An Iterative and Analytical Approach to Optimal Synthesis of a Multiplexer With a Star-Junction

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Abstract—This paper presents an iterative and analytical approach to optimal synthesis of a multiplexer with a star-junction. Two types of commonly used lumped-element junction models, namely, nonresonant node (NRN) type and resonant type, are considered and treated in a uniform way. A new circuit equivalence called phased-inverter to frequency-invariant reactance inverter transformation is introduced. It allows direct adoption of the optimal synthesis theory of a bandpass filter for synthesizing channel filters connected to a star-junction by converting the synthesized phase shift to the susceptance compensation at the junction. Since each channel filter is dealt with individually and alternately, when synthesizing a multiplexer with a high number of channels, good accuracy can still be maintained. Therefore, the approach can be used to synthesize a wide range of multiplexers. Illustrative examples of synthesizing a diplexer with a common resonant type of junction and a triplexer with an NRN type of junction are given to demonstrate the effectiveness of the proposed approach. A prototype of a coaxial resonator diplexer according to the synthesized circuit model is fabricated to validate the synthesized result. Excellent agreement is obtained.

Index Terms—Circuit synthesis, coupling matrix, diplexers, microwave filters, multiplexers.

I. INTRODUCTION

M ICROWAVE diplexers and multiplexers are extensively employed in wireless and satellite communication systems to combine RF signals of different frequency bands into one channel with specified frequency selectivity and isolation requirements. A very common and simple way to combine the multiple channel signals is to directly connect all the channel filters to a star-junction with one common port. Such a connecting scheme makes the multiplexing network simple while maintaining a good microwave performance. The most critical issue in synthesizing such a multiplexer is how to take the interaction among all the channels into account, especially when the frequency bands are spaced close to each other. An analytical approach to synthesis of such a multiplexer is highly desirable in the industry.

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The research efforts to analytically synthesize a multiplexer with a star-junction have never rested over the past three decades. In the early years, the classical circuit synthesis approach was adopted [1], [2]. To compensate for the interaction among the channel filters, the parameters of separately designed channel filters are subject to an appropriate adjustment, but the number of channels and coupling topologies of channel filters are limited and the synthesis result deteriorates as the frequency bands get closer to each other.

In recent years, a more effective and flexible way to synthesize diplexers is proposed in [3]. The relationship between the overall diplexer S-parameters and those of separate channel filters is derived by circuit analysis first. Suitable polynomials describing the characteristics of the diplexer are evaluated by insisting on reflection zeroes assigned *a priori*, and the transfer and reflection functions of each channel filter are derived accordingly. At last the channel filters are synthesized separately by using a well-known coupling matrix synthesis approach [4]. The nonresonant node (NRN) type of junction and the resonant node type of junction are analyzed separately and treated differently in the synthesis approach. In [5], the method for a diplexer with a resonant junction is extended to the synthesis of starjunction multiplexers. In such a type of approach, the roots of high-order polynomials need to be identified. The root-finding process will be a decisive factor that affects the accuracy of the method, and thus the applicable aggregate number of system poles of a multiplexer system becomes limited.

Another existing approach to the diplexer and multiplexer synthesis is to directly apply the multiport coupling matrix synthesis technique, as proposed in [6]. A special class of diplexer polynomials are synthesized analytically in [7] by a similar approach for filter synthesis. The common port of such a diplexer is matched at all frequencies, which simplifies the necessary conditions for the polynomials to be lossless and realizable, but for the most commonly used diplexers and multiplexers, how to analytically synthesize the multiport characteristic functions remains a difficult problem to be solved.

