A Combined Full-Wave CG-FFT Method for Rigorous Analysis of Large Microstrip Antenna Arrays

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***Abstract*—An accurate and efficient technique is presented for the analysis of large microstrip antenna arrays. The technique consists of an amalgamation of a spatially discrete scheme, consisting of the CG-FFT method and the complex discrete image (CDI) technique. The unique feature of this approach is the use of the spatially discrete CG-FFT for analyzing microstrip structures. The aliasing and truncation errors are thoroughly eliminated in this approach. In addition, the grad-div operators are transformed from singular Green's functions to differentiable expansion and testing functions by using Galerkin's procedure, thereby improving the accuracy and the rate of convergence. To show the accuracy and efficiency of this technique, a number of microstrip arrays, including a large microstrip reflectarray, have been studied. It is found that the simulations carried out using this technique are in very good agreement with measurements.**

I. Introduction

MICROSTRIP antennas have been studied extensively using various types of full-wave analysis techniques. The full-wave analysis of large microstrip antenna arrays, however, is still fraught with difficulties due to the fact that it usually involves the solution of a very large linear equation. Some authors have analyzed large microstrip antenna arrays as infinitely large arrays composed of uniform elements based on a Floquet-type representation of fields. But the simplifying assumptions used in this technique render it incapable of dealing with spurious radiation from the network and some other effects. It is felt that, techniques that do not require these simplifying assumptions are likely to be more widely used in the future for designing large microstrip antenna arrays [1].

The conjugate gradient fast Fourier transform (CG-FFT) method is considered to be powerful enough to be used for solving large EM problems. In the conventional CG-FFT procedure [2], analytical forms of the Fourier transform of the Green's functions are used when implementing the convolution theorem. But this procedure requires a large amount of padding of the FFT to reduce aliasing errors because the analytical spectral Green's function $\tilde{G}$ is usually widely spread in the frequency domain. To overcome this difficulty, a spatial discretization scheme has been proposed. As first described in [3], the integral equation is converted to a discrete periodic form and the spatial derivatives are approximated by finite differences, then the discrete cyclic convolution is calculated using a DFT. When using this scheme, the requirement for zero padding is reduced, more importantly, the aliasing and truncation errors are eliminated.

When applying CG-FFT to the analysis of microstrip structures, most researchers use the conventional CG-FFT procedure since the analytical forms of the Green's functions in the spectral domain are readily available. This inevitably leads to the aliasing problems. Efforts have been made by some researchers to reduce the aliasing errors in the application of CG-FFT to the analysis of microstrip structures [5], [6]. The problem, however, has not yet been thoroughly solved.

In this paper, we present an efficient full-wave analysis technique for the application to microstrip structures, which consists of the CG-FFT method combined with the CDI technique. In this technique, the spectral domain Green's functions for microstrip structures are converted into the closed-form spatial Green's functions by means of complex images representations. No full-wave information is lost in the procedure and the computational overhead is negligible. With the closed-forms of spatial Green's functions in hand, the integral equation describing our microstrip problem is discretized accurately in the spatial domain before the discrete Fourier transform is applied. This can eliminate all of the aliasing or truncation problems that exist in the other CG-FFT schemes used for microstrip analysis, thereby resulting in high efficiency and accuracy. In addition, if one implements the Galerkin procedure, the ∇, ∇' operators are moved from the singular Green's functions to the differentiable basis and testing functions, thereby further improving the accuracy and the efficiency. The efficiency and accuracy of this technique are demonstrated by making comparisons with experimental results, as well as other published results. The numerical examples, consisting of a corporate fed planar array and a very large microstrip reflectarray, show that the proposed technique is very suitable for full-wave analysis of large microstrip antenna arrays.

II. The Integral Equation and the Scheme for Discretization

In Fig. 1 is given an example of our microstrip structures. It consists of a 16 element corporate fed microstrip planar array. Let S denote the surface of both antenna array and its feed

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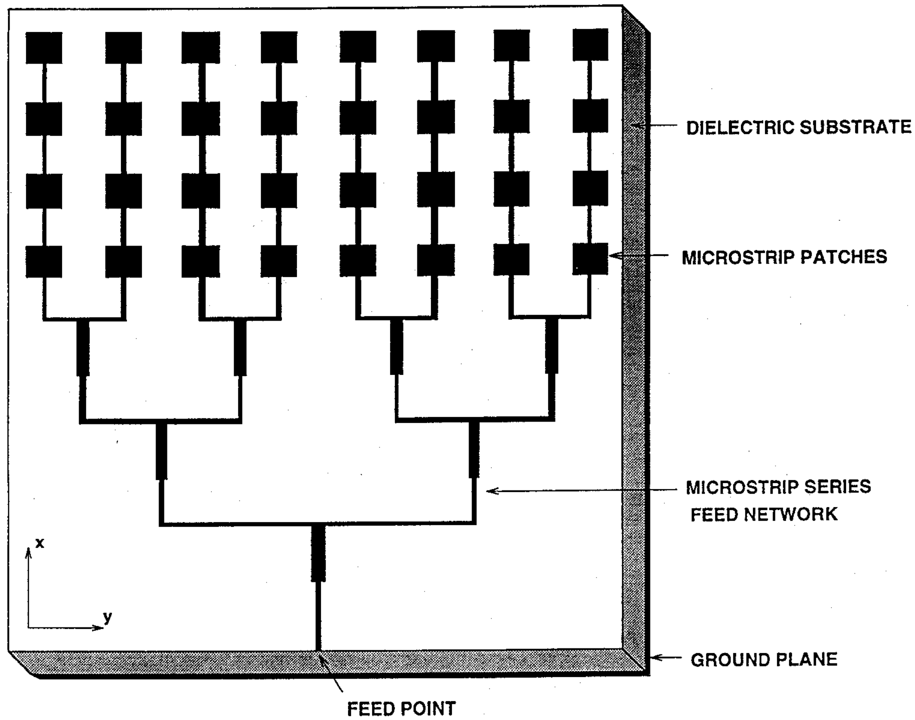


