Exhaustive Synthesis Framework of Coupled Resonator Microwave Bandpass Filters

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***Abstract*— The synthesis of coupled resonator bandpass filters has been a classic theme of research for the past half century. Continuous efforts have been paid to explore new coupling topologies. Needless to say, the ultimate goal for filter synthesis is to find all the viable coupling topologies and their real-valued circuit models. In this article, a rigorous and straightforward framework for exhaustive search of not only all the viable coupling topologies but also all their real coupling matrices for a given filtering function is presented. The proposition that provides sufficient and necessary condition for a viable coupling topology with finite real coupling matrices is proved. The uniqueness of the folded coupling matrix is also proved and is used to establish the simultaneous equations for a well-behaved numerical solution search process. The special cases of the proposition are discussed with illustrative examples. Demonstration examples, including a prototyped filter in a novel "grid" coupling topology, have shown that the framework can systematically find all the viable coupling topologies and their real-valued solutions numerically, through which many new useful coupling configurations can be found, including the "trapezoid" configuration that is highly suitable for a dual-mode realization for a symmetric/asymmetric filter response.**

***Index Terms*— Exhaustive filter topology search, exhaustive solution search, filter synthesis.**

I. Introduction

THE systematic description of a coupled resonator microwave bandpass filter using a coupling matrix can be traced back to 1970 s in the article by Atia and Williams [1] for a dual-mode waveguide filter with a symmetric filter transfer response. Since then, the concept of coupling matrix has been enriched and become the most prevalent circuit model for coupled resonator filters [2], [3] owning to its concise and substantial description of a general physical realization.

A complete synthesis of a coupling matrix constitutes two indispensable components: to construct an optimal rational filtering function for a given specification and to turn the filtering function into the coupling matrix in a desired coupling topology according to the intended physical realization. A pertinent filtering function in the form of rational functions is uniquely defined by three monic polynomials in the low-pass complex frequency plane, namely, $E(s)$, $P(s)$, and $F(s)$, of which $E(s)$ determines the system poles, $P(s)$ prescribes the chosen transmission zeros (TZs), and $F(s)$ specifies the reflection zeros.

The classic theory to synthesize Chebyshev filtering functions and to obtain the coupling matrices by similarity transformations for many practical coupling topologies can be found in many textbooks [4]. When the arrangement of TZs becomes complex, such as a multiband filter, numerical methods can be applied to obtain a filtering function with an "equal-ripple" response [5]. For a multiport filtering network, the filtering function for each channel filter can also be synthesized by using the concept of complex loads [6]. Very recently, a kind of bounded Chebyshev and reduced Chebyshev functions are introduced to acquire a filtering function with TZs that is realizable with an in-line coupling topology [7].

Having had the rational filtering function, one needs to systematically derive a circuit model in terms of frequency invariant coupling matrix in a *canonical* form in the low-pass frequency domain. A topology is considered to be *canonical* if it can be directly obtained from a general class of filtering functions and that the number of couplings in the circuit model is the minimum for the given transfer characteristic. The two well-known classical canonical forms for realizing a symmetric/asymmetric response in a low-pass prototype filter network are the "folded" and "arrow" forms proposed by Bell in 1982 [8]. The $N+2$ transversal "fully canonical" form that describes a general Chebyshev filtering function was proposed by Cameron in 2003 [3], which can be directly obtained from the partial expansion form of the admittance matrix of the filtering function. Interestingly, the number of couplings in the transversal form is the maximum for the given order of the filter N, the number of resonators.

Compared to filtering function synthesis, the development of orthogonal transformations for coupling matrices with practical coupling topologies is relatively immature. Traditionally, the coupling matrix in an intended coupling topology is acquired by applying a series of Givens rotation transformations (or rotation transformations) to the coupling matrix in a canonical form based on a handful of "recipes" [4]. For instance, to realize a symmetric transfer characteristic with an asymmetric dual-mode structure, the recipe for the "Pfitzenmaier" configuration was proposed in 1977 [9]. Although the "extended box" configuration, which was proposed by Cameron in 2002 [10], can realize an asymmetric transfer response with up to $(N \setminus 2) - 1$ TZs by a dual-mode realization, where operator "\" means integer division, other possible dual-mode coupling topologies for asymmetric responses with more TZs are to be explored. For the applications in base stations of wireless communication systems, the cascaded trisection (CT) topology for asymmetric

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responses and cascaded quartet (CQ) topology for symmetric and asymmetric responses by Levy in 1976 [11] and Liang and Zhang in 1999 [12] have shown a great degree of design flexibility in realizing TZs with the payoff of an irregularly shaped layout, which is undesirable for a filter array interfaced with multiple-input multiple-output (MIMO) array antennas. To facilitate the synthesis process, the rotation recipes for the coupling topologies consisting of CT and CQ units were introduced by Tamiazzo and Macchiarella [13] in 2005. Despite that the known coupling topologies have shown their great vitality in various applications, the available topologies are still far from sufficient to meet the increasing demands with regard to layout flexibility and manufacturing simplicity.

To find coupling matrices for unconventional coupling topologies, research efforts have also been paid to direct algebra approaches to find the orthogonal transformation matrix for unconventional coupling topologies, for which no recipes are available. In this regard, the Gröbner basis method has been applied to turn the matrix orthogonal transformation problem into a set of simultaneous multivariable polynomial equations, whose solutions, including the complex ones, can be solved by a sophisticated computer algebra routine [14], [15], [16]. However, the method is very mathematics involved and is difficult to apply for high-order filters without an advanced mathematic tool [17]. Another barrier for the method to enjoy its popularity is that its formulation heavily depends on the coupling topology and tremendous tactics. Similar challenges are also faced by the homotopy continuation-based method, with which a set of polynomial equations for the orthogonal transformation matrix need to be solved numerically provided that if a good start system can be found [18]. Various gradient-based optimization schemes are available to find a coupling matrix in a given coupling topology with a given specification [19], [20], [21]. Having said that, there are two missing parts in all the aforementioned works: 1) the fundamental theory that sufficiently and necessarily ensures the legitimacy of a coupling topology that possesses a finite number of real-valued coupling matrix solutions for a targeted filtering characteristic, and 2) a generic and straightforward numerical framework that enables to find all the possible legitimate coupling topologies and their associated real-valued coupling matrices.

This work is an attempt to make up the missing parts by providing the industry with an exhaustive synthesis framework of coupled resonator bandpass filters: to search for all the legitimate coupling topologies and all their real solutions in terms of normalized coupling matrices. The framework is warranted by a solid mathematic foundation, and the numerical search process is based on a set of optimal mapping relations between a trial solution and a given specifications, leading to a well-behaved objective function for all possible real coupling matrices. The exhaustivity of the real solutions is realized by a dynamic search scheme and is verified by adversarial attack approach.

In this article, several synthesis examples with symmetric and asymmetric filter responses are presented, among which an 8-4 filter and a 9-4 filter cases are presented in detail to illustrate the details of the proposed framework. It will be shown that a number of new and practical coupling topologies can be found through the exhaustive synthesis. A prototype filter with a new "grid" shape coupling topology is designed and tested, showing the usefulness of having a compact and simple coupling topology. A novel class of coupling topologies that are suitable for dual-mode realizations for asymmetric transfer character is also discussed. It is expected that the exhaustive synthesis framework will be found useful for synthesizing coupled resonator bandpass filters with more design options, greater flexibilities, and ampler satisfaction.