In this paper, a different point of view is taken, which focuses on the synthesis of each channel filter rather than the whole multi-port network. All the channel filters are synthesized separately and alternately in an iterative way. Once the iteration procedure converges, all the channel filters are optimally matched in the sense that when they are placed in the context of a multiplexer, all the reflection zeroes are located along the imaginary axis of the complex low-pass frequency domain. Different from the method used for the synthesis of a diplexer with a lumped-element junction [8], a new phased-inverter to frequency-invariant reactance (FIR) inverter transfor-

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Fig. 1. Circuit model of a multiplexer with a star-junction, where the loading effect of a channel filter at the junction port is represented by complex impedance Z_L .

mation is introduced. The approach presented in this paper is general in that a multiplexer with an arbitrary number of channels can be synthesized. Using the alternate and iterative approach, the problem of evaluating high-order polynomials is circumvented while maintaining the highest flexibility of the coupling topology in a channel filter. Additionally, the two types of junction models, namely, the NRN type and the resonant node type, are treated in a uniform way in the proposed synthesis procedure.

Two synthesis examples are presented in the paper: one is a diplexer with a common resonant type of junction and one is a triplexer with an NRN type of star-junction. A coaxial resonator diplexer with a common resonant star-junction is fabricated and tuned according to the synthesized circuit model in the first example. Excellent agreement between the experiment and theory is achieved, which validates the practical value of the synthesized circuit model.

II. OPTIMALLY MATCHED CHANNEL FILTERS

A direct coupled resonator circuit model for a multiplexer with a star-junction is shown in Fig. 1. For a channel filter, the loading effect of the other channel filters at the junction port can be represented by a complex-impedance load Z_L . The diagram suggests that in order to design a well-matched channel filter, one must be able to synthesize a bandpass filter that is matched to a frequency-dependent complex load within a given frequency band at one port and a real unitary load at the other port.

To clarify the theory of a filter with optimal matching to complex impedance, the three filtering systems as shown in Fig. 2(a)–(c) are considered. The system shown in Fig. 2(a) is the target channel filter in a multiplexer expressed in terms of general Chebyshev filter characteristic polynomials. The S-parameter of the filter is denoted by [S] and has an equiripple in-band reflection coefficient with reflection zeroes located on the imaginary axis. The system shown in Fig. 2(b) is the same filter of that in Fig. 2(a), but detached from the multiplexer and measured with a matched load at the two ports. The detached filter responses are denoted by [S'] and are associated to [S] by power wave renormalization theory [9] as

$$S'_{22} = \frac{(1-r_2)(S_{22}-r_2^*)}{(1-r_2^*)(1-r_2S_{22})}$$
$$S'_{21} = \frac{(1-r_2)S_{21}}{\sqrt{\operatorname{Re}(Z_L)}(1-r_2S_{22})}$$
(1)



Fig. 2. Systems of interest. (a) Target system. (b) Detached target system. (c) Approximated detached system.

where

$$r_2 = (1 - Z_L) / (1 + Z_L^*).$$
⁽²⁾

From (1), a set of legitimate relations between the characteristic polynomials defining [S'] and [S] can be found as [10]

$$P'(s)/\varepsilon' = 2\sqrt{\operatorname{Re}(Z_L(s))}P(s)/\varepsilon$$

$$F'(s) = (1 + Z_L(s))F(s) - (1 - Z_L^*(s))E(s)$$

$$E'(s) = (1 + Z_L^*(s))E(s) - (1 - Z_L(s))F(s).$$
 (3)

The three equations in (3) are named transference, matchability, and conservativeness, respectively. If E(s), F(s), and P(s) are polynomials characterizing a general Chebyshev filter and $Z_L(s)$ is arbitrary frequency-variant complex impedance, the right-hand side of (3) needs to be approximated by polynomials, which may not necessarily be monic. However, it is well known that the characteristic polynomials defining the transfer and reflection characteristics of an N + 2 coupled-resonator network must be monic polynomials. That is to say, in general, the system in Fig. 2(b) may not be realized as a coupled-resonator network. It can be shown that the system in Fig. 2(b) can be best approximated by a system [S''] that is realizable by a coupled-resonator network plus an optimal phase shift θ_l , as shown in Fig. 2(c) such that

$$S'_{22} \approx e^{-2j\theta_l} \frac{F''(s)}{E''(s)} \quad S'_{21} \approx e^{-j\theta_l} \frac{P''(s)}{\varepsilon''E''(s)}$$
(4)

in which polynomials E''(s) and F''(s) are *monic* and the phase length θ_l is optimally selected in order to best satisfy the *conservativeness* condition. The procedure of synthesizing the polynomials E''(s), F''(s), and P''(s), the coefficient ε'' , as well as θ_l has been well described in [10] and is not repeated here.

