &= D_1 D_2 D_3 \\ &= I - (a_1+a_2+a_3)Z_M^{-1} - (b_1+b_2+b_3)Z_n^{-1} \\ &\quad +(a_1b_2+a_1b_3+a_2b_1+a_2b_3+a_3b_1+a_3b_2)Z_M^{-1}Z_n^{-1} \\ &\quad +(a_1a_2+a_1a_3+a_2a_3)Z_M^{-2} \\ &\quad +(b_1b_2+b_1b_3+b_2b_3)Z_n^{-2} \\ &\quad -(a_1a_2b_3+a_1a_3b_2+a_2a_3b_1)Z_M^{-2}Z_n^{-1} \\ &\quad -(a_1b_2b_3+a_2b_1b_3+a_3b_1b_2)Z_M^{-1}Z_n^{-2} \\ &\quad -a_1a_2a_3 Z_M^{-3} - b_1b_2b_3 Z_n^{-3} \end{aligned} \quad (15)$$

and the time domain expression of the new boundary condition is

$$\begin{aligned} E^n(M) &= d_1E^n(M-1) + d_2E^{n-1}(M) \\ &\quad -d_3E^{n-1}(M-1) - d_4E^n(M-2) \\ &\quad -d_5E^{n-2}(M) + d_6E^{n-1}(M-2) \\ &\quad +d_7E^{n-2}(M-1) + d_8E^n(M-3) \\ &\quad +d_9E^{n-3}(M) \end{aligned} \quad (16)$$

where

$$d_1 = a_1+a_2+a_3 \quad (17)$$

$$d_2 = b_1+b_2+b_3 \quad (18)$$

$$d_3 = a_1b_2+a_1b_3+a_2b_1+a_2b_3+a_3b_1+a_3b_2 \quad (19)$$

$$d_4 = a_1a_2+a_1a_3+a_2a_3 \quad (20)$$

$$d_5 = b_1b_2+b_1b_3+b_2b_3 \quad (21)$$

$$d_6 = a_1a_2b_3+a_1a_3b_2+a_2a_3b_1 \quad (22)$$

$$d_7 = a_1b_2b_3+a_2b_1b_3+a_3b_1b_2 \quad (23)$$

$$d_8 = a_1a_2a_3 \quad (24)$$

$$d_9 = b_1b_2b_3. \quad (25)$$

Since the new boundary condition (16) and DBC developed in [2] both have nine terms on the right-hand sides of their equations, we conclude that they have similar computational complexity.

COMPUTER SIMULATIONS

We have tested the performance of the new boundary condition (16) by carrying out FD-TD simulations. In Figure 2 is given its reflection coefficient as well as the reflection coefficients of the DBC of [2] and the Mur's first order ABC for a standard rectangular P band waveguide. The waveguide dimensions are 15.8 mm × 7.9 mm and the waveguide frequency range for TE_{10} mode is 12.40 – 18.00 GHz [6].

It is seen from Figure 2 that the absorbing performance of the new boundary condition of (16) is much better than that of the DBC and that of Mur's first order ABC in an extremely dispersive waveguide application. Although the new boundary condition has three poles, we can still guarantee its stability for a waveguide application where there is no DC offset and all three poles lie inside the unit circle.

CONCLUSION

A new stable and very dispersive boundary condition has been presented in this paper. The new boundary condition has much better absorbing performance and a similar computational complexity, compared with the DBC and Mur's first order ABC.

We can further insure the stability of the boundary condition by using

$$\frac{E^n(M)-E^n(M-1)}{\Delta x} + \frac{1}{v_i}\frac{E^n(M)-E^{n-1}(M)}{\Delta t} + \alpha_i E^n(M) \tag{26}$$

instead of (2) to replace the differential factor (3). In case of $\alpha_i \neq 0$, we need only to change (7) and (8) to

$$a_i = \frac{\rho_i}{1+\rho_i(1+\alpha_i\Delta x)} \tag{27}$$

and

$$b_i = \frac{1}{1+\rho_i(1+\alpha_i\Delta x)}, \tag{28}$$

respectively.

