Characterizing Microwave Planar Circuits By Coupled Finite-Boundary Element Method

Ke-Li Wu, Chen Wu and John Litva

Communications Research Laboratory, McMaster University,
Hamilton, Ontario, Canada L8S 4K1

ABSTRACT *A general approach to the analysis of microwave planar structures, specifically intended to treat complicated geometries and dielectric load, is presented. The proposed approach is based upon the coupling of the finite element and boundary element methods. The respective merits of these methods are extracted to yield much faster solution and to enhance computation efficiency. This general approach can handle a problem with mixed electric and magnetic walls, as well as complicated dielectric load such as ferrite materials. Computed and measured data for various complicated devices are compared, showing an excellent agreement.*

1. INTRODUCTION

In the development of electromagnetic field CAD numerical techniques, one has to compromise between accuracy and simplicity. Three dimensional exact analyses are often impractical because of the exceedingly high computer time and vast memory space required. In contrast, a simple model used in the analysis of planar circuits assumes that the circuit is laterally bounded by a magnetic wall, except at the ports. This model is used particularly for solving the microwave planar circuit problems. Several methods have been known for the analysis of planar microwave circuits. When the circuit pattern is as simple as square, rectangular, circular, or annular, the field expansion in terms of resonant models can be used, in which the eigenfunctions satisfying the two dimensional eigenvalue equation together with magnetic wall boundary condition must be known in analytical form. This can be done analytically if the structure has a separable geometry, otherwise, the numerical analysis must be adopted.

From the practical point of view, the analysis of arbitrary shaped planar circuits is more important in the CAD development. The Finite Element Method (FEM)[1] was used to handle the discontinuities of arbitrarily shaped planar circuits, although the computational overhead is large. It should be noted that the dielectric within a planar circuit is assumed mostly homogeneous. The boundary integral method or Boundary Element Method (BEM) [2] seems to be of most efficient, because they reduce the problem from a two-dimensional case to a one-dimensional one. However, it is very difficult to handle the planar circuits involving complicated or anisotropic dielectric load using BEM.

In this paper, the coupled finite boundary element method (CFBM), which was originally developed to the waveguide discontinuity analysis [3], is extended for a general planar circuit structure. The CFBM has the merits of both the FEM and the BEM, and can be used to solve extremely complex problems without requiring excessive computer memory and computation time. Using the method, only the complex media subdomains, thich may consist of lossy or anisotropic materials, need to be treated using FEM. Elsewhere, the BEM is used on the boundary to take into account the circuit configuration. Comparing with the FEM (two-dimensional algorithm) and BEM (one-dimensional algorithm), one can find that the CFBM is a one-and-half dimensional algorithm for planar circuits.

2. THE PLANAR CIRCUIT MODEL

When the spacing d, the distance between upper circuit and lower ground of arbitrarily shaped planar circuit, is much smaller than the wavelength and the spacing material is homogeneous and isotropic, the variation of the z-directed vertical field and the z component of the magnetic field inside the circuit are negligible. Under these assumptions, it can be directly deduced from Maxwell's equation that a two-dimensional Helmholtz equation dominates the electromagnetic field in the planar circuit. Because the circuit is excited symmetrically with respect to the upper and lower ground conductors, the following boundary condition must hold:

$$\frac{\partial E_z}{\partial n} = 0 \qquad (1)$$

where **n** is the outward unit normal of the circuit edge.