II. Mathematical Background

For a given coupled resonator bandpass filter of the N-th order, its two-port admittance matrix Y can be found as

$$\mathbf{Y} = -j\mathbf{B}^T(\mathbf{M}+\omega\mathbf{I})^{-1}\mathbf{B}, \quad \mathbf{B} = [\mathbf{w}_1, \mathbf{w}_N] \tag{1}$$

where $\mathbf{B}$ is an $N \times 2$ interfacing matrix, whose entries are the couplings between the input–output (I/O) and resonators, $\mathbf{M}$ is an $N \times N$ coupling matrix consisting of couplings between resonators with diagonal terms representing the relative frequency shift of each resonator, and $s = j\omega$ with ω being the low-pass frequency. The shape (the pattern of nonzero entries) of $\mathbf{M}$ reflects the coupling topology and can be rearranged through an orthogonal similarity transformation without altering the filter response. Among the few known canonical coupling topologies, the coupling matrix $\mathbf{F}$ in the folded form is of particular interest. The layout pattern (the shape) of $\mathbf{F}$ is shown in (2), in which the asterisks refer to the nonzero sequential couplings, the diagonal pluses refer to the self-couplings, and the off-diagonal pluses refer to possible cross couplings

$$\mathbf{F} = \begin{pmatrix} + & \ast & 0 & \cdots & \cdots & \cdots & 0 & + \\ \ast & + & \ast & 0 & \cdots & 0 & + & + \\ 0 & \ast & + & \ast & 0 & + & + & 0 \\ \vdots & 0 & \ast & + & \ast & + & 0 & \vdots \\ \vdots & \vdots & 0 & \ast & + & \ast & 0 & \vdots \\ \vdots & 0 & + & + & \ast & + & \ast & 0 \\ 0 & + & + & 0 & 0 & \ast & + & \ast \\ + & + & 0 & \cdots & \cdots & 0 & \ast & + \end{pmatrix} \text{For } N \text{ is even} \tag{2a}$$

$$\mathbf{F} = \begin{pmatrix} + & \ast & 0 & \cdots & \cdots & \cdots & \cdots & 0 & + \\ \ast & + & \ast & 0 & \cdots & \cdots & 0 & + & + \\ 0 & \ast & + & \ast & 0 & 0 & + & + & 0 \\ \vdots & 0 & \ast & + & \ast & + & + & 0 & \vdots \\ \vdots & \vdots & 0 & \ast & + & \ast & 0 & \vdots & \vdots \\ \vdots & \vdots & 0 & + & \ast & + & \ast & 0 & \vdots \\ \vdots & 0 & + & + & 0 & \ast & + & \ast & 0 \\ 0 & + & + & 0 & \cdots & 0 & \ast & + & \ast \\ + & + & 0 & \cdots & \cdots & \cdots & 0 & \ast & + \end{pmatrix} \text{For } N \text{ is odd} \tag{2b}$$

with

$$B = \begin{pmatrix} B_{11} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & B_{2N} \end{pmatrix}^T. \tag{3}$$

The coupling matrix in the *folded* form for a given filtering function will be the core component in the exhaustive search framework both for coupling topologies and their real coupling matrices.

The following two propositions lay the mathematic foundation of the framework to ensure the legitimacy of a coupling topology and the sufficient and necessary conditions of the numerical search of all possible real-valued coupling matrices.

Proposition 1: For a given coupled resonator bandpass filter of the N-th order, whose admittance matrix Y for a given filter transfer function is described by the $N \times N$ coupling matrix $\boldsymbol{F}$ in the folded form and its $N \times 2$ interfacing matrix $\boldsymbol{B}$, then the $\boldsymbol{F}$ and $\boldsymbol{B}$ are unique.

Proof: Applying an orthogonal transformation on $\mathbf{F}$ and $\mathbf{B}$ with $\mathbf{P} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N]$, whose orthonormal column vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N\}$ do not change the property of Y. Denote $\mathbf{F}' = \mathbf{P}^T\mathbf{F}\mathbf{P}$ and $\mathbf{B}' = \mathbf{P}^T\mathbf{B}$, specifically

$$\mathbf{F}' = \begin{pmatrix} \mathbf{v}_1^T \cdot \mathbf{F}\mathbf{v}_1 & \cdots & \mathbf{v}_1^T \cdot \mathbf{F}\mathbf{v}_N \\ \vdots & \ddots & \vdots \\ \mathbf{v}_N^T \cdot \mathbf{F}\mathbf{v}_1 & \cdots & \mathbf{v}_N^T \cdot \mathbf{F}\mathbf{v}_N \end{pmatrix} \tag{4}$$

and

$$\begin{aligned} \mathbf{F}\mathbf{v}_1 &= F'_{1,1}\mathbf{v}_1 + F'_{2,1}\mathbf{v}_2 + \cdots + F'_{N,1}\mathbf{v}_N \\ \mathbf{F}\mathbf{v}_2 &= F'_{1,2}\mathbf{v}_1 + F'_{2,2}\mathbf{v}_2 + \cdots + F'_{N,2}\mathbf{v}_N \\ &\vdots \\ \mathbf{F}\mathbf{v}_N &= F'_{1,N}\mathbf{v}_1 + F'_{2,N}\mathbf{v}_2 + \cdots + F'_{N,N}\mathbf{v}_N \end{aligned} \tag{5}$$

where $F'_{i,j}$ is the ith row and jth column element of $\mathbf{F}'$.

It will be proved that if $\mathbf{F}'$ and $\mathbf{B}'$ have the same shapes (or coupling topologies) as those of $\mathbf{F}$ and $\mathbf{B}$, respectively, $\mathbf{P}$ must be a diagonal matrix whose entries are either 1 or -1. In other words, the coupling matrix in the folded form is unique.

Since $\mathbf{B}' = \mathbf{P}^{\mathbf{T}}\mathbf{B}$ must retain the shape, the following equation must be held:

$$\begin{pmatrix} B_{11}v_1(1) & B_{2N}v_1(N) \\ B_{11}v_2(1) & B_{2N}v_2(N) \\ \vdots & \vdots \\ B_{11}v_N(1) & B_{2N}v_N(N) \end{pmatrix} = \begin{pmatrix} B'_{11} & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & B'_{2N} \end{pmatrix}. \tag{6}$$

Then, $v_2(1) = v_3(1) = \cdots = v_N(1) = 0$ and $v_1(N) = v_2(N) = \cdots = v_{N-1}(N) = 0$. Because $(v_1(1), \ldots, v_N(1))$ and $(v_1(N), \ldots, v_N(N))$ are the first and last rows of $\mathbf{P}$, their magnitudes equal to 1. Thus, $v_1(1) = \pm 1$ and $v_N(N) = \pm 1$. Consequently, $\mathbf{v}_1 = (\pm 1, 0, \ldots, 0)^T$ and $\mathbf{v_N} = (0, \ldots, 0, \pm 1)^T$ for $|\boldsymbol{v}_1| = 1$ and $|\boldsymbol{v}_\mathbf{N}| = 1$. By the same token, the following results can be proven by induction:

$$\begin{aligned} \mathbf{v}_k &= \pm\Big(\underbrace{0, \ldots, 0}_{k-1 \text{items}}, 1, 0, \ldots, 0\Big) \\ \text{and } \mathbf{v}_{N-k+1} &= \pm\Big(\underbrace{0, \ldots, 0}_{N-k \text{items}}, 1, 0, \ldots, 0\Big) \\ k &= 1, 2, \ldots, N\backslash 2. \end{aligned} \tag{7}$$

Assume the shape of $\mathbf{F}'$ is the same as that of $\mathbf{F}$, which is illustrated in (2), the first equation in (5) can be expressed as

$$\pm\big(F_{1,1}, F_{2,1}, 0, \ldots, 0, F_{N,1}\big)^T = F'_{1,1}\mathbf{v}_1 + F'_{2,1}\mathbf{v}_2 + F'_{N,1}\mathbf{v}_N \tag{8}$$

which leads to $\mathbf{v}_2 = \pm(0, 1, 0, \ldots, 0)^T$ for F_{21} must be nonzero. The same process can be applied to the last equation of (5), resulting in

$$\begin{aligned} \pm\big(F_{1,N}, F_{2,N}, 0, \ldots, 0, F_{N-1,N}, F_{N,N}\big)^T \\ = F'_{1,N}\mathbf{v}_1 + F'_{2,N}\mathbf{v}_2 + F'_{N-1,N}\mathbf{v}_{N-1} + F'_{N,N}\mathbf{v}_N \end{aligned} \tag{9}$$

or $\mathbf{v}_{N-1} = \pm(0, \ldots, 0, 1, 0)^T$ for $F_{N-1,N}$ must be nonzero.

