Trapezoid Topology for Dual-Mode Bandpass Filters

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***Abstract*— Presently, coupling topologies compatible with dual-mode resonator filters are restricted to either symmetric responses or asymmetric responses with up to *N*\2-1 transmission zeros (TZs). This article introduces a new coupling topology, the trapezoid topology, which is fully compatible with dual-mode resonator filters and capable of realizing both symmetric and asymmetric filter responses with up to *N*\2 TZs. The topology is categorized into two types: singly coupled and doubly coupled input and output (I/O) coupling structures. The available mixed configuration of single- and dual-mode resonators offers the advantage of suppressing harmonic resonances. This generic coupling topology can realize all possible out-of-band TZ arrangements. A mathematical formulation for numerically synthesizing trapezoid filters is also provided. The characteristic of multiple solutions offers users greater design flexibility. To validate the coupling topology and demonstrate its applicability to dual-mode resonator filters, two physical realization examples are presented: an electromagnetically designed 6-3-0 trapezoid filter realized by dual-mode dielectric resonators and a prototyped 8-4-0 trapezoid filter using circular waveguide dual-mode (CWDM) cavities. The simulated/measured filter responses align closely with the synthesized ones. Regarding practical designs, the criteria for selecting solutions and the sensitivities of TZs with respect to coupling elements are also discussed in detail.**

***Index Terms*— Asymmetric filter responses, coupling matrix, coupling topologies, dual-mode filters, trapezoid topology.**

I. Introduction

EXPLORING new coupling topologies for coupled-resonator bandpass filters is always an important topic in the realm of microwave filters due to the consistent increase in the demand for more compact and versatile filter configurations in modern communication systems. With the prevalent adoption of massive multi-input–multioutput (M-MIMO) antenna array technology, with which a large number of high-performance bandpass filters need to be affixed to the back panel of an antenna array, filters tend to have a smaller and smaller footprint to fit the limited real estate. However, compactness and versatileness are sometimes contradictory when the filter order is high and response is asymmetric. To confront this predicament, developing dual-mode filters that can realize both symmetric and asymmetric responses with high rejection roll-off would be highly desirable. The new class of coupling topology presented in this article, namely, trapezoid topology, is an attempt to alleviate this dilemma.

The trapezoid topology is highly compatible with dual-mode resonator filter realizations, permitting great flexibility in space saving and layout simplicity without sacrificing rejection performance. The coupling topology is applicable to the filters with not only symmetric but also asymmetric responses. As far as an asymmetric response is concerned, the topology can provide more transmission zeros (TZs) than any existing known dual-mode compatible topologies. Such attributes are attractive to the applications of not only wireless base stations but also space payloads.

A coupling topology that is compatible with a dual-mode resonator filter must possess the structural feature consisting of at least a pair of coupled-resonator array that is coupled to adjacent pairs of resonator arrays side-by-side without diagonal cross couplings. The first microwave dual-mode bandpass filter can be traced back to the article by Atia and Williams [1]. This article proposed a circular waveguide dual-mode (CWDM) filter with a symmetric filter transfer response. Since then, a handful of coupling topologies have been proposed for dual-mode filter realizations, which include: 1) the folded topology without diagonal cross couplings [2]; 2) the "Pfitzenmaier" configuration [3], with which the input and output (I/O) ports are located in different physical dual-mode resonators, attaining a good I/O isolation; and 3) the cascaded quadruplet (CQ) topology without diagonal cross couplings [4]. These topologies do provide multiple selections for I/O arrangements, the number of TZs, as well as the structural symmetry options. However, they can only realize symmetric filter responses.

To break the limitation and to realize asymmetric filter responses, the "extended box" coupling topology was proposed in 2002 [5]. The topology is formed by cascading of coupled-resonator pairs, permitting a dual-mode resonator filter in an in-line configuration, while the I/O ports are located at the two ends of the filter. The most attractive feature of the extended box topology is its ability to realize both symmetric and asymmetric filter responses. A 6-2-0 CWDM filter with an asymmetric response in the extended box topology is reported in [6]. Here, the term "*N*-*I*-*C*" filter means the *N*th order filter with *I* imaginary TZs and *C* complex TZs. Although being

Manuscript received 20 August 2023; revised 30 October 2023; accepted 14 November 2023. This work was supported by the Postgraduate Scholarship of The Chinese University of Hong Kong. *(Corresponding author: Ke-Li Wu.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TMTT.2023.3335929.

Digital Object Identifier 10.1109/TMTT.2023.3335929

able to realize asymmetric responses, the maximum number of realizable TZs is limited to $N\backslash 2$-1, where N is the order of the filter and "\" refers to integer division. Even though one more TZ is sometimes essential to meet the required rejection specification in practice, the $N\backslash 2$-1 bar has not been broken through by an advanced generic coupling topology that is fully compatible with dual-mode realizations in the past 20 years.

