COMBINING AN AUTOREGRESSIVE (AR) MODEL WITH THE FD–TD ALGORITHM FOR IMPROVED COMPUTATIONAL EFFICIENCY

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Abstract

In this paper, we combine an autoregressive (AR) model with the FD–TD algorithm. It is shown that this greatly improves the computational efficiency of the FD–TD technique. The AR model can be set up using a short segment of the time domain trace generated by FD–TD. Through extrapolation, future segments of the response can be accurately predicted. It is shown that the AR model is more robust and accurate than other prediction techniques used with EM modeling. A number of numerical examples is shown.

1 Introduction

The finite–difference time–domain (FD–TD) method has been widely used to solve electromagnetic problems. It is now highly recommended in simulations of very complex electromagnetic problems[1][2]. With the use of an absorbing boundary, it has been successfully used in open problems such as printed circuits and patch antennas. However, if one uses a Fourier transform to derive an accurate frequency response, based on the FD–TD algorithm, very long time domain records are usually required. Premature termination of a time domain record will result in inaccuracy frequency domain results. To reduce the computational time for FD–TD, people have combined the FD–TD method with a number of signal processing techniques, such as Prony's method[3], the System Identification method[4] and the MUSIC method[5]. In this paper, the autoregressive(AR) method is used to reduce the length of the FD–TD time domain record for accurate frequency results. In short, a short segment of the FD–TD time domain record is used to set up the AR model. The final analysis is carried out on the combined time record, i.e. the original FD–TD trace plus the extension of the trace based on predictions provided by the AR model. It is focused that the hybrid technique, i.e. FD–TD combined with AR, provides highly accurate results with a greatly increased computational efficiency.

2 Autoregressive Model and Operational Procedure

The autoregressive (AR) model[6–7] has found applications in many fields of research. Its set–up is based on using data observed in the past to develope the AR model coefficients. Once the model is established, future realizations can be predicted by present occurrences. In contrast to Prony's method which seeks a deterministic exponentials model to fit the data, the AR model seeks a random model, to the second–order, to fit a statistical data base.

We assuming that time series $u(n), u(n-1), \cdots, u(n-p)$ represents the realization of an autoregressive process of order p if it satisfies the the difference equation,

$$u(n) = -a_1 u(n-1) - a_2 u(n-2), \cdots - a_p u(n-p) + v(n) \tag{1}$$

where $a_1, a_2, \cdots, a_p$ are constants, which and are called AR parameters, and $\{v(n)\}$ is white–noise process. We see that the present value of the process, $u(n)$, is equal to

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a finite linear combination of past values of the process, $u(n-1)$, $u(n-2)$, $\cdots$, $u(n-p)$, plus the error term $v(n)$. Therefore, the present status can be predicted using a linear combination of previous observations. The least square criterion is used when we set up the model.

The AR spectrum tends to have sharp peaks, a feature often associated with high resolution spectral estimation technique. It has been proved that, under the Gaussian noise assumption, the AR spectrum is equivalent to the spectrum which would be derived using Maximum entropy technique. Maximum entroipy extends the information of an existing finite record to the future record.

Before analyzing the data, some precautions must be taken in using the signal obtained from FD–TD method. The first is the the resample FD–TD data must be resampled. To meet the FD–TD stability condition, the FD–TD algorithm usually oversamples the data compared to the needs of the AR method. So the data have been re-sampled where only 1 out of 10 FD–TD samples is in this study for the AR analysis. The second is to use a low pass digital filter to improve the accuracy in AR parameter estimation.

In our proposed approach, much shorter time sequence obtained by FD–TD method is needed to extract S parameters of printed circuits or patch antennas. The most important step in establishing an AR model is to estimate the order of AR model, which is usually determined by final prediction error (FPE) and can be verified by curve fitting.

3 Numerical Results and Discussion

The model is first applied to an edge coupled bandpass filter[8] as shown in Fig.1. In order to obtain an accurate estimate of the S parameters, over 30000 iterations are needed in original FD–TD method. By using the AR approach, the FD–TD only needs 2000-3500 iterations to generate the original data base for the AR process, denoted as $\{x\}$. Resampling the $\{x\}$ at the rate of 1 out of 10, we get a time series $\{x_1\}$. The $\{x_1\}$ is then processed with a low pass filter, using a cut–off frequency

beyond the frequency of our interest, thereby generating the time series $\{u\}$. Finally, $\{u\}$ is used to develope an AR method of order of 50. Fig.2 gives a comparison between the result obtained by FD–TD plus AR model and that by the direct FD–TD computations alone. It should be added that once the AR parameters are set up, the AR model can accurately predict very long time–domain traces of transient waveforms. Fig.3 shows the S parameter calculated by the FD–TD plus AR method, and the direct FD–TD using 3500 FD-TD time domain iteration, as well as a measured result. It is observed that the result obtained using FD–TD plus the AR method agrees very well with the measured results.

The next example[9], as shown in Fig.4, is the application of the proposed FD–TD + AR model to a printed antenna. A similiar procedure is applied to the FD–TD result consisting of 1200–2500 time domain iterations. Fig.5 shows the AR extrapolation compared with FD–TD result. Again, a good agreement can be observed.

4 Conclusion

A digital filtering technique, namely autoregresssive method has been successfully incorporated with the FD–TD method to improve the efficiency of FD–TD algorithm. With the same degree of accuracy, the computation time can be reduced by a factor of 3–5. This method can also be extended to the TLM method and other time domain numerical methods.

5 Acknowledgement

References

[1] X. Zhang, K.K.Mei, "Time–Domain finite–difference approach the calculation of the frequency-dependent characteristics of microstrip discontinuities," *IEEE Tran. Microwave Theory Tech.*, vol.36, pp.1775–1787, 1988.