For N is even, considering the shape of $\mathbf{F}$ shown in (2a), the second equation of (5) gives

$$\mathbf{F}\mathbf{v}_2 = F'_{1,2}\mathbf{v}_1 + F'_{2,2}\mathbf{v}_2 + F'_{3,2}\mathbf{v}_3 + F'_{N-1,2}\mathbf{v}_{N-1} + F'_{N,2}\mathbf{v}_N \tag{10}$$

or

$$\begin{aligned} \pm\big(F_{1,2}, F_{2,2}, F_{3,2}, 0, \ldots, 0, F_{N-1,2}, F_{N,2}\big)^T \\ = F'_{1,2}\mathbf{v}_1 + F'_{2,2}\mathbf{v}_2 + F'_{3,2}\mathbf{v}_3 \\ + F'_{N-1,2}\mathbf{v}_{N-1} + F'_{N,2}\mathbf{v}_N \end{aligned} \tag{11}$$

which means $\mathbf{v}_3 = \pm(0, 0, 1, 0, \ldots, 0)$ for $F_{3,2}$ must be nonzero. By the same token

$$\begin{aligned} \mathbf{F}\mathbf{v}_{N-1} = F'_{2,N-1}\mathbf{v}_2 + F'_{3,N-1}\mathbf{v}_3 + F'_{N-2,N-1}\mathbf{v}_{N-2} \\ + F'_{N-1,N-1}\mathbf{v}_{N-1} + F'_{N,N-1}\mathbf{v}_N \end{aligned} \tag{12}$$

which leads to $\mathbf{v}_{N-2} = \pm(0, \ldots, 1, 0, 0)$ for $F_{N-2,N-1}$ must be nonzero. With $\mathbf{v_1}, \mathbf{v_2},$ and $\mathbf{v_3}$ found, it is trivial to repeat the process till $k = N\backslash 2$. Consequently, (7) is proved for N is even.

For N is odd, only vector $\mathbf{v}_{N\backslash 2+1}$ needs to be determined. Since all the other vectors of $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N\}$ are found in the case when N is even, obviously

$$\mathbf{v}_{N\backslash 2+1} = \pm(\underbrace{0, \ldots, 0}_{N\backslash 2 \text{ items}}, 1, 0, \ldots, 0)^T.$$

The proposition is proved.

Proposition 2: Suppose $\boldsymbol{F} = (F(1), F(2), \ldots, F(N_F)) \in \mathbb{R}^{N_F}$ represents the coupling matrix of N-th order in the folded form for the given filter response character $\mathcal{C}$, $\boldsymbol{F}_0$ is a point of $\boldsymbol{F}$ for the filter response $\mathcal{R}$ of $\mathcal{C}$, and a coupling matrix $\boldsymbol{M} = (M(1), M(2), \ldots, M(N_M)) \in \mathbb{R}^{N_M}$ in a nonfolded form can achieve the same filter character $\mathcal{C}$; then, the sufficient and necessary condition for $\boldsymbol{M}$ to have a unique solution to achieve the same $\mathcal{R}$ in a neighborhood of $\boldsymbol{M}_0$ is $N_F = N_M$, where N_F and N_M refer to the numbers of couplings in $\boldsymbol{F}$ and $\boldsymbol{M}$, respectively, and that the determinant of the Jacobian matrix of $\boldsymbol{F}$ with respect to $\boldsymbol{M}$ in the neighborhood of $\boldsymbol{M}_0$, or $det(\boldsymbol{J}|_{M_0})$, is not zero.

Proof: Given a filtering function of the N-th order in terms of the low-pass domain frequency ω having response $\mathcal{R}$, its seed circuit model $\mathbf{F}_0$ that is a point of $\mathbf{F}$ in the folded form can be emerged directly from the filtering function [8]. For a coupling matrix **M**, the following mapping relation between **F** and **M** is always available [4, pp. 275-278]:

$$\mathbf{F} = \mathbf{Q}_m\mathbf{Q}_{m-1}\cdots\mathbf{Q}_2\mathbf{Q}_1\mathbf{M}\mathbf{Q}_1^T\mathbf{Q}_2^T\cdots\mathbf{Q}_{m-1}^T\mathbf{Q}_m^T \quad (13)$$

where the Givens rotation matrix $\mathbf{Q}_s$ ($s = 1, 2, \ldots, m$) is governed by the rotation angle in the s-th step

$$\theta = \tan^{-1}\left(\frac{M^{(s-1)}(i,\ k)}{M^{(s-1)}(j, k)}\right) k(\neq i, j) = 1, 2, 3, \ldots, N. \quad (14)$$

The indexes i and j in the s-th step as well as the total number of Givens rotations applied depend on the coupling topology of **M**. Therefore, for response $\mathcal{R}$, (13) can be implicitly expressed by the following continuous and differentiable equations:

$$\begin{aligned} F_0(1) &= f_1(M(1), M(2), \ldots, M(N_M)) \\ F_0(2) &= f_2(M(1), M(2), \ldots, M(N_M)) \\ &\vdots \\ F_0(N_F) &= f_{N_F}(M(1), M(2), \ldots, M(N_M)) \end{aligned} \quad (15)$$

whose Jacobian matrix is defined as

$$\mathbf{J} = \begin{pmatrix} \dfrac{\partial f_1}{\partial M(1)} & \cdots & \dfrac{\partial f_1}{\partial M(N_M)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_{N_F}}{\partial M(1)} & \cdots & \dfrac{\partial f_{N_F}}{\partial M(N_M)} \end{pmatrix}. \quad (16)$$

Since $\mathbf{F}_0$ is unique to response $\mathcal{R}$ (*Proposition* 1) and that N_F, the number of couplings in **F** including the mutual- and self-couplings, inclusive, is the minimum number of required couplings for the given $\mathcal{C}$, and when the orders of **M** and **F** are the same, the necessary condition for (15) to be bijective is $N_F = N_M$.

According to the inverse function theorem [22], the sufficient and necessary condition for (15) to have a single-valued solution in the neighborhood of $\mathbf{M}_0$ is $\det(\mathbf{J}|_{M_0}) \neq 0$.

*Corollary 1: For **M** that satisfies the sufficient and necessary condition in Proposition 2 may have multiple but a finite number of real solutions within the finite solution domain.*

Proof: The corollary is obvious as the determinant of Jacobian matrix det(**J**(**M**)) is a nonlinear continuous algebra function of **M**. According to Proposition 2, det(**J**(**M**)) only has finite zeros in the finite solution domain. Therefore, there are finite number of multiple solutions of real or complex valued to (15) for a given $\mathbf{F}_0$.

The significance of (15) is threefold: 1) the Givens rotations to the folded form is always available for any **M**; 2) it warrants a real solution, if there is one, as any **M** corresponds to a unique folded form; and importantly, 3) it is a real domain to a real domain mapping relation. Moreover, (15) provides an efficient means to find all possible real solutions numerically.