Another way to further insure the stability of the new boundary condition is to multiply b_i by a constant slightly less than 1.0 that will force the pole to move toward the

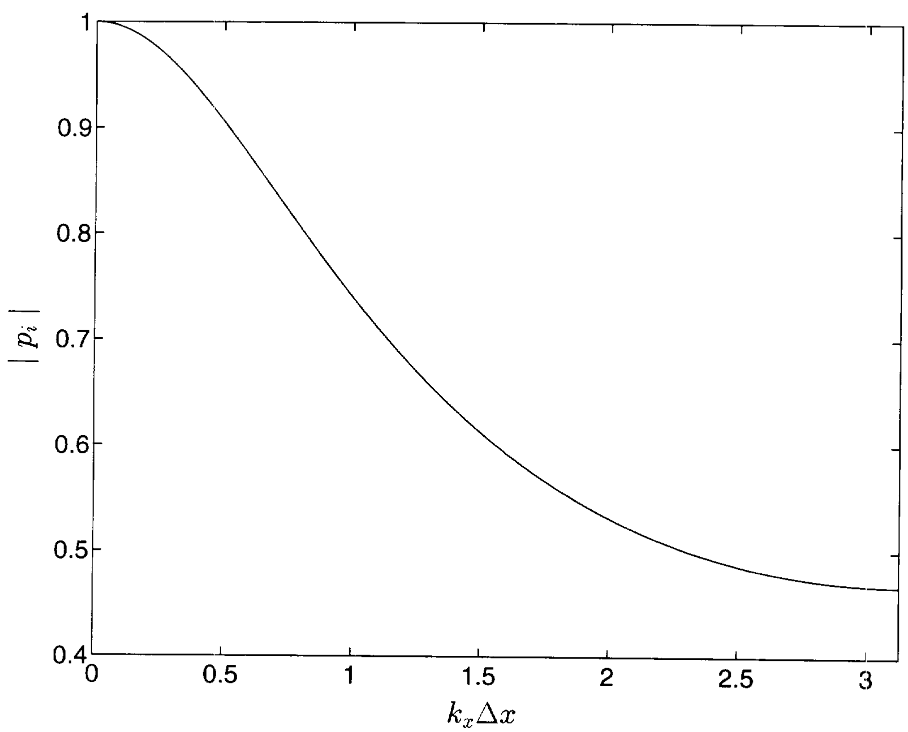

Figure 1: Magnitude of the pole p_i

origin in the Z-plane.

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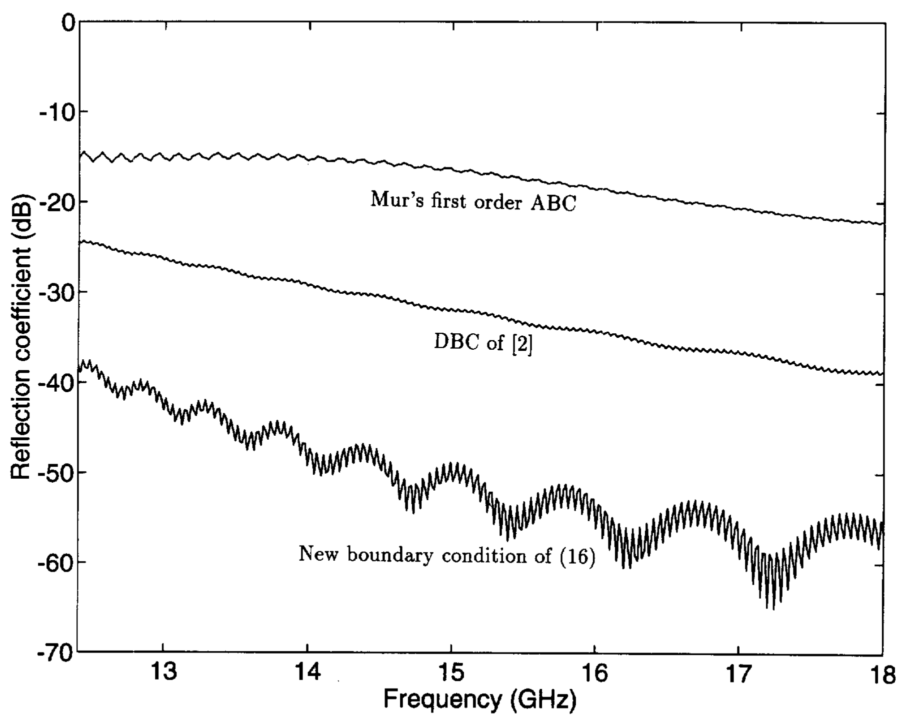

Figure 2: Reflection coefficients against frequency for a waveguide