The generic trapezoid topology permits a dual-mode filter for a symmetric or asymmetric response with up to $N\backslash 2$ TZs. The topology is divided into two categories: singly coupled and doubly coupled I/O structures. By applying the newly proposed exhaustive synthesis framework of coupled-resonator filters [7], the trapezoid coupling topology up to 10th-order with all possible out-of-band TZ arrangements has been verified to be a generic coupling topology with fixed topology patterns. While the literature has referenced two specific instances of the trapezoid topology with singly coupled I/O—a 7-3-0 dual-band filter realized by a single-mode rectangular waveguide cavity filter [8] and the solutions for 8-4-0/9-4-0 trapezoid topologies [7]—the full generality of the topology has not been thoroughly explored, nor has it been physically validated. This article not only presents a comprehensive study of the trapezoid topology for the first time but also mathematically affirms the legitimacy of the topology in general, including the distinction of multiple solutions, the unique characteristics of both odd- and even-order trapezoid filters, and the option of a doubly coupled I/O port. Furthermore, the applicability of the topology to dual-mode resonator filters is validated by both electromagnetic (EM) design and experimental prototypes.

This article will give the full disclosure of the trapezoid topology as a generic topology for dual-mode resonator filters, including both the singly coupled and doubly coupled I/O structure trapezoid topologies for all the possible TZ arrangements. In addition to providing an easy-to-use synthesis method, the criteria for selecting an optimal solution among multiple solutions will also be discussed. The physical realizations using dual-mode resonators are demonstrated by one EM design example of a 6-3-0 trapezoid dual-mode dielectric resonator filter in the C-band and one hardware 8-4-0 trapezoid filter prototyped using CWDM cavities in the *Ku*-band, showing the practical viability of the trapezoid topology. Both examples use the singly coupled I/O coupling structure.

Although the research on dual-mode filters continues to be a hot topic recently, concerning the design approaches [9], [10] and synthesis methods [11], it will be shown that the trapezoid topology is the most capable member in the family of known coupling topologies that are suitable for dual-mode resonator filters. It can be expected that the topology will provide the industry with an attractive option in designing filters and diplexers when both compactness and high rejection requirements are imposed.

II. Topological Features of Trapezoid Filters

The generic trapezoid coupling topologies, which realize $N\backslash 2$ TZs, are shown in Fig. 1 for both even- and odd-

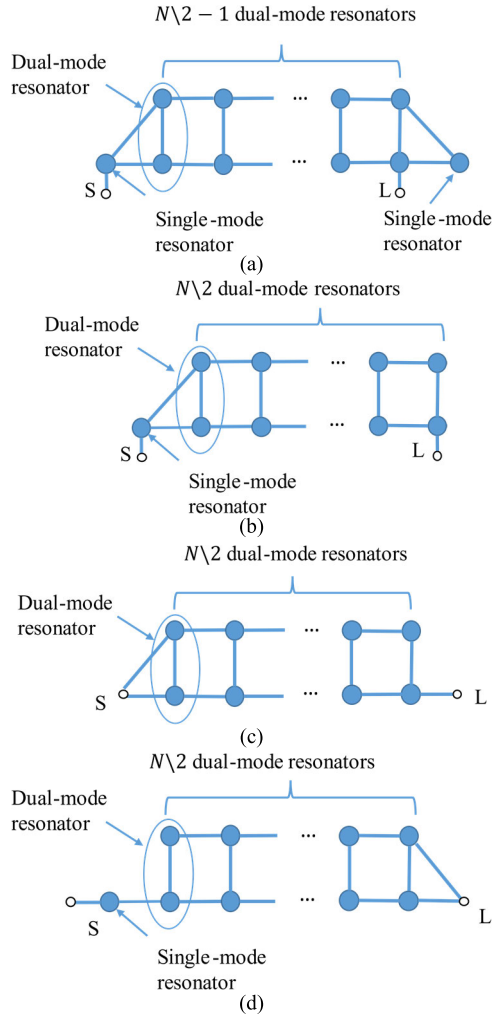


Fig. 1. Trapezoid coupling topology with (a) even order N with singly coupled I/O, (b) odd order N with singly coupled I/O, (c) even order N with doubly coupled I/O, and (d) odd order N with doubly coupled I/O.

orders of N. The topologies fall into two distinct categories: the first includes singly coupled I/O structures as shown in Fig. 1(a) and (b), while the second comprises doubly coupled I/O structures shown in Fig. 1(c) and (d). The trapezoid topology presents the following unique characteristics.

1) *For Singly Coupled I/O Trapezoid Topology:*
   a) The topology consists of $N\backslash 2$-1 dual-mode and 2 single-mode resonators if N is even or $N\backslash 2$ dual-mode and 1 single-mode resonators if N is odd, where the single-mode resonator is always located at an end and is coupled to two modes of the adjacent dual-mode resonator simultaneously.
   b) One port is tapped to a single-mode resonator and the other port is coupled to one of the dual modes in the dual-mode resonator that is either adjacent to the second single-mode resonator (for even N) or the last dual-mode resonator (for odd N).
2) *For Doubly Coupled I/O Trapezoid Topology:*
   a) The topology consists of $N\backslash 2$ dual-mode resonators if N is even or $N\backslash 2$ dual-mode and 1 single-mode resonators if N is odd, where the

single-mode resonator is always located at one end and is coupled to one of the dual modes in the adjacent dual-mode resonator.

b) One port is coupled to the two modes of an end dual-mode resonator simultaneously and the other port is coupled to one of the dual modes in the dual-mode resonator at the other end (for even N) or the single-mode resonator at the other end (for odd N).

