Analysis Of Diode Circuits

Rectifier Circuits: The Half-Wave Rectifier

Application Background

![Block diagram of a dc power supply.](image)

The Half-Wave Rectifier

Using the battery-plus-resistance diode model, we have

\[ v_O = 0, \quad \text{if } v_S < V_{D0} \]
\[ v_O = \frac{R}{R + r_D} (v_S - V_{D0}), \quad \text{if } v_S \geq V_{D0} \]

In many applications

\[ r_D \ll R \quad \text{and} \quad v_O \approx (v_S - V_{D0}) \]
Discussions:

- In selecting diodes for rectifier design, the current handling capability required of the diode determined by the largest current flowing through the diode, should be considered.

- The peak inverse voltage (PIV) that the diode must be able to sustain without breakdown, determined by the largest reverse voltage that is expected to appear across the diode, must be taken into account in the design. Usually, $\text{PIV} = V_S$

- This kind of rectifier circuit does not function properly when the input signal is small.
Analysis Of Diode Circuits

Rectifier Circuits: Full Wave Rectifier

The Full-Wave Rectifier

- PIV = 2VS - VD0

Diodes with twice higher breakdown voltage for design of center-tapped full-wave rectifier than that of half-wave rectifier.

- The full-wave rectifier obviously produces a more “efficient” waveform than that provided by the half-wave rectifier.

In almost all rectifier applications, one opts for the full-wave type.
Analysis Of Diode Circuits

Rectifier Circuits: Bridge Rectifier

Bridge Rectifier

When \( v_s > 2V_{D0} \)

\[
V_O = -V_{D0} + v_s - V_{D0} = v_s - 2V_{D0}
\]

\[
PIV = V_O + V_{D0} = v_s - V_{D0}
\]

When \( v_s < -2V_{D0} \)

\[
V_O = -V_{D0}v_S - V_{D0} = -v_S - 2V_{D0}
\]

\[=|v_s| - 2V_{D0}\]
### Analysis Of Diode Circuits

**Rectifier Circuits: Comparisons Study**

<table>
<thead>
<tr>
<th></th>
<th>PIV</th>
<th>$V_O$</th>
<th>Efficiency</th>
<th>Transformer Turns</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Half-Wave</strong></td>
<td>$V_S$</td>
<td>$V_S - V_D$</td>
<td>About 40%</td>
<td>$n_1:n_2$</td>
</tr>
<tr>
<td><strong>Center-Tapped Full-Wave</strong></td>
<td>$2V_S - V_D$</td>
<td>$V_S - V_D$</td>
<td>About 90%</td>
<td>$n_1:2n_2$</td>
</tr>
<tr>
<td><strong>Bridge Rectifier Full-Wave</strong></td>
<td>$V_S - V_D$</td>
<td>$V_S - 2V_D$</td>
<td>About 80%</td>
<td>$n_1:n_2$</td>
</tr>
</tbody>
</table>
Analysis Of Diode Circuits

Peak Rectifier

(a) Diode cut-off period \((0 < t < T - \Delta t)\)

\[ i_D = C \frac{dv_O}{dt} + \frac{v_O}{R} = 0 \quad \text{with} \quad v_O(t = 0) = V_p \]

Letting \( v_O = Ae^{at} + B, \)

we get \( C \frac{d}{dt} (Ae^{at} + B) + (Ae^{at} + B) / R = 0 \)

or \( Ae^{at} (C \alpha + \frac{1}{R}) + B = 0 \) which means

\( C \alpha + \frac{1}{R} \Rightarrow \alpha = -\frac{1}{RC}; \quad \frac{B}{R} = 0 \Rightarrow B = 0 \)

Therefore, \( v_O(t) = v_O(0)e^{\frac{t}{RC}} = V_p e^{\frac{t}{RC}} \)

At the end of cut-off period \((t = T - \Delta t)\), \( v_O(t = T - \Delta t) = V_p - V_r \)

if \( RC \gg T \) and \( T \gg \Delta t \), we have

\[ V_p e^{-\frac{(T-\Delta t)}{RC}} \approx V_p e^{-\frac{T}{RC}} \approx V_p [1 - \frac{T}{RC}] = V_p - V_r \]

That is \( V_r \approx V_p \left(\frac{T}{RC}\right) = \frac{V_p}{fRC} \quad (1) \)

During the cut-off period, capacitor discharges and

\[ Q_{lost} = C\Delta V = CV_r \quad (2) \]
Analysis Of Diode Circuits

(b) Diode conduction period \((-\Delta t \leq t < 0\))

\[ V_p \cos(\omega(-\Delta t)) = V_p - V_r \]