III. PHASED-INVERTER TO FIR-INVERTER TRANSFORMATION

By following the procedure of synthesizing a general Chebyshev filter with a complex load discussed in Section II, the S-parameters of a channel filter are described in the following form:

$$[S] = \begin{bmatrix} e^{-j2\theta_l} \frac{F(s)}{E(s)} & e^{-j\theta_l} \frac{P(s)}{\varepsilon E(s)} \\ e^{-j\theta_l} \frac{P(s)}{\varepsilon E(s)} & \frac{F_{22}(s)}{E(s)} \end{bmatrix}$$
(5)

where F(s), $F_{22}(s)$, and E(s) are *monic* polynomials, and the highest degree coefficient of P(s) is 1 or j, which depends on the filter order and the order of P(s).

The channel filter described by (5) can be split into two parts: a fixed phase shift followed by a coupled resonator network that is described by F(s), $F_{22}(s)$, and E(s). In waveguide or microstrip type of diplexers or multiplexers, a phase shift can be introduced by adding a piece of transmission line between the junction and a channel filter. However, for a star-junction multiplexer where coaxial cavities are coupled directly to the common junction, a suitable circuit model is a multiport direct coupled-resonator network.

This paper presents the mathematic foundation for the method proposed in [8] for synthesizing the channel filter in a diplexer with a lumped-element junction, which is based on a new circuit equivalence called the phased-inverter to FIR-inverter transformation. The transformation converts the phase shift necessary for optimal matching a channel filter to an FIR that compensate to the susceptance of the star-junction.

The [ABCD] matrix of the frequency-invariant phase shift cascaded with a *J*-inverter can be evaluated as

$$[ABCD] = \begin{bmatrix} \cos\theta_1 & j\sin\theta_1\\ j\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} 0 & j/J_{01}\\ jJ_{01} & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -J_{01}\sin\theta_1 & j\cos\theta_1/J_{01}\\ jJ_{01}\cos\theta_1 & -\sin\theta_1/J_{01} \end{bmatrix}.$$
(6)

Converting the [ABCD] matrix in (6) to the Y matrix, and taking the FIR of the first resonator into account, the overall Y matrix of the circuit in the dashed line box in Fig. 3(a) is

$$[Y]_{A} = \begin{bmatrix} j \tan \theta_{1} & j J_{01} / \cos \theta_{1} \\ j J_{01} / \cos \theta_{1} & j J_{01}^{2} \tan \theta_{1} + j b_{1} \end{bmatrix}.$$
 (7)

It is obvious that the Y matrix for the circuit in the dashed line box in Fig. 3(b) is

$$[Y]_B = \begin{bmatrix} jb_0 & jJ'_{01} \\ jJ'_{01} & jb'_1 \end{bmatrix}.$$
 (8)

Comparing (7) and (8), one can find the equivalence of a phasedinverter and an FIR-inverter by

$$b_0 = \tan \theta_1 \quad J'_{01} = \frac{J_{01}}{\cos \theta_1} \quad b'_1 = b_1 + J^2_{01} \tan \theta_1.$$
 (9)



Fig. 3. Equivalent-circuit models for a channel filter. (a) Phased-inverter form. (b) FIR-inverter form.



Fig. 4. Star-junctions used for multiplexer synthesis. (a) NRN type of junction. (b) Resonant type of junction.

The circuit in the form of Fig. 3(b) with the leading FIR jb_0 is a very useful form for a channel filter because when it is connected to a star-junction as the FIR, jb_0 can be combined with the star-junction model to derive the final multiplexer circuit model.