TABLE I

NUMBER OF TOPOLOGIES IN EACH STAGE AND COMPUTATION TIME FOR TOPOLOGY SEARCH OF FOUR REPRESENTATIVE FILTERING FUNCTIONS

Filter order & No. of TZs	6-2	7-3	8-4	9-4
Initial No.	32,768	2,097,152	268,435,456	68,719,476,736
$N_F = N_M$ rule	6,435	352,716	30,421,755	125,161,770
Minimum-path	71	14,382	1,308,611	8,050,119
Planarity check	71	14,382	1,304,555	8,030,419
Jacobian check	24	4,192	439,558	1,483,276
Isomorph check	7	64	737	1,708
No. of legitimate	7	64	737	1,708
Computing time	53s	15m	14h30m	36h50m

III. EXHAUSTIVE SEARCH OF LEGITIMATE TOPOLOGIES

The exhaustive search process for legitimate coupling topologies begins with listing all the possible coupling topologies for a given order of the filter, including all the possible I/O coupling arrangements regardless of the number of TZs specified. For an N-th order filter, the total number of combinations of couplings between every pair of resonators is $N_P = C_{N+2}^2$, where C_n^m means the number of m-combinations from a given set S of n elements, or $C_n^m = (n!/m!(n-m)!)$. Therefore, the initial number of possible coupling topologies is 2^{N_P}.

The legitimate coupling topologies refer to the topologies that can practically achieve the given filtering function with finite solutions. Specifically, a legitimate coupling topology does not involve any crossover couplings, possesses a finite number of solutions (real or complex valued), and is nonisomorphic. In designing a filter, it is unfavorable to have a physical crossover coupling if not unrealizable; infinite number of solutions leads to design redundancy without constrains; and nonisomorphic implies nonduplications. Therefore, the exhaustive topology search is the process that excludes all the aforementioned nonconformities.

To give numerical details in each exclusion stage and total computation time on a PC with a 16 core CPU, four representative filter transfer characters, i.e., 6-2 (a sixth-order filter with two TZs), 7-3, 8-4, and 9-4 filters, are investigated with details listed in Table I. It needs to be mentioned that a legitimate topology does not necessarily render a real coupling matrix solution.

A. $N_F = N_M$ Rule

Having proved the uniqueness of the folded coupling matrix (Proposition 1) and the sufficient and necessary condition for a coupling matrix **M** to have a finite number of solutions (Proposition 2), it can be concluded that a necessary condition for a coupling topology to be legitimate is $N_F = N_M$. This condition is consistent with the postulation that $2n + m + 1$ should equal the "degrees of freedom," where n is the order of the filter and m is the number of TZs [23]. This necessary condition is rigorously justified in the proof of Proposition 2 and is naturally applied in (15) and (16).

Since a coupling topology can be viewed as an undirected, nonweighted and connected graph, all possible candidate topologies that satisfy the $N_F = N_M$ rule encompass $C_{N_P}^{(N_F-N)}$ possible combinations. For example, for a third-order filter

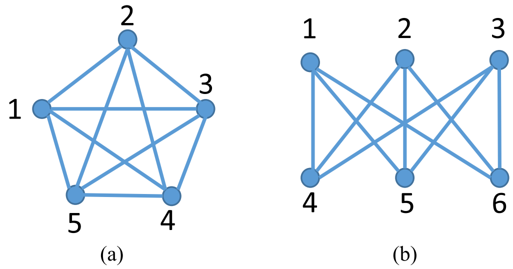


Fig. 1. Topologies of the complete graph. (a) K_5. (b) $K_{3,3}$.

with one TZ, $N_P = 10$ and $N_M = 8$. The total number of possible candidate topologies satisfying the $N_F = N_M$ rule is $C_{10}^{8-3} = 252$.

B. Minimum-Path Rule

Although Proposition 2 provides the sufficient and necessary condition to stipulate a legitimate coupling topology, since numerically validating the $\det(\mathbf{J}) \neq 0$ condition is very costly, any efficient and simple necessary condition for checking the legitimacy of a coupling topology would be helpful in the topology search process. The well-known *minimum-path rule* can serve the purpose very well. The rule states that the possible maximum number of TZs equals the order of the filter minus the number of resonators on the shortest path between two I/O ports [24]. Any coupling topology that complies the $N_F = N_M$ rule but not the minimum-path rule will be filtered out.

C. Planarity of Topologies

Nonplanar coupling topologies with crossover couplings are not considered in the synthesis framework as there are little practice uses. In graph theory, the Kuratowski theorem [25] can be applied to identify the planarity of a coupling topology. The theorem states that *a graph G is planar if and only if G contains a subgraph that is a subdivision of K_5 or $K_{3,3}$*. Here, K_5 is the complete graph on five vertices and $K_{3,3}$ is the complete bipartite graph on six vertices, whose topologies are illustrated in Fig. 1. A subdivision of a graph refers to a graph that is formed by subdividing its edges into paths of one or more edges.

Many algorithms for identifying the existence of subdivisions of K_5 or $K_{3,3}$ are available. A highly efficient linear time testing algorithm is a PC-trees data structure and an adopted graph-reduction technique [26], which is applied in all the examples of this work.

D. Condition of Nonzero Determinant of Jacobian Matrix

Although the necessary condition of $N_M = N_F$ is applied in the topology search process, coupling topologies consisting of redundancy, which causes infinitely many solutions, are inevitable and must be removed. As stated in *Proposition 2*, to ensure the number of solutions to (15) is finite, the $\det(\mathbf{J}) \neq 0$ condition in the solution domain must be ensured. Be noted that this condition does not ensure the existence of real-valued coupling matrix solutions. Two kinds of numerical checks are conducted: a random check for removing the topologies with redundant coupling elements and a specific check for ensuring the existence of real solutions. The specific check is subject to the given transfer function and will be discussed in Section IV. To conduct a random check, a large number of randomly generated coupling matrices, 20 000 for instance, for a coupling topology in a reasonably large solution domain are applied to test $\det(\mathbf{J}) \neq 0$ condition numerically. If all test samples satisfy $\det(\mathbf{J}) = 0$, redundancy in the coupling topology presumably exists. Nevertheless, passing the random check does not warrant redundancy free for real-valued solutions, particularly for the cases with symmetric filter responses, in which redundancy is governed by a hypersurface of coupling elements, which is difficult to capture by a set of random coupling elements.

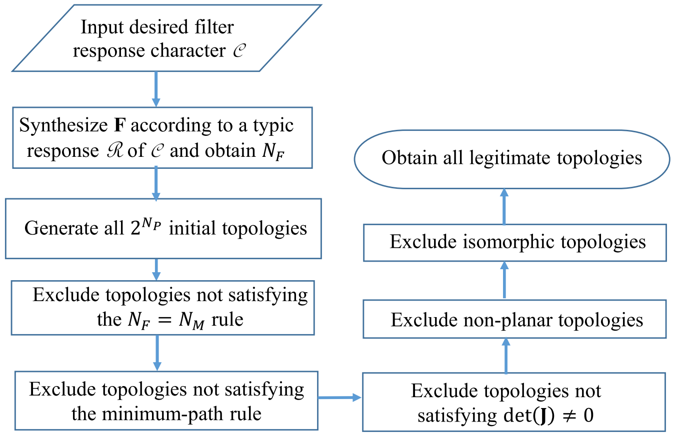


Fig. 2. Flowchart of exhaustive search process for legitimate coupling topologies.

E. Isomorphism of Topologies

Isomorphism is a concept in graph theory. Two graphs G_1 and G_2 are called isomorphic if there is a one-to-one correspondence between the vertices of G_1 and those of G_2, such that the number of edges joining any two vertices of G_1 is equal to the number of edges joining the corresponding vertices of G_2 [25].

