EM scattering of an arbitrary multiple dielectric coated conducting cylinder by coupled finite boundary element method

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Abstract: A coupled finite boundary element method (CFBM) solution is developed for the EM fields scattered by an arbitrary multiple-dielectric coated conducting cylinder. The field in the dielectric coating domain is formulated by the finite element method, while the radiation condition is included by adopting a boundary element method. Numerical results show that the CFBM does not suffer from non-uniqueness in the resonance frequency. An excellent agreement between CFBM solutions and exact solutions for coated circular cylinders is observed.

1 Introduction

The growing interest in better understanding of the scattering behavior of coated conducting cylinders has resulted in many investigations [1–9], including the exact series solution and various approximate solutions. The analytical solution is only suitable in few cases, for instance circular cylinders [1–4]. Most numerical methods solve the scattering from arbitrary cylinders by using the surface equivalence principle and the method of moments [5, 6], in which a coupled set of integral equations is obtained and the boundary conditions are enforced on various interfaces. A recent paper [7] deals with the scattering from multiple layer coated conducting cylinders using the same procedure as that used in Reference 6. Since, for each homogeneous region, the integral equation and boundary condition must be written separately by a detailed inspection process, such a method is very difficult to extend to multimedia-coated cases.

Although the finite element method (FEM) is a powerful numerical technique for solving multimedia and complex geometries, solving the unbounded scattering problems of multimedia objects has so far always been perplexing. The unimoment method [8] takes advantage of a finite element representation of the field inside the object while the scattered field is represented with an eigenfunction series expansion [8], but only the circle appears to be a convenient computational boundary because of the completeness of the eigenfunction. The recent work [9] carried out for electromagnetic scattering, using the hybrid finite element method, has overcome the disadvantage of the unimoment method, which is not efficient for dealing with non-circular cylindrical problems, by introducing two artificial boundaries. However, more finite elements should be used to fill in the additional region formed by those two artificial boundaries, and numerical normal derivatives are involved.

This paper deals with the scattering from multimedia-coated conducting cylinders with the coupled finite boundary element method (CFBM) [10], which takes advantage of both the finite element method and the boundary element method (BEM). The field inside the dielectric ring domain is described by the differential equation, and solved by the finite element method. The boundary element method is used in the outer homogeneous region taking into account the radiation condition. On the other hand, an analytical relationship beween the field and its derivative on the outer boundary is used and no artificial boundary is needed. The importance of this feature is obvious when the scattering object is long and slender.

2 Formulation

The typical problem to be solved is depicted in Fig. 1. The domain of the problem is divided into two regions.

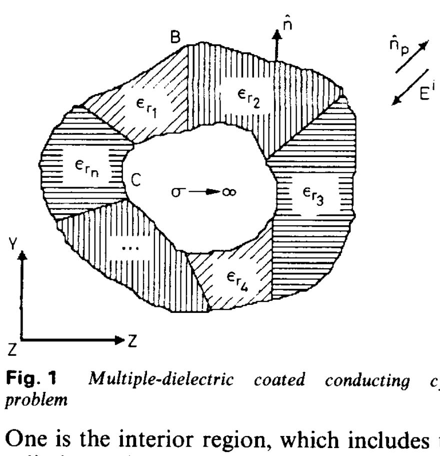

Fig. 1 *Multiple-dielectric coated conducting cylinder scattering problem*

One is the interior region, which includes the conducting cylinder and the dielectric ring domain. In fact, the field inside the conducting cylinder is zero and only the ring-

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shaped domain needs to be considered. Another is the exterior region enclosed by B and the infinite boundary. The former will be treated by the finite element method and the latter by the boundary element method.

With the finite element approach, the primary dependent variables are replaced by a system of discretised variables over the domain under consideration. Therefore, the domain itself is discretised into finite elements. Compatability within the element and between element boundaries is ensured by the choice of the shape function. Within an arbitrarily-shaped triangle, with a time dependence of the form $\exp(j\omega t)$ being implied, the field value u is written in terms of second-order polynomials in this study as

$$u = \{N\}^T\{u\}_e \qquad (1)$$

where $\{u\}_e$ is the field value vector at the nodal point of the element and $\{N\}^T$ denotes the transpose of the shape function vector.

Substituting eqn. 1 into the two-dimensional Helmholtz equation and using a Galerkin procedure, after integrating by parts, results in

$$[A]\{u'\} = \{\psi'\} \qquad (2)$$

which can be rewritten in greater detail as

$$\begin{bmatrix} [A_{RR}] & [A_{RB}] & [A_{RC}] \\ [A_{BR}] & [A_{BB}] & [A_{BC}] \\ [A_{CR}] & [A_{CB}] & [A_{CC}] \end{bmatrix} \begin{Bmatrix} \{u'\}_R \\ \{u'\}_B \\ \{u'\}_C \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [B_B]\left\{\dfrac{\partial u'}{\partial n}\right\}_B \\ [B_C]\left\{\dfrac{\partial u'}{\partial n}\right\}_C \end{Bmatrix} \qquad (3)$$

where subscripts R, B and C denote the nodals in region R and on boundaries B and C, respectively.