IV. MULTIPLEXER SYNTHESIS PROCEDURE

Two types of lossless and reciprocal star-junctions are considered here, they are the NRN type of junction and the resonant type of junction. The low-pass domain models of these two types of junctions are shown in Fig. 4(a) and (b), respectively. Both of them can be used as the junction in a multiplexer circuit model and will be treated in a uniform way in the synthesis procedure.

Before the synthesis starts, the specifications are translated into the low-pass prototype frequency domain with the wellknown frequency mapping

$$\omega = f_0 / BW(f / f_0 - f_0 / f)$$
(10)



Fig. 5. Final stage to combine channel filters with the star-junction. The leading FIRs and the junction FIR are connected in parallel with each other as shown in the dashed line box.

where f_0 is the center frequency and BW is the total bandwidth of the multiplexer, as is the same with the frequency mapping used for multiplexer synthesis in [5].

At the initial stage, doubly terminated general Chebyshev filters are employed as the channel filters and are connected to the star-junction of either type. The preset parameter M_{s0} and initial parameter of the junction model b_0 are assigned. The value for parameter M_{s0} should be set to be realizable and appropriate so that each channel filter can be easily matched within its band.

For each channel filter, the load impedance seen toward the junction can then be found numerically. The impedance is complex and frequency variant. By applying the synthesis procedure given in [10] to a channel filter with the calculated complex load, channel by channel and repeatedly until the response converges, the characteristics of each channel filter can be found in the form of (5). Each channel filter is synthesized separately and then reconfigured to the target coupling topology individually. With the phased-inverter to FIR-inverter transformation introduced in Section III, the phase shift coupled channel filter is converted to the form of an FIR followed by a coupled-resonator network. Since the leading FIRs of all the channel filters are in parallel with the junction's jb_0 , they are combined together to compensate the initial shunt admittance. The final FIR of the synthesized optimal junction is then obtained. This combining process is illustrated in Fig. 5. The synthesized multiplexer is a direct coupled resonator network, which can be fully described by a multiport coupling matrix.

V. ILLUSTRATIVE EXAMPLES

In this section, two synthesis examples are given to demonstrate how to apply the proposed approach in designing a coaxial resonator diplexer and a multiplexer.

A. Diplexer With a Common Resonant Node

Consider the following specifications of a diplexer.

- Channel 1: Passband: 2.478–2.568 GHz; Order: 5; Returnloss level: 22 dB; TZ: 2.622 GHz.
- Channel 2: Passband: 2.620–2.718 GHz; Order: 5; Returnloss level: 22 dB; TZ: 2.574 GHz.



Fig. 6. Target diplexer realization. (a) Routing diagram of the resonators. (b) Layout of the coaxial resonators.

Junction model: a resonant type of junction with $M_{s0} = 1.4$ and initial $b_0 = 0$.

 M_{s0} is the normalized common port I/O coupling. A too small or large value for M_{s0} will set the junction difficult to be matched in a diplexer. An appropriate value is proven to be around 1.4 through many case studies. Since it is a normalized low-pass domain circuit component, its value is independent of the total bandwidth of the device. For a diplexer with a resonant type of junction, the value is viable for a wide range of guard band and the ratio of two channel bandwidths. The FIR b_0 is related to the offset of the resonant frequency of the common resonator with respect to the center frequency of the diplexer. Intuitively the junction should resonate near the center frequency of the device. Therefore, b_0 is initially set to be zero.

The coupling topology and resonator layout of the diplexer are depicted in Fig. 6, where each channel filter contains a triplet to realize the prescribed TZ.

The center frequency f_0 and the bandwidth BW of the diplexer are calculated as $f_0 = \sqrt{2.478 \times 2.718} = 2.595$ GHz and BW = 2.718 - 2.478 = 0.24 GHz, respectively.