Subject to the minimum-path rule [12], for Nth order filters, up to $N\backslash 2$ TZs, including the complex ones, can be realized with symmetric/asymmetric filter responses. For the singly coupled I/O, when $N = 3$ and 4, the trapezoid topology degenerates to a trisection and quartet topology, respectively, and will not be a concern of this work.

The doubly coupled I/O trapezoid coupling topology holds a notable advantage over its singly coupled I/O counterpart in even-order scenarios: it requires one less physical cavity for dual-mode realizations. In addition, the two I/O couplings are coupled to the same dual-mode physical resonator, making the implementation convenient.

It is important to note that for a given filtering function to be synthesized, the trapezoid topology may offer multiple real-valued solutions. The choice of solution is dependent on design preference, which will be discussed in Section V.

As is seen, the trapezoid topology may consist of a single-mode resonator. The mixed composition of single- and dual-mode resonators has an advantage in suppressing lower order harmonic modes to a certain extent as the resonant frequencies of the harmonics for the single- and dual-mode resonators are usually different. It is worth mentioning that although the highest number of TZs that can be achieved by the topology is $N\backslash 2$, a smaller number of TZs can also be realized according to the minimum-path rule.

III. Synthesis of Coupling Matrix

To check the legitimacy and synthesize the trapezoid topology for a given filter transfer function, an exhaustive real solution synthesis routine introduced in [7] can be utilized. The routine consists of finding the coupling matrix $\mathbf{F}$ in the folded form for the given filter transfer function and solving the following simultaneous nonlinear implicit equations that involve coupling matrix $\mathbf{M}$ in the trapezoid form:

$$\begin{aligned} F(1) &= f_1(M(1), M(2), \ldots, M(N_M)) \\ F(2) &= f_2(M(1), M(2), \ldots, M(N_M)) \\ &\vdots \\ F(N_F) &= f_{N_F}(M(1), M(2), \ldots, M(N_M)) \end{aligned} \tag{1}$$

where N_F and N_M refer to the number of couplings in $\mathbf{F}$ and $\mathbf{M}$ (including mutual- and self-couplings), respectively, and $(f_1, f_2, \ldots, f_{N_F})$ is the series of Given's transformations converting $\mathbf{M}$ to its corresponding folded matrix $\mathbf{F}$, which is available in textbooks [13, pp. 275–278]. $N_F = N_M$ is one of the necessary conditions to guarantee that (1) has a finite number of solutions. The functions $(f_1, f_2, \ldots, f_{N_F})$ are continuous and differentiable, whose Jacobian matrix with respect to $\mathbf{M}$ is defined as

$$\mathbf{J} = \begin{Bmatrix} \dfrac{\partial f_1}{\partial M(1)} & \cdots & \dfrac{\partial f_1}{\partial M(N_M)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_{N_F}}{\partial M(1)} & \cdots & \dfrac{\partial f_{N_F}}{\partial M(N_M)} \end{Bmatrix}. \tag{2}$$

For the trapezoid coupling topology of different orders with all possible numbers of TZs, it has been exhaustively checked that the determinant of the Jacobian matrix is always nonzero, implying the legitimacy of the trapezoid topology. Having validated the legitimacy of the trapezoid topology, an exhaustive solution search can be applied in two steps: 1) setting the Frobenius norm $\|\mathbf{F_M} - \mathbf{F_0}\|$ as the error function, where $\mathbf{F_M}$ is the transformed coupling matrix in the folded form from a trial coupling matrix $\mathbf{M}$ in the trapezoid topology and $\mathbf{F_0}$ is the target folded coupling matrix obtained from the given filtering function, and 2) solving (1) with excessive number of initial guesses (sufficiently larger than the number of possible real solutions). The reason to conduct enough number of solution searches is that the coupling matrix in the trapezoid form has at least one but usually multiple real solutions, subject to the order of the filter, the number of TZs, and the arrangement of TZs. When a newly founded solution is close to the boundary of the search domain, the search domain will be adaptively extended until the number of solutions converges. Such an exhaustive solution search can provide all the possible real solutions in a reasonable range of coupling variables. As an alternative approach for synthesizing all possible solutions of coupling matrix $\mathbf{M}$ in the trapezoid form, including the complex ones, the Gröbner basis method [14] that involves solving a set of simultaneous multivariable polynomial equations using a dedicated computer algebra routine [15] can be used.

Table I lists the number of solutions of the trapezoid topology with singly coupled I/O of orders from 5 to 10 and some typical TZ arrangements when the return loss is set to 20 dB. In fact, the number of solutions does not depend on the return loss level. Notice that the trapezoid topology has the flexibility to realize less TZs than $N\backslash 2$ by simply increasing the shortest path or removing some of the cross couplings. For example, two typical 8-3-0 and 8-2-0 trapezoid filters are also listed in Table I. It can be noticed that the number of real solutions not only depends on the order and the number of TZs but also the arrangement of TZs. Similar features can be observed on the trapezoid topology with doubly coupled I/O as revealed in Table II, which lists the number of real solutions of the trapezoid topology of orders from 5 to 10 with doubly coupled I/O and some typical TZ arrangements. All the topologies listed in Tables I and II realize $N\backslash 2$ TZs except in two cases in which one or two couplings are removed.