Therefore, to get reasonably accurate results with a 2-D model, the fringing field must be taken into account by using an effective dielectric constant and by slightly extending the circuit dimensions. There are two basic types of equivalence that can be used to link a real circuit and the parallel-plate model. One is based on the fact that a real transmission line can be modelled using a planar waveguide with equivalent waveguide width $W_{eff}(f)$ and the equivalent filling dielectric constant $\varepsilon_{eff}(f)$. The parameters W_{eff} and ε_{eff} are determined by the conditions that both the phase velocity and the characteristic impedance are the same as that of the microstrip line or stripline. Since the dominant mode of the planar waveguide is a TEM mode, the equality of the phase velocities imposes the condition that the filling dielectric has the same effective permittivity ε_{eff} as the quasi-TEM mode of the real transmission line[4,5]. This is accomplished by varying the guide width $W_{eff}(f)$ and the relative dielectric constant $\varepsilon_{eff}(f)$, while keeping the height h unchanged. Another equivalence which allows one to go from irregularly shaped circuits to a disk resonator which is surrounded by lateral magnetic walls.

There are various techniques that can be used for accurately determining both $W_{eff}(f)$ and $\varepsilon_{eff}(f)$. Since they are well known they will not be discussed here for simplicity. When characterizing microstrip or strip line circuits by means of an equivalent cavity, surrounded by magnetic walls, it is necessary when establishing the equivalent dimensions to take into account the energy stored in the circuit's fringing field. The stored energy can be estimated by assuming that the electric and magnetic fields are constant throughout the increased volume ΔV and are equal to the field at the edge of the circuit. The increased volume ΔV is taken to be equal to the volume corresponding to the static fringe capacitance ΔC between the radius a and the equivalent radius a_{eq} of the fringe edge. For instance, the equivalent radius of a circular disk is

$$a_{eq} = a\{\, 1 + \frac{d}{\varepsilon \pi a^2}\, \Delta C\,\}^{1/2} \tag{2}$$

where the static fringe capacitance ΔC can be derived using Kirchhoff equation, or using Maxwell's function[6] for a better approximation. The equivalent parameters for other geometries can be obtained in a similar way.

It is usually assumed that the equivalent dimensions reflect only the characteristics of the fringing fields when unperturbed by inhomogeneities in the planar circuits, such as a dielectric loads or conductor posts. It can be shown, by comparing the numerical results with the experimental results, that this assumption is fairly reasonable because the equivalent dimensions are determined mostly by the fringing of the outermost fields.

3. GENERAL EQUATION

By combining the matrix equations for FEM and BEM and considering (1) the compatibility and equilibrium of the electric field, and (2) the boundary condition on electric/magnetic wall, following general coupled matrix equation can be obtained:

$$\begin{bmatrix} [H_0, H_1, H_2, \cdots H_M, H_Q, 0] & -[G_0; G_1; \cdots G_M, G_Q] \\ \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} A \end{bmatrix} & \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & D \end{bmatrix} \\ \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & 1 \ddots & \\ 0 & & 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} & \begin{bmatrix} -Z_1 & & 0 \\ & \ddots & \\ 0 & & -Z_M \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \{E_z\}_{\Gamma_0} \\ \{E_z\}_{\Gamma_1} \\ \vdots \\ \{E_z\}_{\Gamma_M} \\ \{E_z\}_{\Gamma_Q} \\ \{E_z\}_{Q} \\ \{\frac{\partial E_z}{\partial n}\}_{\Gamma_{0'}} \\ \{\frac{\partial E_z}{\partial n}\}_{\Gamma_1} \\ \vdots \\ \{\frac{\partial E_z}{\partial n}\}_{\Gamma_M} \\ \{\frac{\partial E_z}{\partial n}\}_{\Gamma_Q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2\delta_{ij} f_{i0} \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{3}$$

In which the following analytical relation on the waveguide ports is used.