With frequency mapping relation (10), the passbands of channel 1 and channel 2 are mapped to the normalized low-pass frequency domain as [-1, -0.2281] and [0.2055, 1], respectively. At the beginning, two Chebyshev filters that are matched to real unitary loads are synthesized using the procedure given in [4]. The low and high channels are renormalized to the two passbands in the low-pass domain, respectively.

When the two bandpass filters are connected to the junction, the complex impedance Z_{L1} seen by channel filter 1 toward the junction is calculated and is plotted in Fig. 7, where the two passbands are marked by shaded areas, and the overall diplexer



Fig. 7. Complex impedance Z_{L1} seen by channel filter 1 toward the junction before iteration begins. The shaded areas indicate the passbands of the two channels.

response appears to be the one shown in Fig. 8(a). With the complex load impedance Z_{L1} known, channel filter 1 is resynthesized by following the synthesis procedure of a filter with one port loaded by Z_{L1} . Having had channel filter 1 updated in the diplexer circuit model, the overall diplexer response is also updated and is shown in Fig. 8(b). It is seen that the matching of channel filter 1 is significantly improved. By the same token, the complex impedance Z_{L2} seen by channel filter 2 is calculated using newly updated channel filter 1. With the circuit model of channel filter 2 updated, the diplexer response is updated again and is shown in Fig. 8(c). This updating process is repeated alternately between the two channel filters until the result converges. In fact, this iterative process converges very quickly. For this example, having had each channel filter updated twice, a satisfactory diplexer response is obtained. The following two updatings of the diplexer response are plotted in Fig. 8(d) and (e), respectively. The coefficients for s^i , the *i*th exponent of complex frequency s, of the characteristic polynomials during the iterative procedure are listed in Table I, where one iteration means updating both channels once.

With the characteristic polynomials for the two channel filters, the coupling matrices of the channel filters are synthesized separately and their phase shifts are transformed to the FIR-inverter form by using the transformation introduced in Section III. The final circuit model of the diplexer is represented by a three-port coupling matrix given in Table II in which all the mainline couplings are set to be positive, P1 is the common port, and that P2 and P3 are the low- and high-frequency channel ports, respectively. Resonators 1–5 constitute channel filter 1, resonators 7–11 belong to channel filter 2, and the resonator numbered 6 is the common resonant node. Initially the common resonator has zero self-coupling, but the value has been modified by the synthesis approach to optimally match all the channel filters.

A coaxial diplexer prototype is designed based on the synthesized three-port coupling matrix. The diplexer is fabricated and tuned according to the synthesized coupling matrix using a sort of computer-aided tuning (CAT) scheme developed in-house. The photograph of the prototype hardware is shown in Fig. 9. The dimension of the diplexer body is $95 \text{ mm} \times 58 \text{ mm} \times 25 \text{ mm}$. The negative coupling is realized



Fig. 8. Iterative process of the diplexer synthesis example. (a) Initial response by connecting two separately synthesized bandpass filters to the junction. (b) First time update of channel filter 1. (c) First time update of channel filter 2. (d) Second update of channel filter 1. (e) Second update of channel filter 2.

by a dumbbell-shape probe, and all the positive couplings are achieved by opening windows between adjacent resonators. The tapped I/O ports are used at the three ports.