To better illustrate the applicability of the trapezoid topology to realize an arbitrary TZ arrangement, the exhaustive solution search of the sixth-order trapezoid filters with singly coupled I/O for all the five possible TZ arrangements outside of the passband, including a pair of complex TZs, is conducted. Fig. 2 shows the filter responses of the sixth-order trapezoid filters of all possible TZ arrangements in the low-pass domain.

TABLE I

NUMBER OF SOLUTIONS FOR TRAPEZOID TOPOLOGIES WITH SINGLY COUPLED I/O OF ORDERS 5–10 AND TYPICAL TZ ARRANGEMENTS

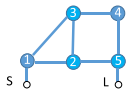
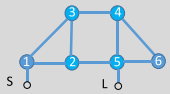
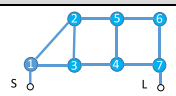
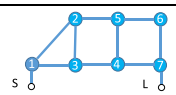
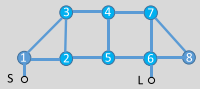
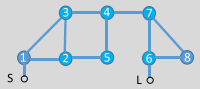
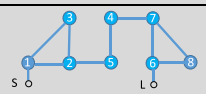
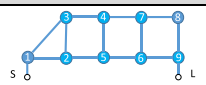
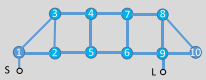
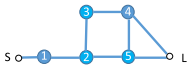
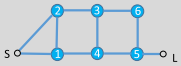
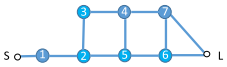
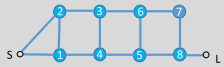
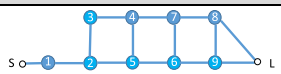
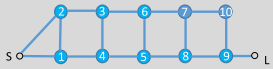
N	Topology	Normalized TZs positions	No. Solutions
5		[-1.3*j*, 1.7*j*]	3
6		[-1.3*j*, 1.4*j*, 1.7*j*]	3
7		[-1.4*j*, -1.3*j*, 1.7*j*]	7
7		[0, 1.2*j*, 1.37*j*]	3
8		[-1.6*j*, -1.3*j*, 1.2*j*, 1.5*j*]	17
8		[-1.5*j*, 1.15*j*, 1.25*j*]	7
8		[-1.3*j*, 1.5*j*]	2
9		[-1.43*j*, 1.15*j*, 1.28*j*, 1.5*j*]	15
10		[-1.8*j*, -1.5*j*, 1.2*j*, 1.4*j*, 1.6*j*]	61

TABLE II

NUMBER OF SOLUTIONS FOR TRAPEZOID TOPOLOGIES WITH DOUBLY COUPLED I/O OF ORDERS 5–10 AND TYPICAL TZ ARRANGEMENTS

N	Topology	Normalized TZs positions	No. Solutions
5		[-1.3*j*, 1.7*j*]	3
6		[-1.3*j*, 1.4*j*, 1.7*j*]	7
7		[-1.4*j*, -1.3*j*, 1.7*j*]	5
8		[-1.6*j*, -1.3*j*, 1.2*j*, 1.5*j*]	27
9		[-1.43*j*, 1.15*j*, 1.28*j*, 1.5*j*]	13
10		[-1.8*j*, -1.5*j*, 1.2*j*, 1.4*j*, 1.6*j*]	113

Only one selected coupling matrix that contains the least number of negative couplings along with the number of solutions for each of the possible TZ arrangements is listed in Table III, for which the return loss is set to 20 dB.

To better utilize these solutions and choose the most suitable one for a particular need, the statistical distributions of the coupling coefficients as well as the sensitivity of each TZ with respect to the coupling coefficients need to be analyzed [16].

IV. EXAMPLES OF DUAL-MODE REALIZATIONS

To demonstrate the realizability of the trapezoid topology using dual-mode resonators, an EM designed 6-3-0 dual-mode dielectric resonator trapezoid filter and a prototyped 8-4-0 filter realized by CWDM cavities are presented in this section. Both examples use the trapezoid topology with singly coupled I/O coupling structure.

A. Dielectric Dual-Mode Resonator Trapezoid Filter

The 6-3-0 trapezoid filter is realized by TM_{11} dual-mode dielectric resonators [17]. The coupling matrix for