Fig. 1. Geometry of a microstrip corporate fed planar array.

network. $\hat{n}$ denotes the unit normal. The electric field integral equation used to solve the surface current distribution $\vec{J}_s$ is

$$\begin{aligned}\hat{n}\times\vec{E}^i(\hat{r}) &= \hat{n}\times(jw\vec{A}(\hat{r})+\nabla\Phi\cdot(\hat{r}))\\ &= \hat{n}\times\left\{\frac{-j\omega\mu}{4\pi}\int\int_S \vec{J}_s\cdot(\hat{r}')\cdot\overline{\overline{G}}_A\cdot(\hat{r},\hat{r}')\,dS'\right.\\ &\quad\left.-\nabla\left[\frac{1}{j\omega}\int\int_S \nabla' G_V(\hat{r},\hat{r}')\cdot\vec{J}_s(\hat{r},\hat{r}')\,dS'\right]\right\}\end{aligned} \tag{1}$$

where $\overline{\overline{G}}_A$ and G_V are the Green's functions for the magnetic vector and charge scalar potentials, respectively. To remove the ∇' operator from the Green's function in the second term of (1), we use a vector identity and Gauss' theorem with the result

$$\begin{aligned}\nabla\left[\frac{1}{j\omega}\int\int_S G_V(\hat{r},\hat{r}')\nabla\cdot\vec{J}_s\cdot(\hat{r}')\,dS'\right.\\ \left.-\oint_{\delta s} G_V(\hat{r},\hat{r}')\vec{J}_s(\hat{r}')\cdot\hat{s}\,dl\right]\end{aligned} \tag{2}$$

where δs is the perimeter of the metal surface and $\hat{s}$ is the outwards unit normal vector. The edge condition guarantees that the normal component of the surface current vanishes on the perimeter of the metal surface. Hence, the last term in the (2) can be eliminated. When the Garlerkin's procedure is used to discretize the integral (1)

$$\langle\vec{E}^s,\vec{f}\rangle = j\omega\langle\vec{A},\vec{f}\rangle+\langle\nabla\Phi,\vec{f}\rangle \tag{3}$$

another ∇ can also be removed from the function Φ which involves the Green's function G_V. Using the same vector identity, as used above, as well as Gauss' theorem, the last term can be written as

$$\begin{aligned}\langle\nabla\Phi,\vec{f}\rangle &= \int\int_S \nabla\Phi\cdot\vec{f}\,dS = -\int\int_S\cdot(\nabla\cdot\vec{f})\Phi\,dS\\ &= -\langle\Phi,\nabla\cdot\vec{f}\rangle.\end{aligned} \tag{4}$$

The purpose for this treatment is to avoid derivatives being applied directly or indirectly to the singular Green's functions. When the well-known rooftop function $\vec{f}=f_x^{m,n}\hat{x}+f_y^{m,n}\hat{y}$ is employed both as basis and expansion functions, the analytical form of its derivative can be obtained before carrying out the discretization. Therefore, the finite difference approximation of the derivatives [5], [6] does not occur in the present scheme. It is noted that both of the ∇'s can also be removed from the Green's functions by invoking the variational principle [4].

Without any loss of generality, the x-component of (3) can be written as

$$\begin{aligned}LJ_{sx} &= \int\int_S E_x^s f_x\,dS\\ &= \frac{j\omega\mu}{4\pi}\int\int_S f_x\left[\int\int_S J_{sx}G_{Ax}(\hat{r},\hat{r}')\,dS'\right]dS\\ &\quad+\frac{1}{j4\pi\epsilon\omega}\int\int_S\frac{\partial f_x}{\partial x}\\ &\quad\cdot\left[\int\int_S\left(\frac{\partial J_{sx}}{\partial x'}+\frac{\partial J_{sy}}{\partial y'}\right)G_V\cdot(\hat{r},\hat{r}')\,dS'\right]dS.\end{aligned} \tag{5}$$

Replacing J_{sx}, f_x with their expanded forms and the analytical form of the derivatives, we find

$$\begin{aligned} LJ_{sx} = &\frac{1}{\Delta x}\sum_{i'=1}^{N+1}\sum_{j'=1}^{M+1}\left\{\frac{c_1}{\Delta x}[I_x(i',j') - I_x(i'-1,j')]\right.\\ &\left.+\frac{c_1}{\Delta y}[I_y(i',j') - I_y(i',j'-1)]\right\}\\ &\cdot[\Gamma_p(i+1-i',j-j') - \Gamma_p(i-i',j-j')]\\ &-c_2\sum_{i=1}^{N}\sum_{j=1}^{M+1} I_x(i',j')\Gamma_{tx}(i-i',j-j') \end{aligned} \quad (6)$$