The matrix $[A]$ can be computed using

$$[A] = \sum_e \iint \left[\frac{\partial\{N\}}{\partial x} \cdot \frac{\partial\{N\}^T}{\partial x} + \frac{\partial\{N\}}{\partial y} \cdot \frac{\partial\{N\}^T}{\partial y} - \varepsilon_r(1 - j\tan\delta)k_0^2\{N\}\{N\}^T \right] dx\, dy \qquad (4)$$

$$[B_{B,C}] = \sum_e{}' \int_{BC} \{N\}\{N\}^T\, d\Gamma \qquad (5)$$

where k_0 is the wavenumber in free space, and δ is the dielectric loss angle.

The integral in eqn. 5 can be calculated analytically, and for the second-order finite elements eqn. 5 can be expressed as

$$[B_{B,C}] = \sum_e{}' \frac{l_e}{15} \begin{bmatrix} 2 & 1 & -0.5 \\ 1 & 8 & 1 \\ -0.5 & 1 & 2 \end{bmatrix} \qquad (6)$$

In the above equations, $\sum_e$ and $\sum_e'$ extend over all the different elements and elements related to boundaries B and C, respectively. The l_e is the length of eth boundary element. In the exterior region, because the scattering wave should vanish at infinity, it can be described using following integral equation:

$$u(\boldsymbol{r}_f) = \int_B \left[G(\boldsymbol{r}_f | \boldsymbol{r}_0) \frac{\partial u(\boldsymbol{r}_0)}{\partial n'} - u(\boldsymbol{r}_0) \frac{\partial G(\boldsymbol{r}_f | \boldsymbol{r}_0)}{\partial n'} \right] d\Gamma \qquad (7)$$

where the fundamental solution

$$G(\boldsymbol{r}_f | \boldsymbol{r}_0) = -\frac{j}{4} H_0^{(2)}(k_0 |\boldsymbol{r}_f - \boldsymbol{r}_0|) \qquad (8)$$

is introduced. Here, $H_0^{(2)}(\cdot)$ is the Hankel function of the second kind and zero order.

Application of conventional boundary element procedure [11, 12] to eqn. 7 produces a matrix equation

$$[H_0]\{u\}_B = [G_0]\left\{\frac{\partial u}{\partial n'}\right\}_B \qquad (9)$$

Both boundary element and finite element are connected on the interface B. To ensure a correct coupling of boundary and finite elements on the interface, conditions of compatibility and equilibrium must be satisfied. The compatibility condition can be reached if both elements have common nodals on the interface and if the shape functions describing the field variation on the interface are identical. The equilibrium is satisfied when the field normal derivatives at the nodal point of the boundary element are equal to that of the finite element on the interface. On the boundary B, the following boundary conditions must be satisfied:

$$\{u'\}_B = \{u\}_B + \{u^i\}_B \qquad (10)$$

$$-V^I\left(\left\{\frac{\partial u'}{\partial n}\right\}_B\right) = V^{II}\left(\left\{\frac{\partial u}{\partial n'}\right\}_B + \left\{\frac{\partial u^i}{\partial n'}\right\}_B\right) \qquad (11)$$

$$u = \begin{cases} E_z & \text{for TM case} \\ H_z & \text{for TE case} \end{cases} \qquad (12)$$

$$V^i = \begin{cases} 1 & \text{for TM case} \\ \dfrac{1}{\varepsilon_{ri}} & \text{for TE case} \end{cases} \qquad i = I \text{ or } II \qquad (13)$$

where ε_{rI} and ε_{rII} are the relative permittivities of the interior and the exterior, respectively. In addition, the incident wave is expressed as

$$u^i = u_0 \exp(jk_0|\boldsymbol{\rho}|) \qquad (14)$$

The co-ordinate ρ is chosen in the direction of the incident wave, and it is easy to obtain

$$q^i = \frac{\partial u^i}{\partial n} = -jk_0 \boldsymbol{n} \cdot \boldsymbol{n}_\rho u^i \qquad (15)$$

which can be expressed as

$$\{q^i\}_B = [D]\{u^i\}_B \qquad (16)$$

Considering the boundary conditions on the conducting boundary C:

$$\begin{cases} \{u'\}_C = 0 & \text{for TM case} \\ \left\{\dfrac{\partial u'}{\partial n}\right\}_C = 0 & \text{for TE case} \end{cases} \qquad (17)$$