$$E_z(x^{(i)}=0, y^{(i)})|_{\Gamma_i} = 2\delta_{ij} f_{i0}(y^{(i)}) - \sum_{m=0}^{\infty} \frac{1}{j\beta_{im}} \int_0^{W_i} f_{im}(y_0^{(i)}) f_{im}(y^{(i)}) \cdot \frac{\partial E_z}{\partial n}(y_0^{(i)})|_{\Gamma_i} dy_0^{(i)} \tag{4}$$

where

$$f_{im} = \begin{cases} \sqrt{\dfrac{2}{W_i}} \cos(m\pi y / W_i) & m=1,2,\ldots \\ \sqrt{\dfrac{1}{W_i}} & m=0 \end{cases} \tag{5}$$

$$\beta_{im} = \sqrt{k_0^2 - (m\pi/W_i)^2} \qquad m=0,1,2,\ldots \tag{6}$$

In the above equation, [1] is an identity matrix, [0] is an empty matrix, $\{E_z\}_{\Gamma_i}$ and $\{\partial E_z/\partial n'\}_{\Gamma_i}$ (i =0,0',1,2,.... M, Q) correspond to electric fields and their normal derivatives at the nodal points related to boundary Γ_0, $\Gamma_{0'}$, Γ_1, $\Gamma_2 \ldots \Gamma_M$, Γ_Q, which are indicated in Fig.1, and {0} is a null vector.

In practices, all ports described in above general equation are matched and the magnitude of the incident field in port j is unit. The solution of the matrix equation determines the scattering electric field distribution across each port. Using the orthogonality of the modes in a planar waveguide, the scattering parameters of the fundamental mode can be determined.

It is interesting to note that, if there is no dielectric inhomogeneity range Q, the general matrix equation is degenerated to the equation for planar circuits with inductive conductor post. If it is assumed further that there is no conductor post in the planar circuit, the general equation becomes the original form for homogeneous planar circuits[2].

4. NUMERICAL EXAMPLES

As the first example, a microstrip circular disk filter is considered as a planar circuit with arbitrary shape. The fringing fields were included in the calculations by assuming that the circuit is bounded by a magnetic wall shifted by some distance from the real edge of the circuit. A square dielectric load with $\varepsilon_r = 10.2$ is placed at the center of the disk as shown in Fig.2. Comparison of the predicated by CFBM and measured results indicates that the CFBM is applicable to complex microstrip planar devices. Considering the inevitable radiation loss by this open structure, the negligible discrepancy can be expected.

It has been discussed that if there is no dielectric inhomogeneity involved, the general formula will be degenerated to a BEM-based algorithm. A typical example for this special case is a four-port band-pass filter, using half-wavelength sections as series resonators and shunt posts. The filter was originally synthesized using Chebyshev filter theory with ripple level of 1.0 dB, and constructed using a four-fort Duroid substrate with thickness = 0.49 mm and ε_r =2.43 [7]. However, in the present analysis, the filter is considered as a whole system consisting of four circular electric walls and magnetic side walls. As shown in Fig.3, the CFBM result agrees well with the results of multipole expansion method (not shown here) and the measurement result presented in [7].

The last example is a Y junction with a TT1-109 triangular ferrite post and the filling substrate of $\varepsilon_r = 2.2$ shown in Fig.4. In the analysis both magnetic losses and dielectric losses are neglected. The S-parameters obtained using the CFBM are also shown in Fig.4, where a wide band isolation performance can be observed. Comparing to the CFBM analysis of waveguide ferrite circulator in [3], the only difference in the planar circuit analysis is that the electric wall is replaced by the magnetic wall. It is found that by a number of numerical test, the dimensions of the ferrite triangular will affect the performance of the circulator, while the dimensions of the cavity determine the working frequency. In this example, only 16 triangular second order finite elements are used to get a convergent result.

5. CONCLUSION REMARKS

A generalized coupled finite and boundary element method has been shown to be applicable to microwave planar circuit problems. The planar waveguide model is used in developing the technique. The technique takes advantage of the strengths of the finite element and boundary element methods. Thus, it can handle complicated and arbitrarily shaped planar circuits with a small computational overhead. The validity of the method was confirmed by comparing the CFBM results, either with published results or with experimental results. The performance of a Y-junction circulator with an equilateral triangular ferrite post was also investigated. For all the numerical examples presented above, the power conservation condition has been found to be satisfied to an accuracy of $\pm10^{-5}$ to $\pm10^{-4}$ within the frequency band of the dominant mode.