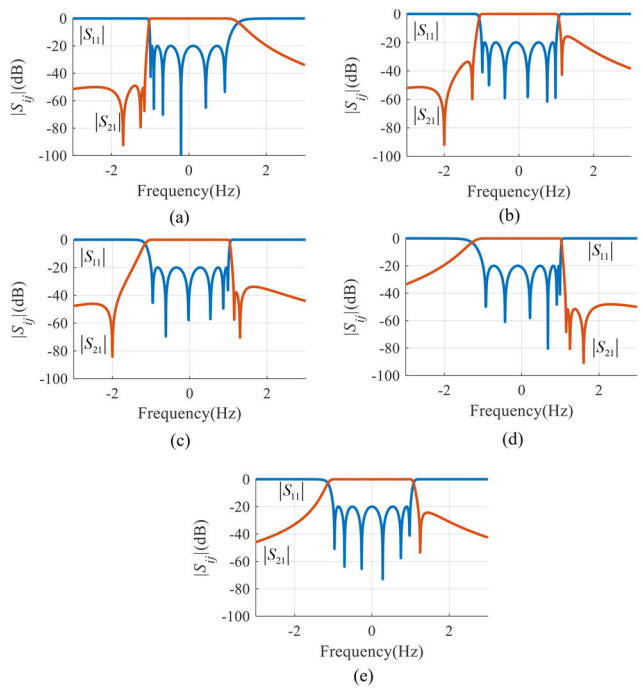


Fig. 2. Filter responses in low-pass frequency domain of the 6-3 trapezoid filters with five possible TZ arrangements: (a) $(-1.75j,\ -1.25j,\ -1.15j)$; (b) $(-2j,\ -1.25j,\ 1.15j)$; (c) $(-2j,\ 1.15j,\ 1.3j)$; (d) $(1.15j,\ 1.25j,\ 1.65j)$; and (e) $(-1-1j,\ 1-1j,\ 1.25j)$.

TZ-A 3 listed in Table III is chosen and the center frequency and bandwidth are 1.955 and 0.05 GHz, respectively. The designated TZs of the filter are located at 1.905, 1.984, and 1.987 GHz. The same size of dielectric resonator is used for both the single- and dual-mode resonators in the HFSS EM model. As shown in Fig. 3(a), each dielectric puck is situated at the center of a metal cavity. In the EM model, the full

TABLE III

SELECTED COUPLING MATRICES FOR THE 6TH-ORDER FILTERS WITH FIVE TZ ARRANGEMENTS

One of The Solutions Based on The Least Number of Negative Couplings for Each TZ Arrangements					
TZ-A ID	TZ-A 1	TZ-A 2	TZ-A 3	TZ-A 4	TZ-A 5
TZs	-1.75*j*	-2*j*	-2*j*	1.15*j*	(-1, -1*j*)
	-1.25*j*	-1.25*j*	1.15*j*	1.25*j*	(1, -1*j*)
coupling	-1.15*j*	1.15*j*	1.3*j*	1.65*j*	1.25*j*
M_{11}	-0.0588	-0.0011	0.0281	0.06	0.0127
M_{22}	-0.3873	-0.0032	-0.1427	0.381	0.5478
M_{33}	0.123	0.1769	0.0494	-0.1368	-0.0505
M_{44}	0.8025	-0.3459	0.1266	-0.8098	-0.0056
M_{55}	-0.0588	-0.0011	0.0281	0.06	0.0127
M_{66}	0.9868	0.3604	-0.9312	-0.9876	-0.5895
M_{S1}	1.0081	0.9899	0.9968	1.0085	0.9985
M_{12}	0.1565	0.1426	0.8246	0.1678	0.7492
M_{13}	-0.8434	-0.8089	0.1233	0.8421	-0.3723
M_{23}	0.5228	0.5139	0.5864	0.5231	0.5192
M_{25}	-0.7402	-0.694	-0.1062	-0.7444	0.0997
M_{34}	0.2506	0.2848	0.644	0.2436	0.5386
M_{45}	0.345	0.432	0.7091	0.3406	0.4238
M_{46}	-0.0626	0.889	0.2147	0.0607	0.5075
M_{56}	0.2625	0.0793	0.4256	0.2591	0.7144
M_{5L}	1.0081	0.9899	0.9968	1.0085	0.9985
No. real solutions	1	3	3	1	5

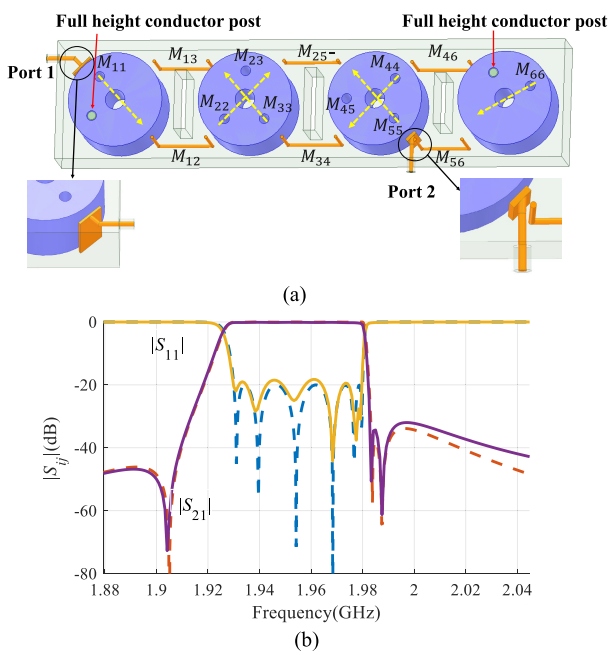


Fig. 3. (a) Physical model of the 6-3-0 trapezoid filter using single- and dual-mode dielectric resonators. (b) Comparison of *S*-parameters of the EM simulated (solid lines) and the synthesized (dashed lines).

height dielectric puck is with relative permittivity $\varepsilon_r = 39.5$, dielectric loss tangent $\tan\delta = 2.5 \times 10^{-5}$, a height of 12 mm, and a diameter of 11.9 mm. The inner dimension of the perfect electric conductor (PEC) cavity is 30 × 30 × 12 mm. In the filter layout, the first and last physical resonators are single-mode resonators, in which one of the two degenerate TM_{11} modes is disabled by inserting a full height conductor post through a frequency tuning hole, as shown in Fig. 3(a). In each dual-mode resonator, there are three through holes in the dielectric puck, two of which are for tuning the frequencies of the two degenerate modes and one is for adjusting the coupling between them, all of which are controlled by tuning screws from the top lid.