Using Taylor series expansion of \(\cos(x) \approx 1 - \frac{x^2}{2} + \ldots\) for small \(x\)

we have

\[ V_p \left[1 - \frac{(\omega \Delta t)^2}{2}\right] = V_p - V_r \quad \text{or} \]

\[ (\omega \Delta t) = \sqrt{\frac{2V_r}{V_p}} \quad (3) \]

That is to say

\[ \Delta t = \left(\frac{1}{2\pi f}\right)\sqrt{\frac{2V_r}{V_p}} \quad (4) \]

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**Rectifier Circuits: The Peak Rectifier**

- **Fig. 3.41** Voltage and current waveforms in the peak rectifier circuit with \(CR \gg T\). The diode is assumed ideal.
Analysis Of Diode Circuits

Rectifier Circuits: The Peak Rectifier

\[ Q_{\text{supplied}} = \text{charge accumulated by } C \]
\[ \text{during diode charging period} \]
\[ (-\Delta t \leq t \leq 0) \]
\[ = \frac{i_c}{\Delta t} \]
\[ = Q_{\text{lost}} = C V_r \]
\[ \Rightarrow \]
\[ i_c = \frac{C V_r}{\Delta t} \]
\[ \Rightarrow \]
\[ i_c \approx C V_r \left[ 2\pi f \sqrt{\frac{V_p}{2V_r}} \right] \]
\[ \therefore \]
\[ i_c \approx \frac{C V_r}{\Delta t} \cdot C V_r \left[ 2\pi f \sqrt{\frac{V_p}{2V_r}} \right] \]
\[ i_{\text{avg}} = i_c + i_L \]
\[ (\omega: R \ i_L = \text{load current}) \]
\[ = C V_r 2\pi f \sqrt{\frac{V_p}{2V_r}} + \frac{V_p}{R} \]
\[ = \frac{V_p}{R} \left[ C \left( \frac{V_p}{f C R} \right) \right] \cdot 2\pi f \sqrt{\frac{V_p}{2V_r}} \cdot \frac{R}{V_r} + 1 \]
\[ \Rightarrow \]
\[ i_{\text{avg}} = I_L \left[ 2\pi f \sqrt{\frac{V_p}{2V_r}} + 1 \right] \]
\[ \therefore \]
\[ i_{\text{avg}} = I_L \left[ 1 + \pi \sqrt{\frac{2V_p}{V_r}} \right] \]
Analysis Of Diode Circuits

Rectifier Circuits: The Peak Rectifier

\[ i_D = i_C + i_L \]
\[ = c \frac{dv_C}{dt} + i_L \]

Since \( V_0 \approx V_I \) with ideal load, \( V_D = 0 \). Thus,
\[ i'_{0,\text{max}} = c \frac{dv_C}{dt} \left|_{t=\Delta t} \right. + \frac{V_P - V_R}{R} \]
\[ = c \frac{d}{dt} [V_P \cos \omega t] \left|_{t=\Delta t} \right. + \frac{V_P - V_R}{R} \]
\[ = \omega C V_P \left[ -\sin \omega (\Delta t) \right] + \frac{V_P - V_R}{R} \]

\[ i'_{0,\text{max}} = \omega C V_P \sin \omega \Delta t + I_L \]

where \( I_L \approx \frac{V_P}{R} \) since \( V_P \gg V_R \)
for \( \omega t \) very small, \( \sin \omega t \approx \omega t \)

\[ i_{D_{\text{max}}} = \omega C V_p \omega t + I_L = \left( \frac{V_p}{R} \right) \left( \frac{R}{V_p} \right) \omega C V_p \omega t + I_L \]

\[ = I_L \left[ \frac{R}{V_p} \omega C V_p \omega t + 1 \right] \]

\[ = I_L \left[ \frac{WRC \omega t + 1}{WRC} \right] \]

\[ \Rightarrow \omega t = \sqrt{\frac{2 V_F}{V_p}} \]

\[ WRC = \left( \frac{V_F}{V_p} \right) \pi \]

\[ = I_L \left[ \frac{2 \pi \left( \frac{V_F}{V_p} \right) \sqrt{\frac{2 V_F}{V_p}}}{2 \pi} + 1 \right] \]

\[ = I_L \left[ \frac{\sqrt{2 V_F}}{V_p} + 1 \right] \]

\[ \Rightarrow i_{D_{\text{max}}} = I_L \left[ 1 + 2 \pi \sqrt{\frac{2 V_F}{V_p}} \right] \]
In designing a Peak-Rectifier, it is important to choose the right size of capacitor to be able to recharge for the lost charge during the diode cut-off period.

It is also important to determine the maximum and average diode current during the diode charging period so that appropriate diode can be selected for the circuit design. This is illustrated in the following example.