According to the definition, a direct approach to identify an isomorphism of two topologies is to change the labels to the resonators of one topology and to check if the newly labeled topology is isomorphic with the other. To check the isomorphism of two N-th order coupling topologies, there are $N!$ different arrangements to be considered at most. This process is time-consuming and is placed at the very end of the topology search process.

F. Legitimate Coupling Topologies

Having removed all the isomorphic topologies, the topologies remained are the legitimate ones. The flowchart in Fig. 2 concludes the exhaustive search process for legitimate coupling topologies.

To search for all the legitimate coupling topologies cost-effectively, the $N_M = N_F$ condition is stipulated in the first place to screen out overwhelming useless possibilities followed by the minimum-path rule check. As the process for checking planarity is relatively efficient, it is applied before checking the determinant of Jacobian matrix and removal of the isomorphic topologies.

It should be mentioned that in the topology search, the most general response of the given filter characteristic must be considered to obtain a *seed* folded coupling matrix, from which N_F is obtained. Particularly, when the target filter response is symmetric, the response for the seed folded coupling matrix needs to be asymmetric by setting the corresponding diagonal cross couplings to numerical zeros. This treatment is understandable as the solution search process will not exactly follow the path for a symmetric response, so does the filter tuning in practice.

Optionally, additional constrains can be applied to the final list of legitimate coupling topologies, such as a maximum numbers of couplings associated to a resonator and that to I/O ports. In all the case studies in this article, only one coupling associated to an I/O port is assumed.

As seen from Table I, the $N_M = N_F$ rule not only provides a necessary condition for topology search but also serves as the first coarse sieve that greatly reduces the burden in the search process.

G. Additional Constrains Among Couplings

It is known that there exist some coupling topologies that have finite solutions but $N_M > N_F$, such as the well-known transversal topology [3]. In such a case, at least one additional constrain among couplings exists. Taking the transversal coupling matrix of a 3-1 filter as an example, the constrain of $(M_{S1}\times M_{1L}+M_{S2}\times M_{2L}+M_{S3}\times M_{3L}) = 0$ and $M_{SL} = 0$ must be satisfied to comply with the TZ arrangement. Such an additional constrain among coupling elements is difficult to apply in practice and is not considered in this framework.

IV. Exhaustive Solution Search

Having had all the legitimate coupling topologies for a given filtering character $\mathcal{C}$, numerical search for all possible real solutions of a given specific response $\mathcal{R} \in \mathcal{C}$ can be conducted using (15) for the coupling topologies of interest. As said in Section III, passing the random check of $\det(\mathbf{J}) \neq 0$ condition does not warrant redundancy free for real-valued coupling matrices, for which redundancy is governed by a hypersurface of coupling elements. Additionally, in most of the cases, the solution to (15) is not unique. In this framework, the numerical solution search is conducted using the Levenberg–Marquardt method [27] with an adoptive search domain. An adversarial attack process [28] is applied to make sure that all the solutions have been found in a sufficiently large search domain. The approximate solutions of which $N_M < N_F$ for certain coupling topologies and the special case for $N_M = N_F$ in which redundancy exists for real-valued coupling matrices will also be discussed. A viable coupling topology is defined as a legitimate topology that possesses at least one real solution to (15) for the given response $\mathcal{R}$.

A. Objective Function for Solution Search

An efficient numerical search for all real solutions of a given legitimate coupling topology requires defining a smooth and well-defined objective function. Toward this goal, two advantages of using the folded coupling matrix as the seed coupling matrix are taken: the uniqueness of the folded coupling matrix (Proposition 1) and the known systematic sequence of Givens rotations to the folded coupling matrix from a coupling matrix in any topology (Proposition 2). In other words, (15) is applicable for any legitimate coupling matrix to form the objective function, with which the search for a real solution becomes minimizing the norm of $\|\boldsymbol{F}_M - \boldsymbol{F}_0\|$, where $\boldsymbol{F}_M$ is the transformed coupling matrix in the folded form from a trial coupling matrix $\mathbf{M}$ described by functions f_i on the right-hand side of (15) and $\mathbf{F_0}$ is the seed folded coupling matrix on the left-hand side of (15) obtained from the given filter transfer function.

TABLE II

Some Examples for the Topologies With Real Solutions

Filter Order	Location of TZs	Number of topologies with real solutions
6-2	$1+1.5j,\ 1-1.5j$	6
	$-1.3j,\ 1.2j$	7
7-3	$1+1.5j,\ 1-1.5j,\ 1.3j$	61
	$-1.6j,\ -1.3j,\ 1.2j$	64
8-4	$1+1.5j,\ 1-1.5j,\ 1.3j,\ 1.5j$	721
	$-1.6j,\ -1.3j,\ 1.2j,\ 1.5j$	737
9-4	$1+1.5j,\ 1-1.5j,\ 1.3j,\ 1.5j$	1,676
	$-1.6j,\ -1.3j,\ 1.2j,\ 1.5j$	1,708

B. Real Solutions for Responses With Complex Pair of TZs

It is intuitive that for the same filtering character $\mathcal{C}$, different filtering responses $\mathcal{R}$ may have different numbers of real solutions. For some legitimate topologies, real-valued coupling matrices for a given $\mathcal{R}$ may not exist. The number of legitimate topologies for some TZ arrangements of four filter characters listed in Table I is given in Table II, showing that the number of legitimate topologies with real-valued coupling matrices depends on the locations of TZs (or $\mathcal{R}$) regardless of the return loss level. Particularly, a complex pair of TZs require additional constrains. Therefore, there are less viable topologies than those with only imaginary TZs. For example, a cascaded CT topology cannot realize a pair of complex TZs, whose $\mathcal{R}$ is reflected in the seed coupling matrix in the folded form on the left side of (15).

C. Adoptive Search Domain

In the machine learning area, adversarial attack is a popular method to verify the convergence of an optimized neural model by fabricating a large amount of test data that violate the statistical assumption. In an adversarial attack process, a large dataset that has different statistical distribution with the training and validation dataset is generated and given to the neural model to prove the robustness of the model.

The concept of adversarial attack is utilized in the solution search process to verify the exhaustiveness of the solution set for a given seed folded coupling matrix and coupling topology. In the solution search, an excessive number of initial guesses (much larger than the number of possible real solutions) are casted into (15) for solutions. If a new solution is found to be located near the boundary of the variables, the search domain is adoptively expanded accordingly until the number

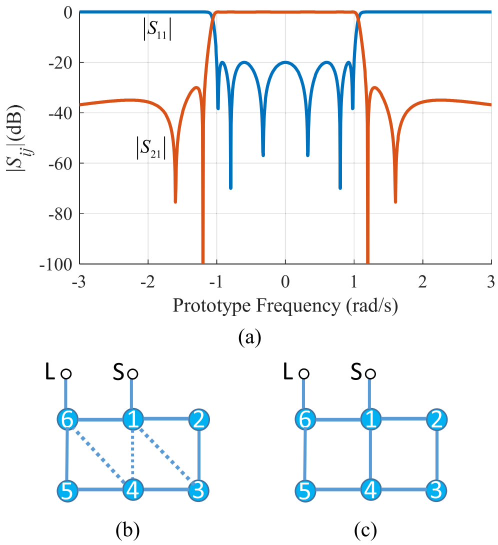


Fig. 3. (a) Symmetrical response of a 6-4 filter in the low-pass domain. (b) Target coupling topology for the 6-4 filter. (c) Coupling topology for the 6-4 filter in the Pfitzenmaier topology.