To realize the intercavity coupling, a rectangular loop formed by a folded metal wire with two ends short-circuited to the top lip of the cavities is adopted, as shown in Fig. 3(a). The height of the metal loop is the main controlling factor of the coupling value. The negative coupling M_{25} is realized by altering the polarization directions of the two degenerate modes in the relevant cavity, such as cavity 3 in this example. The I/O coupling is realized by a grounded strip loop whose magnetic field is coupled to that of electrical resonator 1 or 5. Notice that since electrical resonator 1 needs to be coupled to resonators 2 and 3 simultaneously, the dielectric puck for resonator 1 is deliberately tilted to realize different values of M_{12} and M_{13}, so does the dielectric puck for resonator 6.

The simulated *S*-parameters of the fully designed EM model of the 6-3-0 trapezoid filter and those by the synthesized coupling matrix given in the column of case TZ-A 3 in Table II are superposed in Fig. 3(b), showing very good agreement. A slight deviation of the rejection in the higher frequency band compared to the synthesized one is due to the dispersion effect.

B. CWDM Cavity Trapezoid Filter

The second example is a *Ku*-band 8-4-0 CWDM cavity trapezoid filter for the steepest asymmetric rejection response that a CWDM filter can realize. The internal structure and the photograph of the prototyped CWDM trapezoid filter are shown in Fig. 4(a) and (b), respectively. The target coupling matrix is listed in Table IV, which is selected from nine possible solutions due to its highest yield rate and that the maximum absolute value of all its couplings is the minimum among all the solutions. The center frequency and bandwidth of the filter are 12.388 and 0.06 GHz, respectively. The TZs of the filter are placed at 12.341, 12.421, 12.428, and 12.442 GHz. The filter is realized by five 3π electrical-long CWDM resonators with radius $R = 13.0$ mm. Two probes are intruded perpendicularly into physical resonators 1 and 4 separately. The thickness of each coupling iris is chosen to be 0.5 mm. Notice that the first and last resonators are single-mode cavities in which one of the degenerate TE_{113} modes is disabled by a centered and end-shorted halfway long thin horizontal metal diaphragm. Cavities 2–4 all have three tuning screws. The vertical and horizontal ones are for tuning the frequencies of the two orthogonal modes and the one in ±45° is for controlling the coupling of the two degenerate modes. As shown in Fig. 4(a), a long slot on the iris between cavities 1 and 2 inclines with an angle to control the couplings of a single-mode cavity and the two degenerate modes in the coupled dual-mode cavity, so does the iris between cavities 4 and 5. The comparison of the synthesized and measured narrowband *S*-parameters is shown in Fig. 4(c), showing very good agreement.