		Initial		А	fter 1 st Iteratio	n	After 2 nd Iteration			
$s^i, i =$	Low	frequency cha	annel	Low	frequency cha	nnel	Low frequency channel			
	E''(s)	F''(s)	P''(s)	E''(s)	F''(s)	P''(s)	E''(s)	F''(s)	P''(s)	
0	0.0402 -	0.0404i	0.2223	0.0478 -	-0.0097 +	0.2223	0.0490 -	-0.0076 +	0.2223	
	0.00301			0.01091	0.04971		0.01471	0.03031		
1	0.1100 - 0.4563i	0.4393	1.0000i	0.5434i	0.4933 + 0.1191i	1.0000i	0.5467i	0.3033 + 0.1017i	1.0000i	
2	-1.5722 -	1 77(0)		-1.8907 -	0.4133 -		-1.8795 -	0.3662 -		
	1.1358i	-1.//691		0.8700i	1.8209i		0.9345i	1.8575i		
3	-2.9844 +	2 2549		-2.7102 +	-3.2627 -		-2.7795 +	-3.3146 -		
	2.0255i	-3.3348		2.4743i	0.5155i		2.4410i	0.4652i		
4	0.8606 +	2 0750;		1.0713 +	-0.2068 +		1.0515 +	-0.1886 +		
	2.9759i	2.97391		2.8837i	2.8837i		2.9079i	2.9079i		
5	1	1		1	1		1	1		
	$\varepsilon'' = 1$	117.4921, $\theta_l =$	0 rad	$\varepsilon'' = 112$	$\theta_l = 0.0$	910 rad	$\varepsilon'' = 117.4085, \theta_l = 0.0857 \text{ rad}$			
, i	High	frequency cha	annel	High	frequency cha	innel	High frequency channel			
s , <i>i</i> –	E''(s)	F''(s)	P''(s)	E''(s)	F''(s)	P''(s)	E''(s)	F''(s)	P''(s)	
0	0.0321 +	-0 0330j	-0.1779	0.0397 +	-0.0047 -	-0 1779	0.0393 +	-0.0053 -	-0.1779	
	0.0081i	-0.05501		0.0159i	0.0425i	-0.1779	0.0165i	0.0423i		
1	0.0651 +	0 3810	1.0000i	0.0015 +	0.4489 -	1.0000j	-0.0047 +	0.4459 -	1.0000i	
	0.4083i	0.5610		0.4900i	0.0816i	1.00001	0.4883i	0.0864i		
2	-1.5104 +	+1 6186i		-1.7953 +	0.3277 +		-1.7969 +	0.3413 +		
	0.9599i	1.01001		0.7879i	1.7210i		0.7685i	1.7095i		
3	-2.7846 -	-3 1766		-2.6043 -	-3.1610 +		-2.5825 -	-3.1445 +		
	2.0300i	5.1700		2.4248i	0.4391i		2.4336i	0.4539i		
4	0.8854 -	-2.9050i		1.0710 -	-0.1830 -		1.0768 -	-0.1884 -		
	2.90501			2.8442i	2.84421		2.83631	2.83631		
5	1	1		1	1		1	1		
	e''=	93.5852, $\theta_l = 0$	0 rad	$\varepsilon'' = 95$	$.5842, \theta_l = 3.03$	557 rad	$\varepsilon'' = 94.3930, \ \theta_l = 3.0542 \ \text{rad}$			

 TABLE I

 Synthesized Polynomials in Each Iteration of Fig. 8

 TABLE II

 FINALLY SYNTHESIZED THREE-PORT COUPLING MATRIX

	P1	P2	P3	1	2	3	4	5	6	7	8	9	10	11
P1	0	0	0	0	0	0	0	0	1.4000	0	0	0	0	0
P2	0	0	0	0.6568	0	0	0	0	0	0	0	0	0	0
P3	0	0	0	0	0	0	0	0	0	0	0	0	0	0.6665
1	0	0.6568	0	0.6183	0.3498	0	0	0	0	0	0	0	0	0
2	0	0	0	0.3498	0.6229	0.2387	0.0799	0	0	0	0	0	0	0
3	0	0	0	0	0.2387	0.4779	0.2344	0	0	0	0	0	0	0
4	0	0	0	0	0.0799	0.2343	0.6246	0.2940	0	0	0	0	0	0
5	0	0	0	0	0	0	0.2940	0.6194	0.7892	0	0	0	0	0
6	1.4000	0	0	0	0	0	0	0.7892	-0.0023	0.7988	0	0	0	0
7	0	0	0	0	0	0	0	0	0.7988	-0.6089	0.3017	0	0	0
8	0	0	0	0	0	0	0	0	0	0.3017	-0.6151	0.2374	-0.0921	0
9	0	0	0	0	0	0	0	0	0	0	0.2374	-0.4466	0.2421	0
10	0	0	0	0	0	0	0	0	0	0	-0.0921	0.2421	-0.6131	0.3600
11	0	0	0.6665	0	0	0	0	0	0	0	0	0	0.3600	-0.6078