TABLE III
SOLUTIONS FOR THE PFITZENMAIER COUPLING TOPOLOGY

Couplings	3 Solutions for The 6-4 Filter		
	1	2	3
M_{11}	0	0	0
M_{22}	0	0.8704	-0.8704
M_{33}	0	-0.0790	0.0790
M_{44}	0	0.0790	0.0790
M_{55}	0	-0.8704	-0.8704
M_{66}	0	0	0
M_{S1}	0.9886	0.9886	0.9886
M_{12}	0.6601	0.5167	0.5167
M_{13}	0	-0.6296	0.6296
M_{14}	-0.4772	0	0
M_{16}	0.0903	0.0903	0.0903
M_{23}	0.8934	0.3090	0.3090
M_{34}	0.3764	0.6092	0.6092
M_{45}	0.6213	0.3090	0.3090
M_{46}	0	0.6296	-0.6296
M_{56}	0.8145	0.5167	0.5167
M_{6L}	0.9886	0.9886	0.9886

Each "0" refers to a value smaller than 10^{-15}.

of solutions converges. Notice that for the same coupling matrix, there are 2^N different sign arrangements subject to designer's preference. Therefore, to avoid any sign ambiguity in the solution search process, only the absolute values of couplings are compared in identifying a new solution.

D. Special Cases for $N_M < N_F$

According to Proposition 2, a coupling matrix **M** in any coupling topology with a finite number of solutions (including the complex valued ones) must satisfy the $N_M = N_F$ condition. However, under certain approximation or additional constrains, some special cases in which $N_M < N_F$ are found to be useful. There are two categories of such cases: 1) additional constrains are applied, such as a topology for a symmetrical response; and 2) the coupling matrices with some negligible couplings (usually in the order of 10^{-4}).

For the case of category 1, a symmetric response will be treated as an asymmetric response to obtain the seed folded coupling matrix by setting the corresponding diagonal cross couplings to value zeros. Therefore, the condition of $N_M = N_F$ is still applied in the solution search. It happens that some coupling variables in some solutions turn to be next to zero numerically or in the order of 10^{-15}, which are zeros in the sense of a numerical solution. Taking a 6-4 Pfitzenmaier filter, which is a favorable topology for a dual-mode waveguide filter with a symmetric response [15], as an example. The target symmetric response is shown in Fig. 3(a). The exhaustive solution search starts with the target topology shown in Fig. 3(b) that is legitimate for an asymmetric response with nonzero cross couplings M_{13} and M_{46}. Three real solutions are found by the solution search and are listed in Table III, one of which is without cross couplings M_{13} and M_{46} as the topology shown in Fig. 3(c), which is the solution obtained by Givens rotations [4]. It appears that $N_M < N_F$ since some couplings are numerically zero.

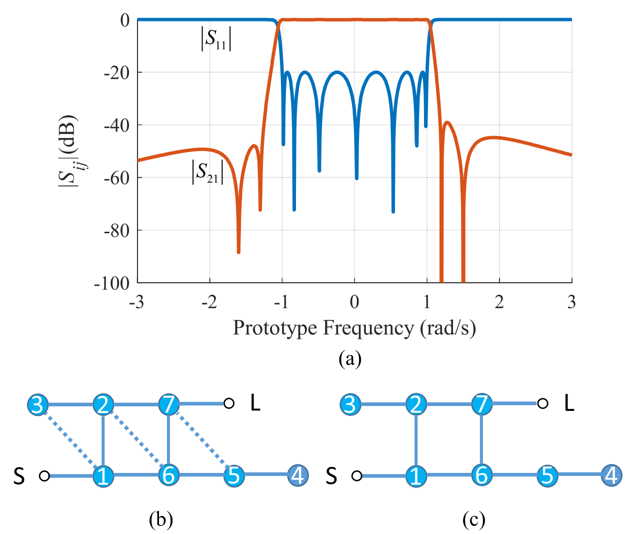


Fig. 4. (a) Asymmetric response of a 7-4 filter in the low-pass domain. (b) Target coupling topology that satisfies the $N_M = N_F$ condition. (c) Coupling topology with an approximate solution in the cul-de-sac topology.

For the case of category 2, some couplings in the solution satisfying the $N_M = N_F$ condition are numerically negligible. Using a 7-4 filter as an example, whose asymmetric response is shown in Fig. 4(a) with the TZs located at $-1.6j$, $-1.3j$, $1.2j$, and $1.5j$ in the low-pass domain. Total 206 legitimate topologies can be found, among which the topology shown in Fig. 4(b) is chosen for the solution search, resulting in four real solutions in total as listed in Table IV, among which solution 1 contains negligibly small M_{13}, M_{26}, and M_{57}, whose magnitudes are in the order of 10^{-6}. Intuitively, these couplings can be neglected without affecting the filter response, resulting

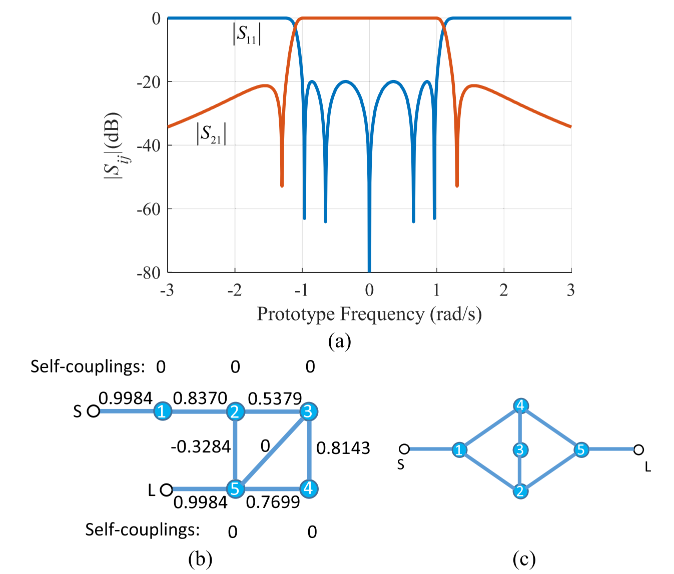


Fig. 5. (a) Symmetric response of a 5-2 filter in the low-pass domain. (b) Normalized seed coupling matrix in folded form. (c) Legitimate coupling topology with redundancy.

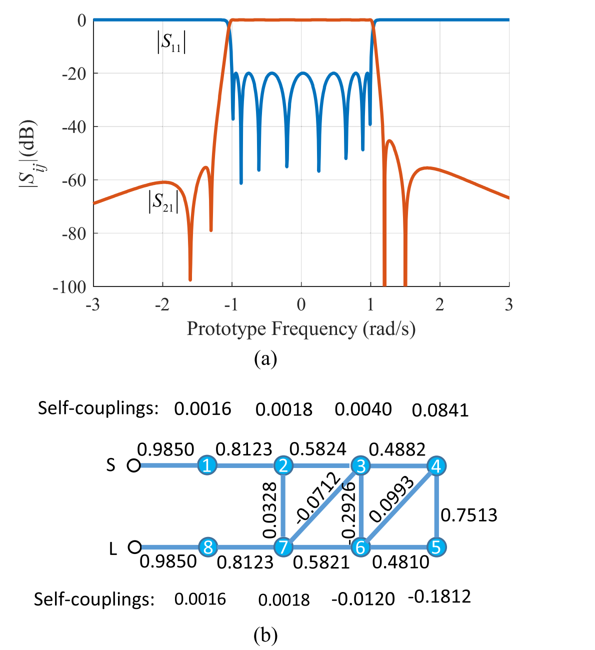


Fig. 6. (a) Asymmetric response of an 8-4 filter. (b) Normalized seed coupling matrix in folded form.