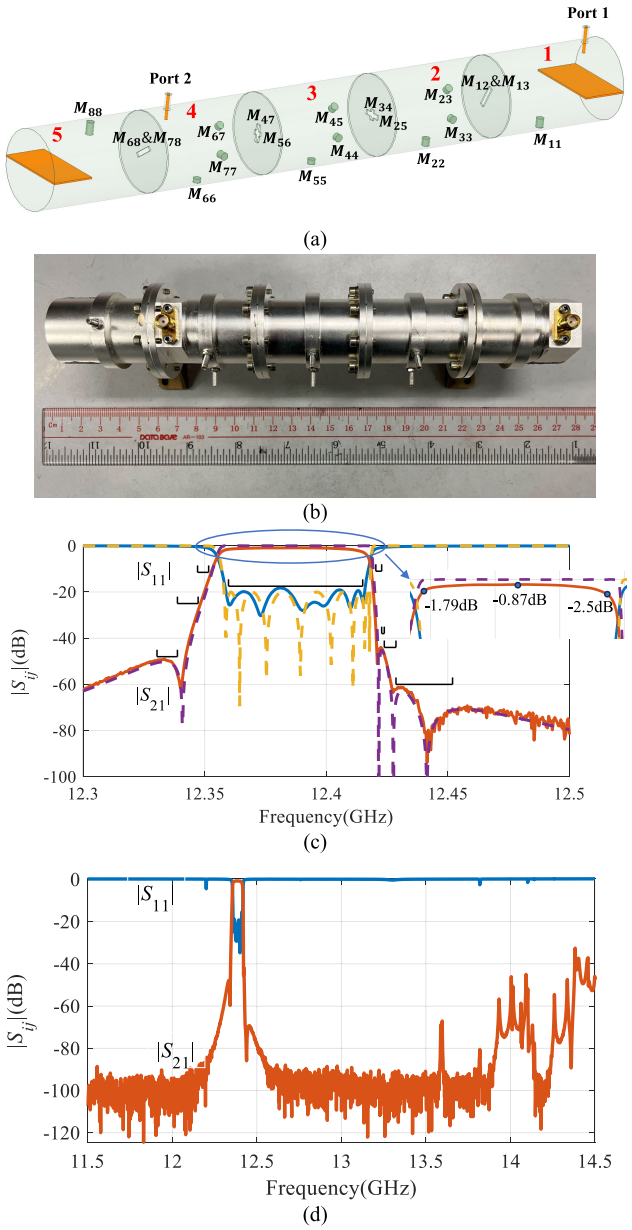


Fig. 4. (a) EM model of the 8-4-0 CWDM trapezoid filter. (b) Photograph of the prototyped 8-4-0 filter. (c) Comparison of S-parameters of the measured (solid lines) and the synthesized (dashed lines) with specs. (d) Measured wideband S-parameters.

TABLE IV
SYNTHESIZED COUPLING MATRIX FOR THE CWDM 8-4-0 FILTER

M_{11}	M_{22}	M_{33}	M_{44}	M_{55}	M_{66}	M_{77}
0.0112	0.0046	-0.5596	0.0382	0.0504	0.0112	0.144
M_{88}	M_{S1}	M_{12}	M_{13}	M_{23}	M_{25}	M_{34}
-0.6259	0.9873	0.4141	0.7035	0.4879	-0.6229	0.1068
M_{45}	M_{47}	M_{56}	M_{67}	M_{68}	M_{78}	M_{6L}
0.5227	-0.5762	0.0739	0.4539	0.6744	0.4294	0.9873

The measured wideband S-parameters are also provided in Fig. 4(d). Due to the combination of single- and dual-mode cavities, whose lower order harmonic frequencies are different, suppression of harmonic modes of a trapezoid filter to a certain extent can be expected. With reference to the 8-2-0 *Ku*-band CWDM filter reported in [18] for a symmetric response, which consists of four CWDM cavities with the same cavity diameter and works in nearly the same frequency band as the 8-4-0 trapezoid filter, the measured $|S_{21}|$ of the prototyped trapezoid filter in the rejection band up to 14.5 GHz is below -35 dB, whereas that in [18, Fig. 4], it is barely below -10 dB.

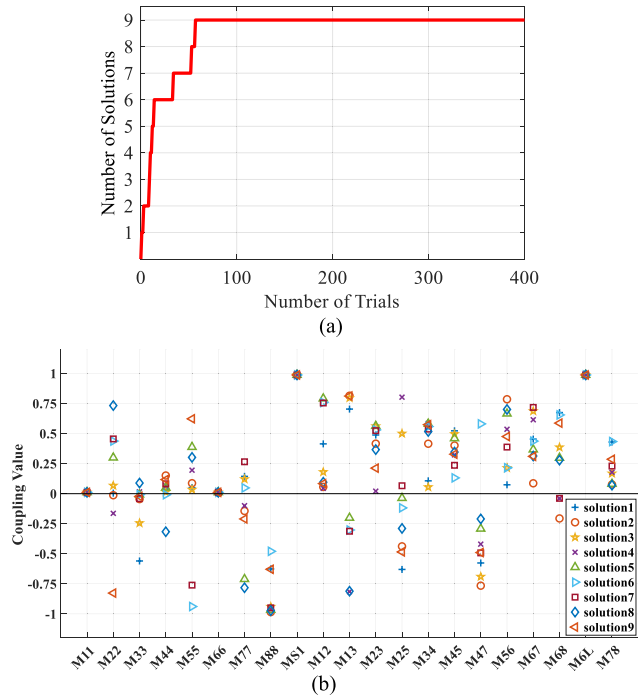


Fig. 5. Multiple solutions of the 8-4-0 trapezoid filter: (a) convergence of solution search and (b) distribution of coupling coefficients.

V. CHOICES OF MULTIPLE SOLUTIONS

As mentioned in Sections II and III, there are multiple real solutions for many trapezoid filters subject to the given filtering responses. To choose the most suitable solution for a physical realization, the analysis of the solution distribution and the sensitivity of the coupling elements to the TZs need to be considered.

A. Distribution of Solutions

The distribution of all the solutions for a given filter response can reveal a lot of useful information about solutions. Having had the exhaustive solution search, whose convergence is evidenced in Fig. 5(a), the distribution of the nine solutions of the 8-4-0 filter with respect to the filter response shown in Fig. 4(c) is presented in Fig. 5(b). The total computation time for the 400 trials is less than 20 s on a PC with 16-core CPU. Among the nine solutions, solution 1, which is listed in Table IV, is chosen for the prototyped filter partially because the maximum absolute value of all its couplings is the minimum among all the solutions. This feature provides convenience in the physical realization. Solution 6 is also worth mentioning for that it is the only solution with a positive M_{47}, which gives a unique identity and avoids the

TABLE V
YIELD RATES BY MONTE CARLO ANALYSIS OF THE 8-4-0 TRAPEZOID FILTER (10 000 SAMPLES)