$$\Gamma_{tx}(i,j) = \int\int_S f_x^{i,j}\left[\int\int_S f_x^{i',j'} G_{Ax}\, dS'\right] dS \quad (7)$$

$$\Gamma_p(i,j) = \int\int_S \Pi^{i,j}\left[\int\int_S \Pi^{i',j'} G_V\, dS'\right] dS \quad (8)$$

where $c_1 = 1/4\pi\epsilon\omega, c_2 = j\omega\mu/4\pi$ and the $\Pi^{i,j}$ is the 2-D unit pulse function defined over the $(i,j)^{th}$ charge expansion segment. Now the integral equation is totally discretized and is expressed by the 2-D discrete sequencies $I_x(i,j), \Gamma_{tx}(i,j)$ and $\Gamma_p(i,j)$. The latter two sequencies are called discrete Green's functions. It is not difficult to find that the operation of (6) involves several two-dimensional linear convolutions. To emploit the efficiency of the FFT for carrying out the convolutions, the 2-D discrete sequencies are expanded to periodic ones with cyclical periods of $2M$ and $2N$ in each dimension. After applying the convolution theorem, the operation of (6) can be carried out as

$$\begin{aligned} LJ_{sx} = -\mathcal{F}_D^{-1}&\left\{\frac{c_1}{\Delta x}\left[\frac{1}{\Delta x}(1-\delta_x^*)\tilde{I}_x + \frac{1}{\Delta y}\cdot(1-\delta_y^*)\tilde{I}_y\right]\right.\\ &\left.\cdot(\delta_x - 1)\tilde{\Gamma}_p + c_2\tilde{I}_x\tilde{\Gamma}_{tx}\right\} \end{aligned} \quad (9)$$

where $\delta_x = e^{j\pi\omega_i/N+2}, \delta_y = e^{j\pi\omega_j/M+2}$. The $\tilde{I}_x(\omega_i,\omega_j), \tilde{\Gamma}_p(\omega_i,\omega_j), \tilde{\Gamma}_{tx}(\omega_i,\omega_j)$ are the 2-D DFT's of their corresponding quantities. Following the same procedure, the operation LJ_{sy} can be carried out in the same way.

From the above description, it is clear that the discrete Green's functions are the key set of parameters used by this algorithm. Therefore, to carry out the spatial discretization procedure, the spatial domain Green's function must be known *a priori*. This poses no difficulty for free space problems, where the spatial Green's function is a very simple function. But for microstrip structures, where the Green's functions are much more complex, this forms a major obstacle to our implementation of the spatial discretization scheme. This issue will be described more fully in the next section.

III. The Application of CG-FFT to Microstrip Problems

A. Background

For most applications of CG-FFT to the analysis of microstrip structures, the analytical form of the spectral domain Green's function are used to avoid having to solve the time-consuming Sommerfeld integrals. But as mentioned above, this results in aliasing problems. To reduce the aliasing errors in [5], a window function is applied over the spatial Green's function and then the resulting function is made to be periodic. But the aliasing is not eliminated as it was claimed in the paper. The error is associated with the evaluation of the dyadic Green's function which invloves integrations in spectral domain from $-\infty$ to ∞. To carry out the integration numerically, some truncation of the spectrum must be implemented. Consequently, the window function is not band limited in space, which then introduces aliasing problems when the equivalent periodic function is generated. In [6], a double FFT scheme has been tried, where the spectral Green's functions are transferred to the spatial domain using FFT's, truncated and then transferred back to the spectral domain with zero padding applied to make the functions twice their original data sets. In this manner, the analytical spectral Green's functions are replaced by spectral cyclic ones. It was reported that a considerable saving in computing time is realized by using this procedure. But it is obvious that the number of the FFT's used in this approach has to be very large and the aliasing errors, caused by the truncation in the spectral and spatial FFT, still exist.

As we know that the spatially discrete scheme described in the last section can eliminate the aliasing error, now it is expected that if the closed-form spatial Green's function for a microstrip problem can be derived, the discrete system should yield a much more accurate and efficient solution. With this intention, a combinational CG-FFT technique is proposed here. The new technique consists of the spatially discrete scheme of the CG-FFT method combined with the CDI technique. In this technique, the closed-form spatial Green's functions for microstrip structures are derived from their spectral domain Green's functions by means of complex images representations. The details of the derivation can be found in [7], which are outlined below.

B. The Closed-Form Spatial Domain Green's Functions

The Green's function corresponding to the vector and scalar potential's can be written in the form of the Sommerfeld integral

$$G_{A,V} = \frac{1}{4\pi}\int_{PSI} k_\rho H_0^{(2)}(k_\rho\rho)\tilde{G}_{A,V}(k_\rho)\, dk_\rho \quad (10)$$

where $\tilde{G}_{A,V}$ stands for the spectral-domain vector potential and scalar potential Green's functions associated with a microstrip structure, PSI denotes the path of the Sommerfeld integral. To derive the spatial domain Green's functions from their spectral forms, it is necessary to perform the inverse Hankel transform. Generally, this inverse cannot be solved analytically. But the complex exponentials can be transferred to the spatial domain analytically using the Sommerfeld identity

$$\frac{e^{-jkz}}{r} = \int_{-\infty}^{\infty}\frac{1}{j2k_z}e^{-jk_z z}H_0^{(2)}(k_\rho\rho)k_\rho\, dk_\rho. \quad (11)$$

Mathematically, the idea underlying the derivation is the use of Prony's method to simulate the spectral domain Green's functions by a series of complex exponential functions. The Sommerfeld identity is then used to convert these functions into a closed-form version in the spatial domain. This unique treatment not only makes the inversion of the spectral domain Green's function very efficient, but also provides for a very clear physical meaning. In the method, a closed-form Green's function for a microstrip structure is divided into three parts $G = G_0 + G_{sw} + G_{ci}$. G_0 represents the contribution from a few quasi-dynamic images which dominates in the near-field region. G_{sw} represents the contribution from surface wave which dominates in the far-field region of the substrate surface. G_{ci} represents the contribution from the complex images related to leaky waves and important in the intermediate region. The procedure that is followed in deriving our closed-form Green's fuinctions consists of identifying and extracting the above terms from the original Green's functions in the inverse Hankel transform form.