The synthesized response is superimposed on the measurement result of the prototype in Fig. 10. The insertion losses at center frequencies of both channels are about 0.6 dB and the average quality factor is estimated to be 2000. Although the response of the synthesized circuit model is lossless, good agreement is achieved, which validates of the synthesized circuit model.

B. Triplexer With an NRN Type of Star-Junction

In [5], multiplexers are synthesized with a resonant type of star-junction with excellent quasi-equiripple response. In this synthesis example using the proposed approach, an NRN type of star-junction will be used for multiplexer synthesis.

The coupling topology of each channel filter and the connection with an NRN star-junction node in the triplexer are shown in Fig. 12. The specifications for each channel filter are as follows.

- Channel 1: Passband: 697–717 MHz; Order: 6; Returnloss level: 22 dB; TZ: 728 MHz.
- Channel 2: Passband: 733–760 MHz; Order: 8; Returnloss level: 22 dB; TZ: 717 MHz, 778 MHz.
- Channel 3: Passband: 776–799 MHz; Order: 7; Returnloss level: 22 dB; TZ: 765 MHz.

Initial junction model: an NRN type of junction with $M_{s0} = 1.3$ and $b_0 = 0$.

In an NRN type of junction, M_{s0} basically functions as an admittance inverter and the FIR b_0 represents a shunt reactive element. A good initial value of M_{s0} can be flexible in this type of junction. Any value from 1 to 2 for this triplexer case can be used and good matching for all channel filters can be achieved.



Fig. 9. Photograph of the diplexer prototype with top lid removed.



Fig. 10. Measured and synthesized responses of the diplexer example. (a) Magnitude of S_{11} , S_{12} , and S_{13} . (b) Magnitude of S_{22} , S_{33} , and S_{23} .

The three channel filters are synthesized, alternately and repeatedly, as a general Chebyshev filter with a frequency variant complex load. Having had each channel filter updated three times, the overall triplexer response is found well converged, as shown in Fig. 11. The synthesized coupling values are marked on the coupling topology shown in Fig. 12. It should be mentioned that the second channel filter has two TZs, which is realized as a cascaded-triplets topology, the matrix transformation



Fig. 11. Synthesized transfer and reflection characteristics of the triplexer.



Fig. 12. Coupling topology of channel filters and their connections to an NRN type of star-junction in a triplexer. Solid circles represent resonators. The hollow circle is the NRN. Solid lines represent J-inverters.

technique reported in [11] is used to obtain the coupling matrix. An NRN junction can be realized by a wired T-junction or a detuned resonator. An experimental validation to realize an NRN by a nonresonant coaxial resonator has been reported in [12].

VI. CONCLUSION

A general approach to the synthesis of a multiplexer circuit model with a common star-junction has been proposed. A key contribution of this work is the creation of the "phased-inverter to FIR-inverter" transformation, which allows a direct adoption of the synthesis theory of a bandpass filter with a complex load to synthesis of a multiplexer with a star-junction. Two types of most commonly used star-junctions have been considered.

Since each channel filter is dealt with individually and alternately, the new method can handle multiplexers with a high channel number with good accuracy. The synthesis approach not only provides the coupling matrix for each channel filter, but also an optimal admittance component associated to the starjunction in the sense that the reflection zeroes of each channel filter are located on the imaginary axis.

The proposed approach has been demonstrated by two synthesis examples, including a diplexer with a resonant type of star-junction and a triplexer with an NRN type of star-junction. A prototype diplexer is built and tuned according to the synthesis result for validating the synthesized circuit model.

It is observed that for contiguous channel multiplexers, the proposed approach can still offer a good starting point for further optimization.

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