[2] C.Wu, K.L.Wu, Z.Bi, J.Litva, "Accurate characterization of planar printed antennas using finite-difference time–domain method ", *IEEE Tran. Antenna Propagat.* vol. AP–40, pp526–534, May 1992.

[3] W.L.Ko, R. Mittra, "A combination of FD–TD and prony's methods for analyzing microwave integrated circuits", *IEEE Tran. Microwave Theory Tech.* vol.39, pp2176–2181, Dec. 1991.

[4] W.Kuempel, I.Wolff, " Digital signal processing of time domain field simiulation results using the system identificaiton method", *IEEE MTT–S*, 1992, pp973–976.

[5] Z.Bi, Y.Shen, K.L.Wu, J.Litva, " Fast finite-difference time–domain analysis of resonators using digital filtering and spectrum estimation technique", *IEEE Tran. Microwave Theory Tech.* vol.40, pp1611–1619, August 1992.

[6] S. Haykin, Nonlinear Method of Spectral Analysis, Springer-Verlag, 1979.

[7] S.M.Kay, S.L.Marple, Jr., "Spectrum Analysis– a modern perspective" *Proc. IEEE*, vol. 69, pp1380–1419, Nov. 1981.

[8] C.J.Railton, S.A.Meade, "Fast rigorous analysis of shielded planar filters", *IEEE Tran. Microwave Theory Tech.*, vol. 40, pp978–985, May 1992.

[9] C.Wu, K.L.Wu, Z.Bi, J.Litva, "Modeling of coaxial-fed microstrip patch antenna by finite difference time domain method", *Electro. Lett*, 1991, 19, pp1691–1692.

Figure 1: (a). Plan of edged–coupled bandpass filter. (b). Elevation of filter. $l_1 = 6.36$mm, $l_2 = \frac{b}{2} = 8.48$mm, $a = 11.62$mm
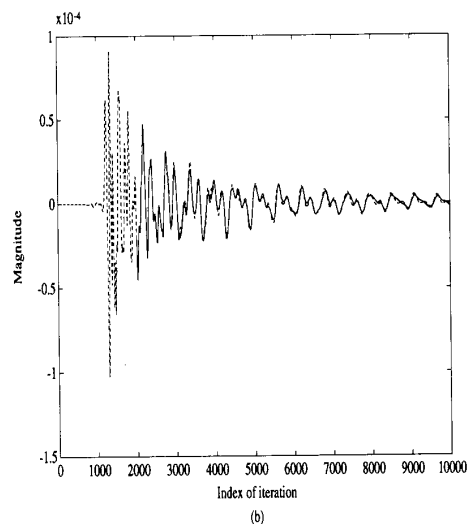

Figure 2: AR curve fitting and extrapolation compared with FD–TD method. —— FD–TD + AR method - - FD–TD records
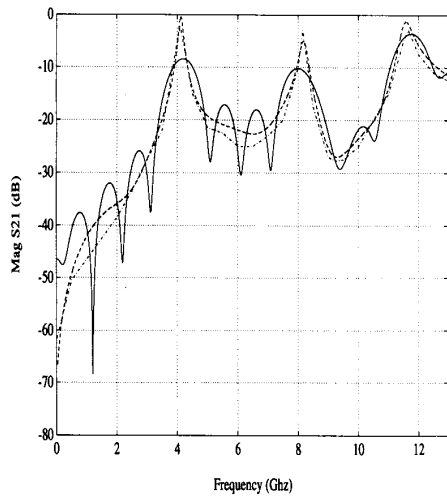
Figure 3: Edged–coupled microstrip filter scattering parameter S_{21}
.   — 1–3500 records FD–TD.   - - 1–3500 FD–TD records
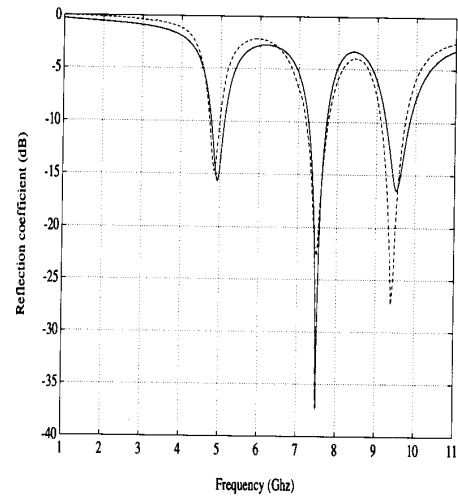plus AR method   -.- measured result

Figure 5: Reflection coefficient of the xoaxial–fed patch antenna.
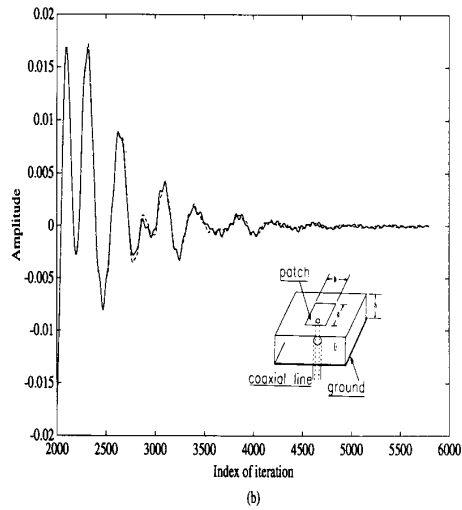— 1–2500 FD–TD records plus AR method.   - - measurement
result.

(b)

Figure 4: AR curve fitting and extrapolation using patch antenna
FD–TD records from 1200 to 2500.   — AR curve fitting and
extrapolation   - - FD–TD records