C. The Combinational Approach

The amalgamation of the complex discrete image technique with the discrete CG-FFT method takes place at the point where the spatial discrete Green's function is calculated. For a given microstrip problem, the information needed for the implementation of the complex discrete image technique is the working frequency f, dielectric constant ϵ_r and the thickness h of the substrate. Then the closed form spatial Green's functions are derived for this microstrip structure. With these analytical functions in hand, the integral equation describing the microstrip problem is discretized in the spatial domain using rooftop functions and the Garlerkin's procedure. The discrete Green's functions $\Gamma_{Ax,y}, \Gamma_p$ defined in (7) and (8) are calculated for each of the small current and charge cells which cover the microstrip metallic surface. The efficiency of the procedure is such that the computing time required for generating the discrete sequence is negligible compared to the time taken to implement the CG-FFT algorithm. Only one set of images needs to be found for a given substrate, regardless of the configuration of the microstrip structure above it. The spatially discrete CG-FFT scheme then can be employed in a straightforward manner to the resultant system. This scheme overcomes the difficulty inherent to carrying out the numerical evaluation of the Sommerfeld integration, at the same time, achieves high efficiency and accuracy by avoiding the aliasing errors that plague the other CG-FFT scheme of the microstrip analysis.

IV. Numerical Results

For the purpose of benchmarking the rate of convergence, a typical problem, namely scattering of a normally incident plane wave from a rectangular microstrip patch antenna, is taken as the first example. In Fig. 2, the co-pol and cross-pol current distributions are shown and compared with the results obtained by MoM along the $A - A'$ and $B - B'$ cuts. Excellent agreement is obtained. The convergence rates of the proposed scheme and those given in [5] are compared in

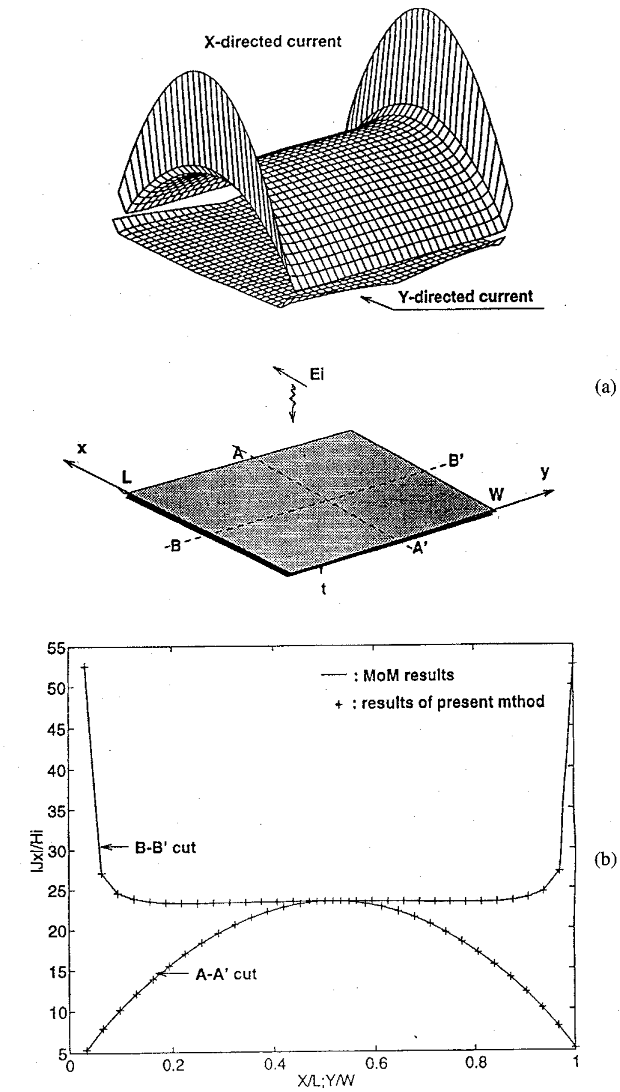


Fig. 2. Current distribution on a rectangular patch antenna illuminated by X-directed electrical field; $W = 15$ mm, $L = 10$ mm, $\epsilon_r = 10.2$, $t = 1.27$ mm, frequency = 3.12 GHz.

Fig. 3. It is clear that the method, proposed here, shows a much better rate of convergence than that in [5]. To demonstrate the accuracy of the proposed method, a line fed microstrip patch antenna is analyzed. The input impedances, which are known as a good metric for testing the accuracy of an analytical technique, are derived from the standing wave current on the feed line. The results over the frequency range 4–7GHz are given in Fig. 4. They are compared with measured data as well as results obtained using FDTD technique in [8]. The comparison shows that the proposed technique has very good accuracy.