Substituting eqns. 10 and 11 into eqn. 3 and merging eqns. 3 and 9 into one matrix equation, the final matrix equation is obtained for both interior and exterior regions:

$$\begin{bmatrix} [A_{RR}] & [A_{RB}] & [0] \\ [A_{BR}] & [A_{BB}] & -[B_B] \\ [0] & [H_0] & [G_0] \end{bmatrix} \begin{Bmatrix} \{u\}_R \\ \{u^s\}_B \\ \{q^s\}_B \end{Bmatrix} = \begin{Bmatrix} -[A_{RB}]\{u^i\}_B \\ ([B_B][D] - [A_{BB}])\{u^i\}_B \\ \{0\} \end{Bmatrix} \qquad (18)$$

for the TM-polarised case, and

$$\begin{bmatrix} [A_{RR}] & [A_{RC}] & [A_{RB}] & [0] \\ [A_{CR}] & [A_{CC}] & [A_{CB}] & [0] \\ [A_{BR}] & [A_{BC}] & [A_{BB}] & -[B_B] \\ [0] & [0] & [H_0] & [G_0] \end{bmatrix} \begin{Bmatrix} \{u\}_R \\ \{u^s\}_C \\ \{u^s\}_B \\ \{q^s\}_B \end{Bmatrix} = \begin{Bmatrix} -\begin{bmatrix} [A_{RB}] \\ [A_{CB}] \end{bmatrix} \{u^i\}_B \\ ([B_B][D] - [A_{BB}])\{u^i\}_B \\ \{0\} \end{Bmatrix} \quad (19)$$

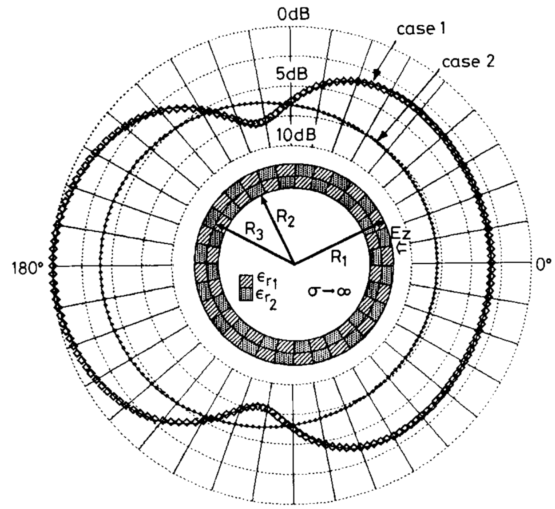

Fig. 2 *TM wave scattering field magnitudes of a composite-dielectric coated circular conducting cylinder*

◇ case 1; $R_2 = 0{\cdot}4\lambda$
+ case 2; $R_2 = 0.1\lambda$
$R_1 = 1.25R_2$, $R_3 = (R_1 + R_2)/2$

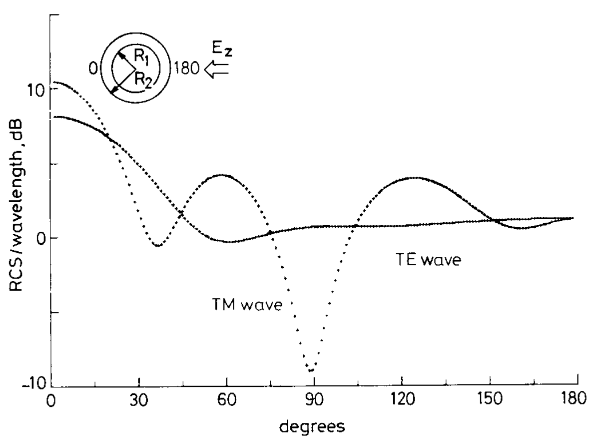

Fig. 3 *Comparison of exact solutions and CFBM solutions for radar cross-sections of dielectric coated circular cylinder*

$\varepsilon_c = 2.59$; $R_1, R_2 = 0.4, 0.5$ wavelengths
——— exact solutions
+ + + CFBM solutions

for the TE-polarised case. By solving the eqn. 18 or eqn. 19, the scattered field and its normal derivative on the boundary B can be obtained provided that the incident wave is given. Once these quantities are known, the far field or radar cross section (RCS) can be easily obtained by using the asymptotic expression of the Hankel function.

