Modeling Imprecise Detection Parameters as Fuzzy Numbers for a Rician Target in Rayleigh Background

S. W. Leung, J. W. Minett, M. K. Lee, and Y. M. Siu

Contact Author: Peter S. W. Leung

Department of Electronic Engineering
City University of Hong Kong
Tat Chee Avenue
Kowloon Tong
Hong Kong

Telephone: (+852) 2788 7757
Fax: (+852) 2788 7791
Email: eeswl@cityu.edu.hk

Modeling Imprecise Detection Parameters as Fuzzy Numbers for a Rician Target in Rayleigh Background

S. W. Leung, J. W. Minett, M. K. Lee, and Y. M. Siu
Department of Electronic Engineering
City University of Hong Kong
Tat Chee Avenue
Kowloon, Hong Kong

Abstract—The performance of a radar detector may become inaccurate when key parameters of the system are imprecise. Imprecise parameters are modeled here as fuzzy numbers. The Fuzzy Integrator detector is constructed for a Rician target in Rayleigh background. Results indicate that the precision of the detection performance decreases as parameter imprecision increases. The false alarm rate is particularly sensitive to parameter imprecision at low false alarm rate; 5% imprecision in the parameters can give rise to more than 300% imprecision in the false alarm rate at nominal false alarm rates below 10^{-5}.

I. INTRODUCTION

Detectors are generally constructed to use classical hypothesis test procedures, for example the Neyman-Pearson test [1]. However, such procedures are designed to be applied only when the parameters on which the hypotheses depend are known precisely. Many practical radar detection systems cannot assess phenomena as precisely as desired, yet parameter imprecision is not generally modeled. This may cause the detection model to poorly match the actual detection environment, resulting in inaccurate predictions of the detection performance.

In a series of papers [2, 3, 4, etc.], Saade and Schwarzlander investigated the effects of fuzzy (imprecise) parameters on the performance of likelihood ratio decision rules in several detection scenarios. In particular, they examined detection of a fixed amplitude signal in Gaussian noise having fuzzy variance using a Neyman–Pearson decision rule [2]. They identified that the likelihood ratio of hypotheses depending on fuzzy parameters would also be fuzzy, sometimes preventing a binary detection decision from being made. Other authors have also examined aspects of signal detection and decision-making using fuzzy logic, including Dziech and Gorzałczany [5], Son et al. [6], and Boston [7].

In [8], Minett and Leung extended the approach of Saade and Schwarzlander to apply the Neyman–Pearson hypothesis test procedure to a detection scenario in which any number of the detection parameters, such as noise variance and signal amplitude, could be imprecise. Furthermore, by applying the Extension Principle for fuzzy numbers [9] to the decision rule, both the false alarm rate and detection rate of the detector could be obtained as fuzzy numbers, explicitly identifying the imprecision of the detector's performance. Later, in [10], they continued this work to develop the Fuzzy Integrator with fuzzy M-out-of-N decision rule to perform signal integration and data fusion with an arbitrary number of imprecise parameters.

The effects of parameter imprecision on detection performance of the Fuzzy Integrator will be investigated here for a Rician target in Rayleigh background. This work goes beyond that carried out in [9] by analyzing the effects of parameter imprecision down to nominal false alarm rate 10^{-6}.

II. SUMMARY OF THE FUZZY INTEGRATOR

We now briefly summarize the structure of the Fuzzy Integrator [10]. When no target signal is present in the test bin, the signal returns, z, of the interference are assumed to have probability density function (p.d.f.) $f(z;\boldsymbol{P}_X)$, which depends on a set of parameters with values $\boldsymbol{P}_X = (p_{X_1}, \ldots, p_{X_k})$. However, when a target signal is present in the test bin, the signal returns of the target-plus-interference are assumed to have p.d.f. $g(z;\boldsymbol{P}_Y)$, which now depends on the parameters with values $\boldsymbol{P}_Y = (p_{Y_1}, \ldots, p_{Y_k})$. We denote the hypothesis that not target is present by H_0 and the hypothesis that a target is indeed present by H_1. The likelihood ratio function, Λ_Z, is defined as

$$\Lambda_Z(z;\boldsymbol{P}_X,\boldsymbol{P}_Y) := \frac{g(z;\boldsymbol{P}_Y)}{f(z;\boldsymbol{P}_X)}. \tag{1}$$

The value of the likelihood ratio for imprecise parameter values $\boldsymbol{P}_X$ and $\boldsymbol{P}_Y$ is obtained by applying the Extension Principle for fuzzy numbers [9] and is the fuzzy set with membership function

$$\mu_{\Lambda(z)}(t) = \sup_{t\,=\,\Lambda(z;\boldsymbol{P}_X,\boldsymbol{P}_Y)} \left\{ \min \begin{bmatrix} \mu_{\tilde{\boldsymbol{P}}_X}\left(p_{X_1}\right),\ldots,\mu_{\tilde{\boldsymbol{P}}_X}\left(p_{X_k}\right), \\ \mu_{\tilde{\boldsymbol{P}}_Y}\left(p_{Y_1}\right),\ldots,\mu_{\tilde{\boldsymbol{P}}_Y}\left(p_{Y_k}\right) \end{bmatrix} \right\}. \tag{2}$$

For a set of N observations of the signal returns in the test bin, $\boldsymbol{z} := \{z_1, \ldots, z_N\}$, the Fuzzy Integrator implements the decision rule

$$\sum_i^N \Omega\left(\Lambda_{\boldsymbol{z}}\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right),\gamma\right) \mathop{\gtrless}_{H_0}^{H_1} M\,, \tag{3}$$

where M is the crisp (non-fuzzy) global threshold and Ω is the fuzzy threshold obtained by applying a fuzzy ordering method, such as total distance ordering [10], to order the likelihood ratio with respect to the crisp threshold, γ. Fig. 1 shows a comparison of the crisp threshold, γ, and the fuzzy threshold, Ω. Ω measures the preference for deciding that a target is present. More information can be found in [10].