The most important characteristics of the CG-FFT method is its potential for solving electrically large EM problems, but very few applications of this method to the analysis of large microstrip arrays have been reported in literature so far. In this paper, to demonstrate the strength of the hybrid CG-FFT

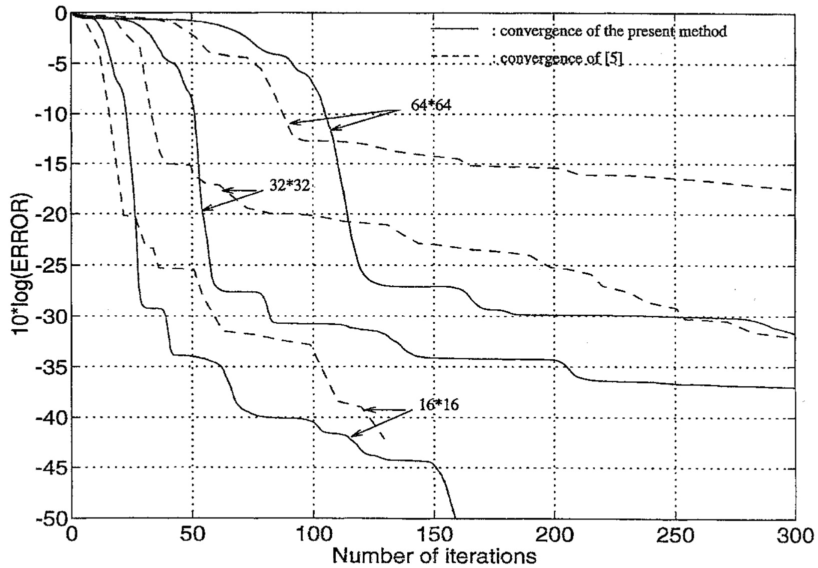


Fig. 3. Comparison of rates of convergence for the proposed approach and the approach given in [5]. Both techniques were used to calculate scattering by a rectangular patch (as shown in Fig. 2) at three different discretization rates (16 by 16, 32 by 32, 64 by 64).

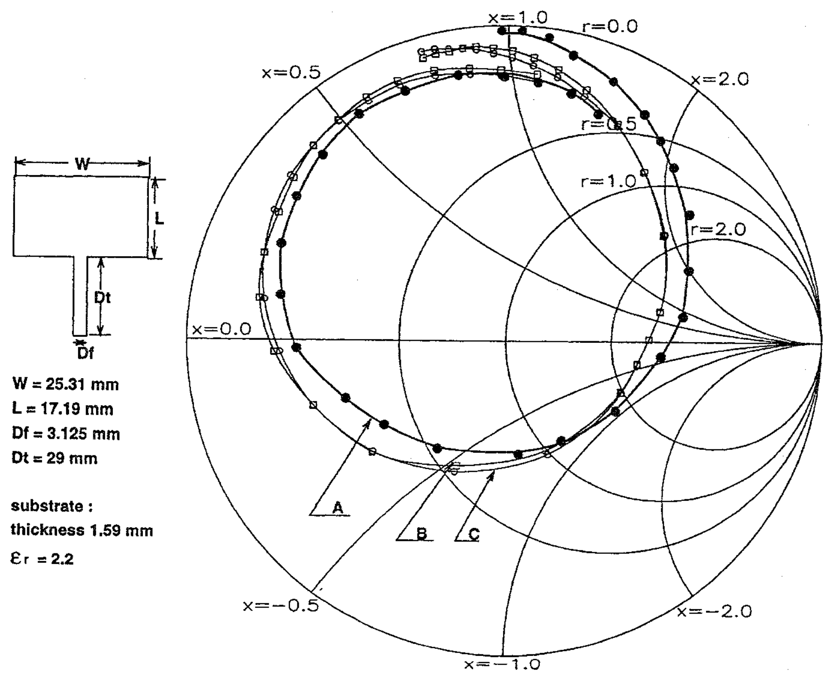
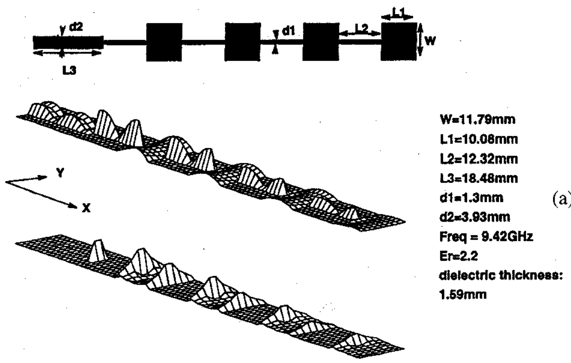


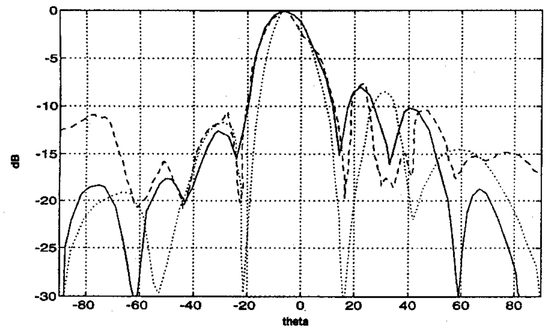
Fig. 4. The input impedance of a rectangular microstrip patch antenna (frequency = 4 − 7 GHz clockwise). A: Results obtained by the present method; B: FD-TD results from [8]; C: Measured results.

method, a microstrip corporate feed planar array and a very large microstrip reflectarray are analyzed.