Implementation of the proposed method is straightforward and requires much less computer resources than other models that have been used for planar circuits. Obviously, it can become a practical tool in engineering applications. In fact, a user-friendly package, named PLANAR MICROWAVE CIRCUIT SIMULATOR, which runs on an IBM PC, has been developed. All the numerical results presented in this paper were calculated by this package. It has been found that the method performs better than other widely used methods for inhomogeneous circuits, such as the finite difference time domain method, FEM and multipole expansion method.

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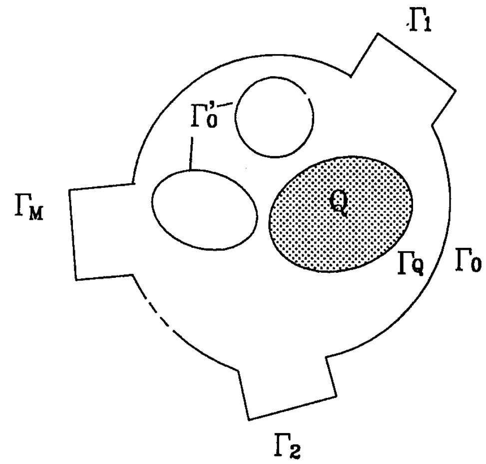

Fig. 1 A general planar circuit geometry and the computation regions.

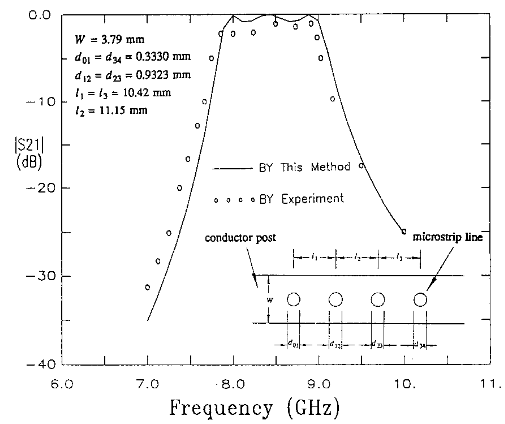

Fig. 3 Computed and measured $|S_{21}|$ for a four-post band-pass direct-coupled cavity filter.

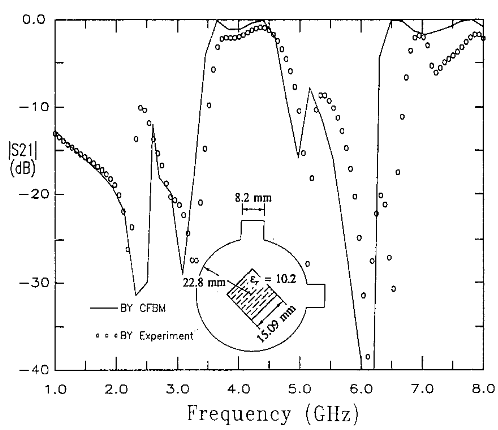

Fig. 2 Computed and measured transmission coefficients for a microstrip circular disk filter with a square dielectric load.

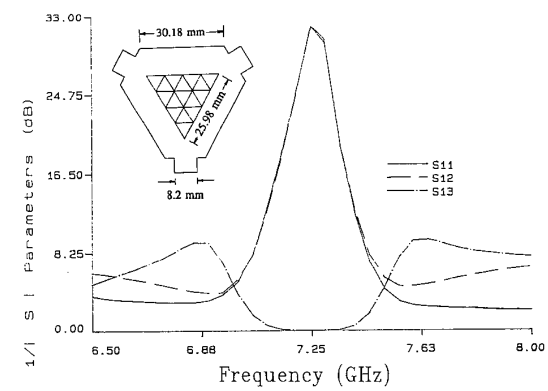

Fig. 4 Scattering parameters of a planar Y-junction circulator with an equilateral triangular ferrite post.