Range of Perturbation	Sol. 1	Sol. 2	Sol. 3	Sol. 4	Sol. 5
±0.0025	28.96%	7.58%	15.66%	6.91%	8.94%
±0.001	83.8%	31.42%	54.83%	25.41%	41.23%
Range of Perturbation	Sol.6	Sol. 7	Sol. 8	Sol. 9	
±0.0025	8.59%	8.39%	6.7%	7.07%	
±0.001	28.9%	30.15%	38.93%	28.04%	

TABLE VI
SENSITIVITY OF TZS OF THE 8-4-0 FILTER

TZ1	M_{56}	M_{34}	M_{45}	M_{47}
	$4.7909j$	$2.0723j$	$-2.0388j$	$-1.2202j$
TZ2	M_{56}	M_{67}	M_{12}	M_{34}
	$-40.3525j$	$19.0576j$	$-16.3407j$	$12.9332j$
TZ3	M_{56}	M_{67}	M_{12}	M_{78}
	$20.121j$	$-18.6502j$	$16.5924j$	$14.9613j$
TZ4	M_{23}	M_{34}	M_{12}	M_{78}
	$5.6121j$	$-5.054j$	$-4.0675j$	$-3.2862j$

TZ1 to TZ4 are ordered in the sequence of 12.341, to 12.421, 12.428, 12.442 GHz.

ambiguity in model extraction during the EM design and tuning process. Another criterion in selecting a solution is the resonant frequencies of the resonators. For example, the self-couplings in solution 2 are mostly located near zero except for M_{88}, which is a single-mode cavity anyway. This may lead to an easier tuning process in a dual-mode physical realization.

B. Monte Carlo Analysis

The Monte Carlo analysis is a useful tool for choosing the right coupling matrix that leads to a high yield in mass production. The analysis provides an estimation of the yield rate when a random perturbation is applied on every coupling element of the chosen coupling matrix with respect to the acceptance specification. If the yield rate is high, the corresponding filter realization is less sensitive to the physical variation, which means that it is more stable and easier to tune. Table V shows the results for the Monte Carlo yield analysis for the perturbation range of ±0.0025 and ±0.001 of the nine solutions using 10 000 random samples for each solution against the specifications marked in Fig. 4(c). It can be seen that the yield rates for the perturbation range of ±0.0025 and ±0.001 are 28.96% and 83.8%, respectively, for solution 1, which are the highest among all the solutions. This is another reason why solution 1 was chosen as the design target coupling matrix for the prototyped 8-4-0 CWDM trapezoid filter.

C. Sensitivities of TZs

Unlike the cascaded coupling topologies, such as the CQ section, whose TZs are controlled by a few dedicated coupling elements, the coupling elements controlling the TZs of a trapezoid filter are scattered. To understand the dominant elements of a particular TZ, the sensitivity of a TZ defined by $\partial \mathrm{TZ}_k/\partial M_{i,j}$ can depict the controlling route of the coupling path between two I/O ports, where TZ_k is the position of the kth TZ. At the fine-tuning stage of a trapezoid filter toward the target coupling matrix, the sensitivities of all the TZs with respect to the coupling elements are useful-to-know information in order to better control the rejection slope. Table VI lists the four most sensitive coupling elements of solution 1 together with the sensitivity slopes of the four TZs of the 8-4-0 trapezoid filter, which are counted from 12.341 to 12.421, 12.428, and 12.442 GHz. It can be seen that the coupling M_{56} is the most sensitive coupling element to the four TZs.

VI. CONCLUSION

Exploring new and beneficial classes of coupling topologies remains a significant and engaging topic in the field of microwave bandpass filters. This article introduces a novel and generic class of coupling topology, the trapezoid topology, along with its appealing features and synthesis method. This new coupling topology is highly adaptable for dual-mode resonator filter realization, facilitating both symmetric and asymmetric filter responses. The trapezoid topology can achieve all possible TZ arrangements up to $N\backslash 2$ TZs, offering one more TZ than the maximum number achievable by existing dual-mode compatible coupling topologies for asymmetric responses. The trapezoid topology's general features are thoroughly explained. It has been demonstrated that the number of trapezoid filter solutions depends on the order, the number of TZs, and the TZ arrangement of the filtering function. Two dual-mode filter realization examples, an electromagnetically designed 6-3-0 trapezoid filter with TM_{11} dielectric dual-mode resonators and a prototyped 8-4-0 trapezoid CWDM cavity filter, have shown that the trapezoid topology can be physically realized in various forms. Moreover, the multisolution attributes are extensively examined through an 8-4-0 trapezoid filter. By performing the Monte Carlo yield analysis, the most robust solution with the highest yield rate can be selected. The sensitivities of coupling elements with respect to the positions of TZs can provide a clear understanding of how to manage the TZs during the design and fine-tuning stages. It is anticipated that the trapezoid coupling topology will offer the filter industry an enhanced option for designing dual-mode filters with symmetric and asymmetric responses.

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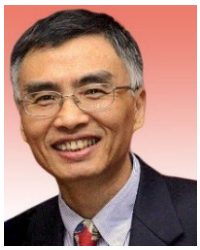

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