To start, one branch of the corporate feed planar array shown in Fig. 1 is simulated. For a δ-gap excitation source located near the end of the feed line, the current distribution and the radiation pattern are calculated and shown in Fig. 5. To study the effects of the feed network, the pattern is calculated again with the radiation from the feed networks suppressed. These results are compared with the measured results. The effect of the feed-line network can be deduced from this comparison. To continue, the complete array in Fig. 1 is simulated. The current distribution is depicted in Fig. 6. The radiation patterns

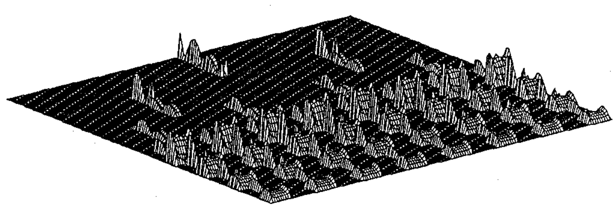


(a)

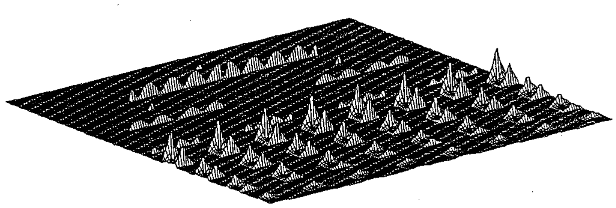


(b)

Fig. 5. Current distribution and radiation pattern of a four-element series fed microstrip antenna array. (a) Current distribution: X-directed (upper) and Y-directed (lower). (b) Radiation patterns: Solid line—simulated result with feed network; dashed line—measured results; dotted line—simulated result without feed network.



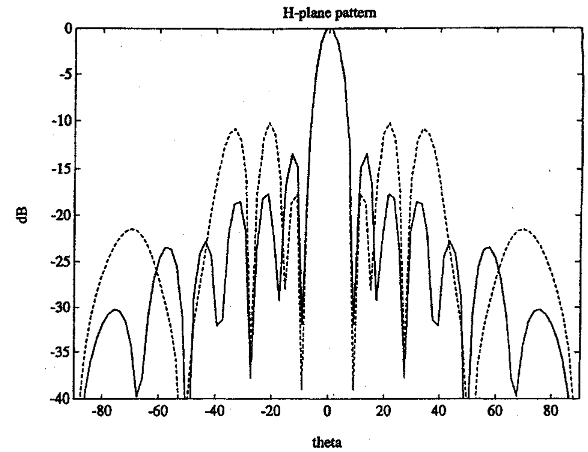
(a)



(b)

Fig. 6. Current distribution on the microstrip corporate fed planar array shown in Fig. 1. Dielectric thickness = 1.59 mm; $\epsilon_r = 2.2$; frequency = 9.42 GHz, dimensions of patches are the same as those in Fig. 5. (a) X-directed current; (b) Y-directed current.

with and without the contributions from the feed networks are illustrated in Fig. 7. The effect of the feed network are easily deduced from this figure.



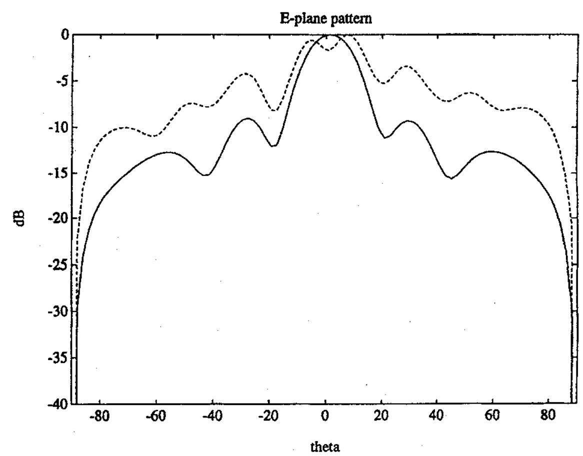


Fig. 7. Radiation patterns of the microstrip corporate feed planar array shown in Fig. 1. Solid/dashed lines: without/with the current distribution from the feed network.

As a final example, a very large microstrip reflectarray is simulated. The principles underlying these arrays were first described by [9]. To date only a few experimental results have been reported. It is difficult for the infinite array analysis technique to accurate predict the fields backscattered from thousands of nonuniformly illuminated microstrip patches with unequal lengths of microstrip transmission lines. It has been demonstrated [10] that the full-wave technique described in this paper is suitable for the modeling studies of such large arrays. Fig. 8 depicts the layout of a microstrip reflectarray. The contour map of the current phase distribution on the array is shown in Fig. 9. The 3-D view of the current distribution is depicted in Fig. 10. It is found that the current distribution has a strong pattern all over its layout which shows clear mapping relation to the distribution of the tails over the array. We can say that it is the pattern, combined with the radiation pattern of the feed antenna, that generates the desired coherent phase front of the reradiated field from the array in the desired direction. Fig. 11 shows the calculated H- and E-plane patterns of this array. It is seen that the main beam is pointed to the designed direction. It worth noting that, the number of the unknown involved in this simulation are approximately $400 \times 500 \times 2 = 400\,000$. Using the proposed