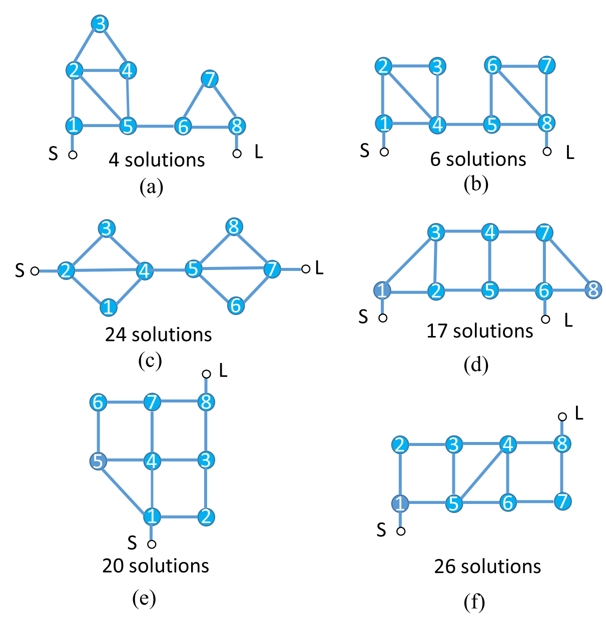


Fig. 7. Some of coupling topologies for the 8-4 filter response. (a) Cascaded quintet- and tri-section. (b) Cascaded quadruplet sections. (c) Two cascaded modified quadruplets. (d) "Trapezoid" configuration. (e) "Grid" configuration. (f) "Full-box" configuration.

in the 7-4 cul-de-sac topology [see Fig. 4(c)] [10]. The same result can be obtained by similarity transformations, justifying the omission of the cross couplings.

E. Special Redundant Cases for $N_F = N_M$

Although the random check of $\det(\mathbf{J}) \neq 0$ condition in the search for the legitimate topologies has been passed to ensure the existence of finite solutions, including complex valued solutions, redundancy may still exist on a hypersurface for real-valued solutions. Such redundancy is understandable particularly for symmetric filter responses. For instance, a symmetric response and the folded coupling matrix of a 5-2 filter are shown in Fig. 5(a) and (b), respectively. The coupling topology of Fig. 5(c) is considered to be legitimate in the exhaustive topology search. Applying (15) leads to 13 simultaneous equations, including three redundant equations: $c_r^2 M_{22} + s_r^2 M_{44} = 0$, $s_r^2 M_{22} + c_r^2 M_{44} = 0$, and $s_r c_r (M_{22} - M_{44}) = 0$, where $s_r = \sin\theta$, $c_r = \cos\theta$, and $\theta = -\tan^{-1}(M_{14}/M_{12})$. Obviously, the first two equations present a degree of redundancy when the third equation is satisfied. In another word, the equation $M_{22} - M_{44} = 0$ defines the hypersurface on which $\det(\mathbf{J}) = 0$ for each solution. For such a redundant case, the number of solutions will never converge.

V. More Synthesis Examples

To better illustrate the usefulness of the exhaustive filter synthesis, two more practical examples are presented in this section. A 9-4 filter with multiple coupling topologies and solutions is thoroughly investigated, among which the solution with the 3×3 grid coupling topology is realized using coaxial combline resonator filter with a practical specification.

A. 8-4 Filter With an Asymmetrical Response

Given the asymmetric response of an 8-4 filter shown in Fig. 6(a) in conjunction with its seed folded coupling matrix in Fig. 6(b), total 737 legitimate topologies are found, but only six of them are sketched in Fig. 7, including three known and three new coupling topologies. The three known topologies are the cascaded quintet- and tri-sections [see Fig. 7(a)], two cascaded quadruplet sections [see Fig. 7(b)], and two cascaded modified quadruplets [see Fig. 7(c)] [29], respectively. The three new topologies are the "trapezoid" configuration [see Fig. 7(d)], which is suitable for a dual-mode realization for an asymmetric response with $(N\backslash 2)$ TZs; the "grid" configuration [see Fig. 7(e)], which is suitable for a compact realization without diagonal cross couplings; and

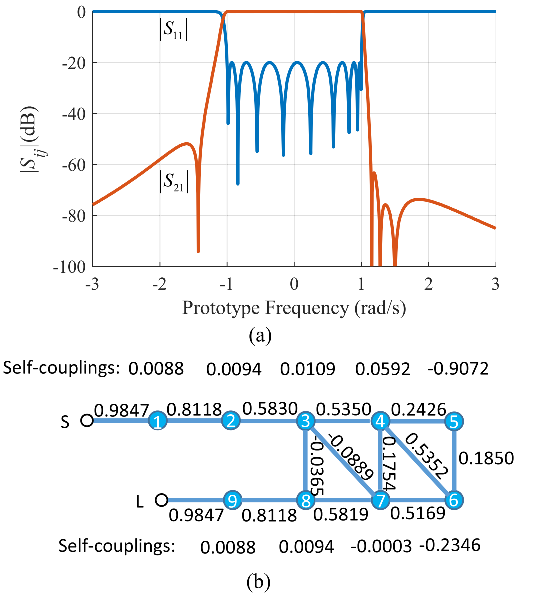


Fig. 8. (a) Asymmetric response of the 9-4 filter. (b) Normalized seed coupling matrix in folded form of the 9-4 filter.

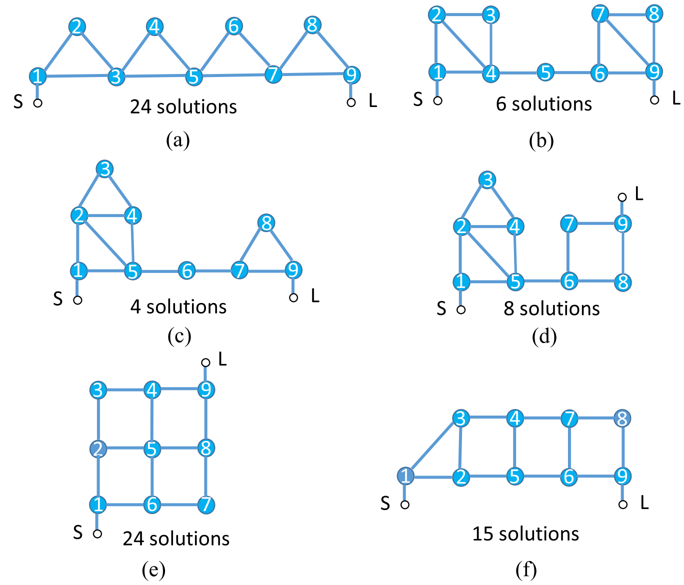


Fig. 9. Some of the coupling topologies for the 9-4 filter. (a) Cascaded tri-section. (b) Cascaded quadruplet. (c) Cascaded quintet- and tri-section. (d) Cascaded quintet and box sections. (e) "Grid" configuration. (f) "Trapezoid" configuration.

the "full-box" configuration [see Fig. 7(f)], which can realize one more TZs than that of the conventional extended box configuration [10]. The total number of real coupling matrices for each of the topologies are also found by the exhaustive solution search and are denoted next to each topology.

B. 9-4 Filter With an Asymmetric Response

The asymmetric response of a 9-4 filer is shown in Fig. 8(a) with its seed coupling matrix in folded form shown in Fig. 8(b). It can be found, through the topology search, that there are 1708 legitimate coupling topologies in total to realize the response, each of which is with a distinct feature. Four known and two newly founded coupling topologies are selected and are listed in Fig. 9, together with their total number of real solutions.

The configurations depicted in Fig. 9(a)–(d) are conventional coupling configurations that can be obtained with known receipts of Givens rotation transformations, including cascaded tri-sections, cascaded quadruplets, cascaded quintet- and tri-sections, and cascaded quintet- and box-sections. The configuration in Fig. 9(e) is the 3 × 3 "grid" configuration and the one in Fig. 9(f) is again the "trapezoid" configuration that is suitable for a dual-mode realization for an asymmetric filter response with $(N\backslash 2)$ TZs.