3. Numerical results and discussion

Numerical results of TM wave scattering field magnitude of a composite dielectric-coated circular conducting cylinder are shown in Fig. 2. It is interesting to note that

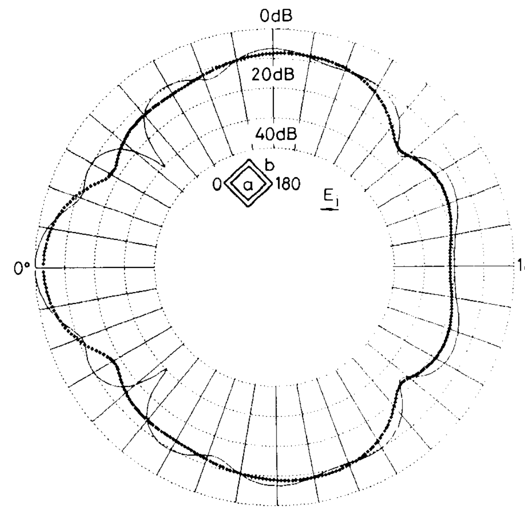

Fig. 4 *TM wave scattering magnitude of dielectric coated square conducting cylinder*

One layer coated bistatic case; $a = 40.0$ mm, $b = 54.0$ mm, $f = 10.0$ GHz
——— $\varepsilon_r = 1.0$
+ + + $\varepsilon_r = 2.3$

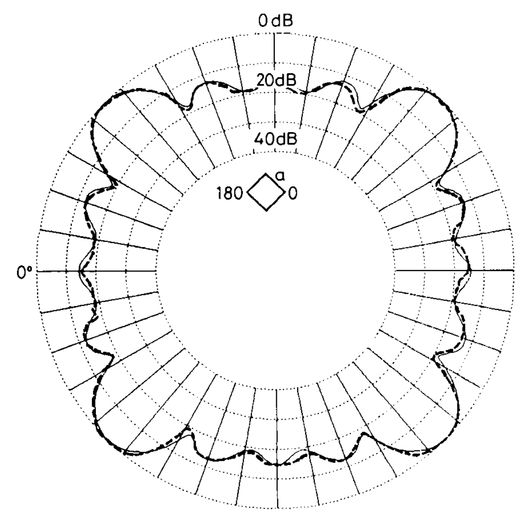

Fig. 5 *CFBM and experimental results of TM wave monostatic scattering by a conducting square cylinder*

Monostatic case; $a = 50.8$ mm, $f = 10.0$ GHz
– – – – experiment
——— CFBM

the scattering pattern of a composite dielectric coating is the same as that of a homogeneous dielectric coating with the average relative permittivity of composite coating. To check the program, a one layer dielectric coating case with $\varepsilon_{r1} = \varepsilon_{r2} = 2.59$ is analysed for both TM and TE cases, as shown in Fig. 3. The agreement between the CFBM results and exact results is excellent.

Fig. 4 shows the TM wave scattering magnitude of far field pattern for a dielectric-coated square conductor cylinder. To show the applicability of CFBM to coated problems, the extreme case of a coated square cylinder with $\varepsilon_r = 1.0$ is investigated. This situation is equivalent to the scattering of a perfectly conducting cylinder without coating. The result is shown in Fig. 5 in comparison with experimental results, and good agreement is observed.

A comparison of the analytical solution and the CFBM solution at the resonant frequency [13] for the scattering of a perfectly conducting circular cylinder can be obtained by setting the ε_r of the dielectric coating to unity in the CFBM program. There are significant agreements between these two solutions both for TM and TE cases as is shown in Fig. 6. This shows that the CFBM

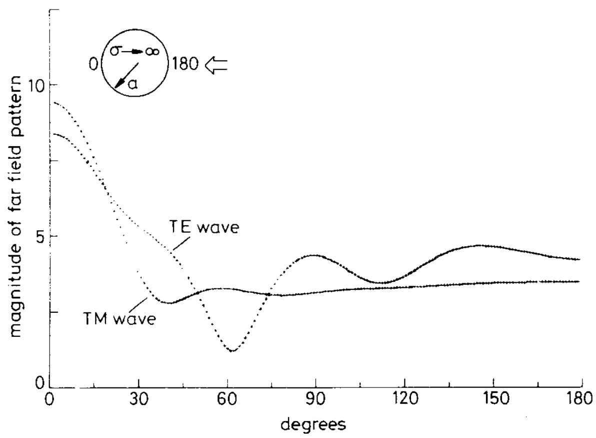

Fig. 6 *Comparison of exact and CFBM solutions at resonant frequency for scattering field by a conducting circular cylinder*

Resonance scattering case

—— CFBM solutions } $k * a = 3.8317$

+ + + exact solutions

does not suffer from the non-uniqueness in the resonance frequency.

Although the dielctric material is isotropic in all cases considered above, the method proposed here can be extended to cover some anisotropic cases, for example the case of anisotropy involving the coupling between x and y directions. In addition, the method may be combined with the spectral domain technique to treat more complicated problems, such as scattering of certain coated objects above or within a multilayered medium by using the recently developed discrete image theory [14].

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