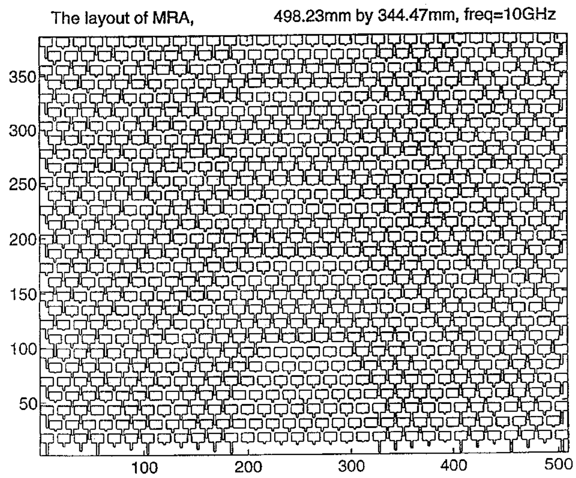


Fig. 8. The layout of a microstrip reflectarray with a total of 914 elements which is designed for 10 GHz. The main beam is directed to $(\theta, \phi) = (-30°, 0)$. Substrate: $\epsilon_r = 2.32$, thickness $h = 2.36$ mm. The x and y coordinates represent the sequence number of the unknowns in each direction.

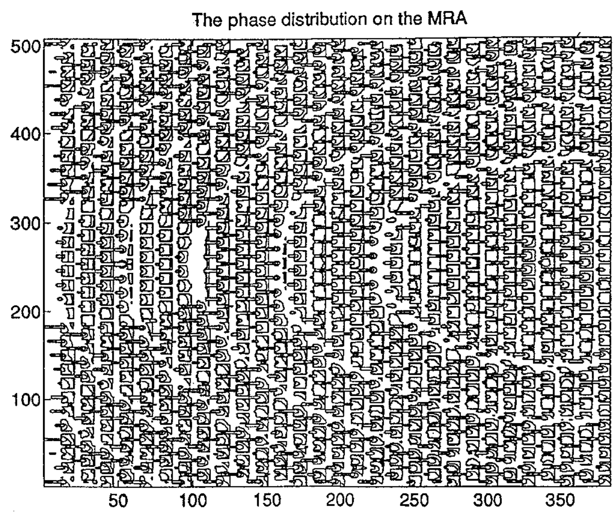


Fig. 9. The contour map of the current phase distribution on the array given in Fig. 8. (Please note this graph is rotated by 90°, with respect to Fig. 8.)

scheme, the convergence rate of 10^{-3} is reached after 6000 iterations which takes about 67 CPU-hours on a SUN-SPARC server. But for MoM with direct inversion scheme, this number means a $400\,000 \times 400\,000$ matrix needs to be stored and processed using a computer with at least 320 Gb memory. This is totally beyond the capacity of any existing computer in the world.

V. Conclusion

An accurate and efficient hybrid full-wave analysis technique for analyzing large microstrip antenna arrays has been presented. It combines the CG-FFT method with the complex discrete image technique. Since the closed-form spatial Green's function is used in the spatial discretization of the integral equation, the proposed method can eliminate all of the aliasing and truncation errors experienced by other CG-FFT techniques. By transferring the grad-div operators from the Green's function to the differentiable basis and testing functions, the accuracy and rate of convergence for the algorithm are further improved. The convergence rate

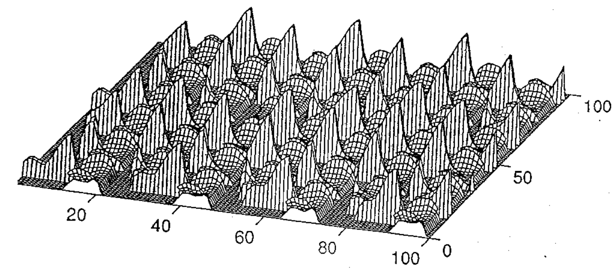


Fig. 10. 3-D view of the current magnitude distribution on a corner of the array shown in Fig. 8.