To demonstrate the realizability of the new "grid" configuration, a prototype filter with the center frequency of 3.92 GHz and bandwidth of 0.31 GHz is designed using electromagnetic (EM) simulation and manufactured using combline coaxial resonators. Since there are 24 real-valued coupling matrix solutions for the topology, the solution whose distance from the rest of solutions is the largest is chosen and is listed in Table V. The EM model and the photograph of the internal structure of the prototyped filter are shown in Fig. 10(a) and (b), respectively, and the synthesized and measured responses of the prototyped filter are superimposed in Fig. 11, showing good agreement. A slightly higher sidelobe in the low rejection band is caused by very weak parasitic couplings.

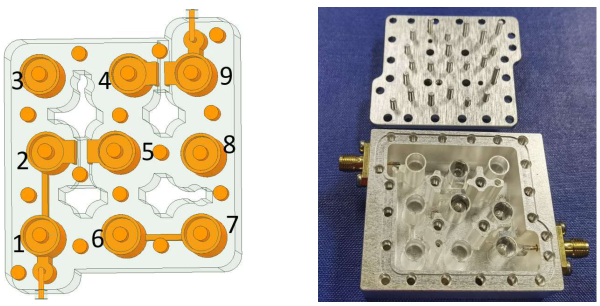
It is worth mentioning that, unlike the cascaded coupling topologies, in which controlling elements of TZs can be clearly identified, in a noncascaded topology, such as the "grid" configuration, each TZ is contributed by multiple coupling elements in an obscure order. Therefore, it is advantageous to

TABLE IV
SOLUTIONS FOR THE COUPLING TOPOLOGY IN FIG. 4(B)

Couplings	4 Solutions			
	1	2	3	4
M_{11}	0.0022	0.0022	0.0022	0.0022
M_{22}	0.0236	0.0678	-0.3003	0.3014
M_{33}	-0.078	-0.1223	0.642	-0.7412
M_{44}	-0.1876	-0.0859	-0.8333	0.7355
M_{55}	0.1555	0.0749	0.2931	-0.3021
M_{66}	-0.0162	-0.0371	0.0957	-0.0963
M_{77}	0.0022	0.0022	0.0022	0.0022
M_{S1}	0.9869	0.9869	0.9869	0.9869
M_{12}	0.5577	0.5555	0.0972	0.1005
M_{13}	**-1.54*10^-6^**	-0.0342	-0.017	-0.0159
M_{16}	0.5951	-0.5962	0.8096	-0.8093
M_{23}	0.7265	0.7216	0.6429	0.5428
M_{26}	**1.38*10^-6^**	-0.0323	-0.358	-0.3907
M_{27}	-0.5577	0.559	-0.5488	0.5729
M_{45}	0.9085	0.922	0.4495	0.558
M_{56}	0.4198	0.4169	0.4161	0.4039
M_{57}	**7.91*10^-7^**	0.0301	-0.5914	0.567
M_{67}	0.5951	0.5931	0.1192	0.1244
M_{7L}	0.9869	0.9869	0.9869	0.9869

TABLE V
NORMALIZED COUPLING MATRIX FOR THE PROTOTYPED 9-4 FILTER

M_{11}	M_{22}	M_{33}	M_{44}	M_{55}	M_{66}	M_{77}
0.0088	-0.0057	-0.9508	0.3947	0.1589	0.0296	0.0591
M_{88}	M_{99}	M_{S1}	M_{12}	M_{16}	M_{23}	M_{25}
-0.7387	0.0088	0.9847	0.6132	0.5320	0.2167	-0.3946
M_{34}	M_{45}	M_{49}	M_{56}	M_{58}	M_{67}	M_{78}
0.1327	0.4428	-0.6596	0.2826	0.2779	0.8359	0.0519
M_{89}	M_{9L}					
0.0519	0.9847					



(a) (b)

Fig. 10. Prototyped 9-4 filter in the 3 × 3 grid configuration using combline resonators. (a) Internal model for EM design. (b) Photograph of the prototyped 9-4 filter (inner dimensions: 40 × 40 × 15 mm^3).

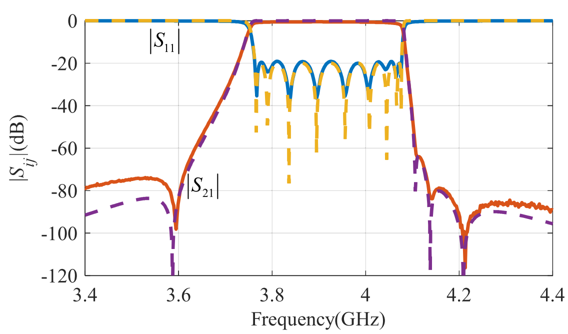


Fig. 11. Synthesized and measured responses (solid lines refer to the measured and dashed lines refer to the synthesized) of the prototyped 9-4 filter in the 3 × 3 grid configuration.

know the sensitivity of each TZ in terms of coupling elements beforehand. Such sensitivity information can be obtained by numerical partial derivatives.

C. Convergence of the Solution Search Process

In previous sections, several new coupling topologies, as shown in Figs. 7(d)–(f) and 9(e) and (f), are found to be useful. To show the convergence of the solution search process, Fig. 12 plots the convergence curves of the number of solutions found in the adversarial attack process versus the number of trials. In each trial, a relatively independent initial value is tossed into the Levenberg–Marquardt method-based solver for (15). All these coupling topologies numerically reach their convergent solutions within 3000 trials, which takes less than 2 min on an ordinary PC.

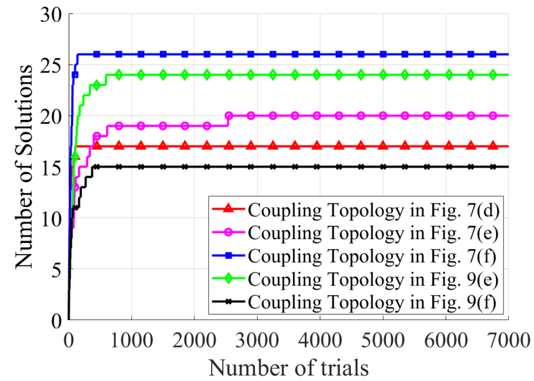


Fig. 12. Convergence curves of the number of solutions versus the number of trials for the topologies in Figs. 7(d)–(f) and 9(e) and (f).

VI. CONCLUSION

Exhaustive synthesis of coupled resonator filters is the ultimate goal to maximize the design flexibility for realizing a given filter response. This work has made an attempt toward this direction by proposing a rigorous and straightforward framework for exhaustive searching of not only all the viable coupling topologies but also all the real coupling matrix solutions. To lay the theoretic foundation, the proposition that provides the sufficient and necessary condition for a legitimate coupling topology is proposed and proved. The uniqueness of the folded coupling matrix is also rigorously proved for the first time to support the proposition and to justify the unique and differentiable simultaneous equations for a well-behaved numerical solution search. Various special cases of the proposition are discussed with illustrative examples. The demonstration examples, including a prototyped 9-4 filter in a novel "grid" coupling topology, have shown that the proposed synthesis framework can systematically find all the legitimate coupling topologies, including many novel topologies with attractive features such as the "grid" configuration and the "trapezoid" configuration that is highly suitable for a dual-mode realization for a symmetric/asymmetric filter response. It is expected that the framework will provide the industry with a useful, comprehensive and straightforward tool for filter synthesis. Having had the framework for dispersion-less coupling matrices, the research on exhaustive synthesis of dispersive coupling matrices for a given filter transfer function is under way.

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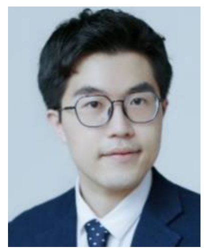

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