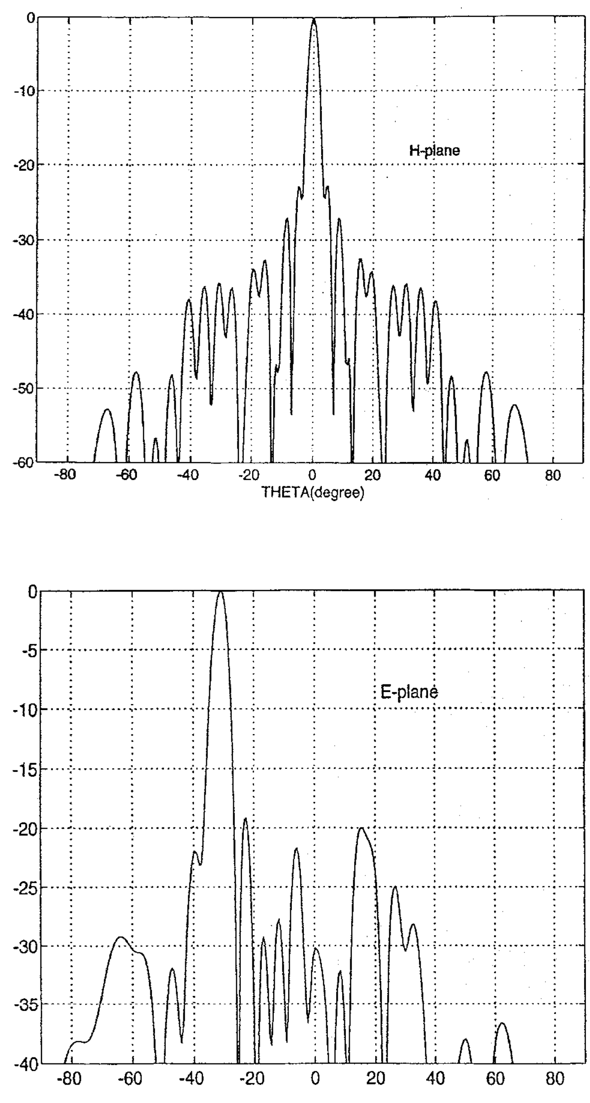


Fig. 11. The calculated radiation patterns of the microstrip reflectarray. Upper: H-plane pattern, lower: E-plane pattern.

and the accuracy are shown by making comparisons between the results presented here and other published results, as well as the measurements. The numerical and experimental examples in this paper for microstrip arrays show that the proposed technique is very suitable for full-wave analysis of large microstrip antenna arrays. Finally, we remark that the proposed technique is not limited to the analysis of single layer microstrip stuctures as suggested by the examples given in this paper, it can treat multilayer problems if the cooresponding spatial Green's functions are used.

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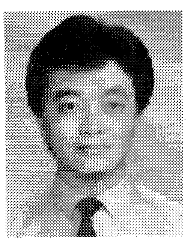
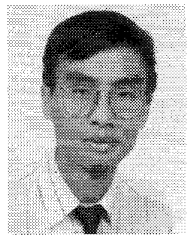


Yuan Zhuang received the B.S. degree from Tsing Hua University, Beijing, China, in 1986, and the M.S.E.E. from the Dalian University of Science and Technology (DUST), Dalian, China, in 1989, all in electrical engineering.

From 1989–1990, he was a teaching and research assistant at DUST, involved in the research of microwave technology in medical applications. From 1991–1993, he was a Ph.D. student at the Communications Research Laboratory (CRL), McMaster University, Hamilton, Canada, engaged in the research of microstrip antennas for mobile communications and EM numerical modeling. He joined Bell-Northern Research (BNR) Ltd., Ottawa, Canada, in 1994, where he is a Member of Scientific Stuff in the Fiber World Division engaged in the R&D of OC3/12 optical transmission network. He is currently also a part-time Ph.D. candidate at CRL. His research interests include wireless communications and application-specific integrated circuits (ASIC's) design.



Ke-Li Wu (M'90) received the B.S. and M.S.E. degrees from the East China Institute of Technology, China, in 1982 and 1985, respectively, and the Ph.D. degree from Laval University, Canada, in 1989, all in electrical engineering.

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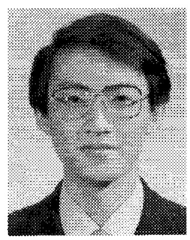
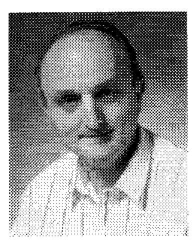
Dr. Wu has published more than 20 journal papers. He was a recipient of an URSI Young Scientist Award, in 1992. He contributed to *Finite Element and Finite Difference Methods in Electromagnetic Scattering,* Vol. 2 of *Progress in Electromagnetics Research* (Elsevier, 1990), and to *Computational Electromagnetics* (North-Holland, 1991).



Chen Wu (M'90) received the B.S. and M.Eng. degrees in electrical engineering from the East China Normal University, Shanghai, China, in 1983 and 1986, respectively.

From 1986–1989 he was a Teaching Assistant, and then a Lecturer, at the East China Normal University, and was engaged in the investigation of TV radio wave propagation in urban and suburban Shanghai, as well as numerical modeling of electromagnetic problems. In 1989, he joined Wireless Technology Group (WTG), Communications Research Laboratory, McMaster University, Hamilton, ON, Canada, as a Research Engineer. He is currently involved in research on integrated antennas, RF repeater, RF/ID, and microwave circuits design for PCS applications. He is also interested in developing full-wave modeling tools for electromagnetic simulation and applying signal processing and neural network techniques for EM modeling.



John Litva (M'82–SM'92) is currently a Professor in electrical and computer engineering at McMaster University, Hamilton, Ontario, Canada, and an Advisory Professor at the Nanjing University of Science and Technology, in Nanjing, China. His research is directed toward applications in satellite and terrestrial communications, and in particular, PCS. At present, his interests are centered on digital beamforming for satellite communications, intelligent antennas for PCS, electromagnetic modeling, and propagation measurements, modeling, and simulations for PCS.

Dr. Litva is the holder of the Microwave Antenna Chair sponsored by Spar Aerospace and NSERC, as well as being a Thrust Leader in the Telecommunications Research Institute of Ontario (TRIO). TRIO is a university-industry based Center of Excellence, funded by the Province of Ontario, whose mandate is to conduct research in support of the telecommunications industry in Ontario. He is also the Director of the Communications Research Laboratory (CRL), which is a separate research institute located at McMaster University. The CRL carries out research in the areas of signal processing, software engineering, microwaves, computer modeling, and antennas. He has published over 100 papers and has lectured widely on selected topics in numerical techniques for analyzing microstrip antennas, digital beamforming, nonlinear and adaptive beamforming, integrated antennas, and simulation and analyses of multipath signals.