The false alarm rate of the Fuzzy Integrator is defined as

$$\Pr\left(\sum_i^N \Omega\left(\Lambda_{\boldsymbol{z}}\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right),\gamma\right) \geq M \,\middle|\, H_0\right), \tag{4}$$

while the detection rate is defined as

$$\Pr\left(\sum_i^N \Omega\left(\Lambda_{\boldsymbol{z}}\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right),\gamma\right) \geq M \,\middle|\, H_1\right). \tag{5}$$

Under both hypotheses, the distribution of the likelihood ratio, $\Lambda_Z\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right)$, and hence of $\sum_i^n \Omega\left(\Lambda_{\boldsymbol{z}}\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right),\gamma\right)$, is imprecise. Therefore both the false alarm rate and detection rate are also imprecise; their membership functions can be calculated by again applying the Extension Principle for fuzzy numbers, much like in (2).

III. Fuzzy Integration for a Rician Target in Rayleigh Background

In this section, we investigate the effects of parameter imprecision on the detection performance of the Fuzzy Integrator for a non-fluctuating target in Gaussian interference. At the output of a band-pass filter, as is well known, the envelope of interference only has Rayleigh distribution with p.d.f. [11]

$$f_X\left(x;\sigma^2\right) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{4}$$

while the envelope of the target-plus-interference has Rice distribution with p.d.f. [11]

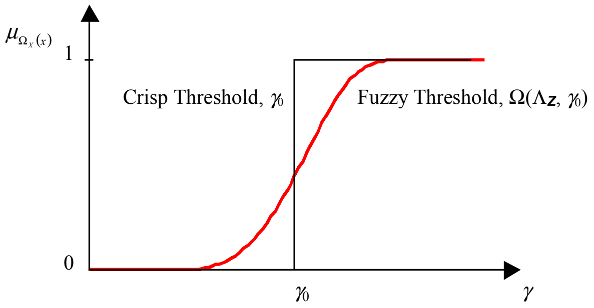

Fig. 1. Comparison of the crisp threshold and fuzzy threshold.

$$f_Y\left(y;\mu,\sigma^2\right) = \frac{y}{\sigma^2}\exp\left(-\frac{y^2+\mu^2}{2\sigma^2}\right) I_0\left(y\frac{\mu}{\sigma^2}\right), \tag{5}$$

where μ is the amplitude of the target, σ^2 is the interference variance, and $I_0(\cdot)$ represents the modified Bessel function of the first kind of order zero. The likelihood ratio function is

$$\Lambda\left(z_i;\mu,\sigma^2\right) = \exp\left(-\frac{1}{2}\frac{\mu^2}{\sigma^2}\right) I_0\left(z_i\frac{\mu}{\sigma^2}\right). \tag{6}$$

However, now suppose that the target signal amplitude, μ, and the noise variance, σ^2, are imprecise, modeled as triangular fuzzy numbers [12]. The fuzzy likelihood ratio can be obtained from the above likelihood ratio function by applying the Extension Principle for fuzzy sets using (2).

Fig. 2 shows the receiver operating characteristics (ROCs) of the Fuzzy Integrator at *SNR* 0dB ($\mu = \sigma = 1$) when there is no imprecision (the *Nominal* curve) and when there is 5% imprecision in both the signal amplitude and noise variance (the region enclosed by the two *Support* curves). The curve was generated by Monte-Carlo simulation with up to 50 million samples per point (higher false alarm rates required fewer samples). So, for example, when there is 5% imprecision in the parameter values at nominal false alarm rate 2.27 × 10^{-6}, the actual false alarm rate lies in the interval $\left[3.60\times10^{-7}, 9.85\times10^{-6}\right]$. Thus for 5% parameter imprecision, the actual false alarm rate has more than one order of magnitude of imprecision. The detection rate is affected similarly.

Fig. 3 displays in more detail the imprecision of the false alarm rate when there is imprecision in both the signal amplitude and noise variance. At high nominal false alarm rates, about 10^{-1}, parameter imprecision has little effect on the precision of the actual false alarm rate. However, as the

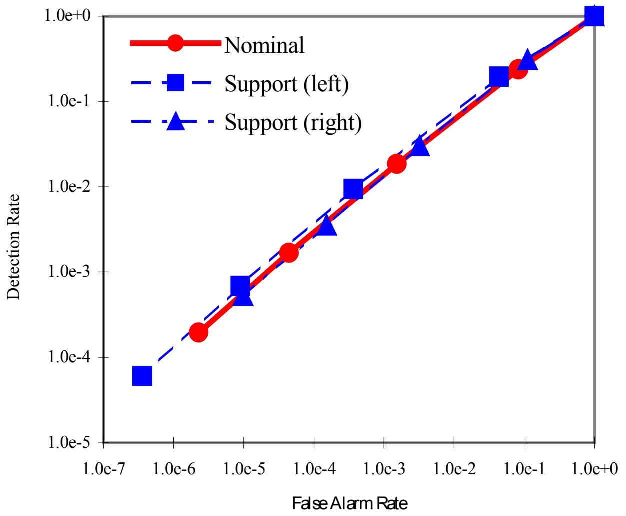

Fig. 2. Receiver operating characteristics of the Fuzzy Integrator for parameter imprecision 5% , SNR = 0dB.

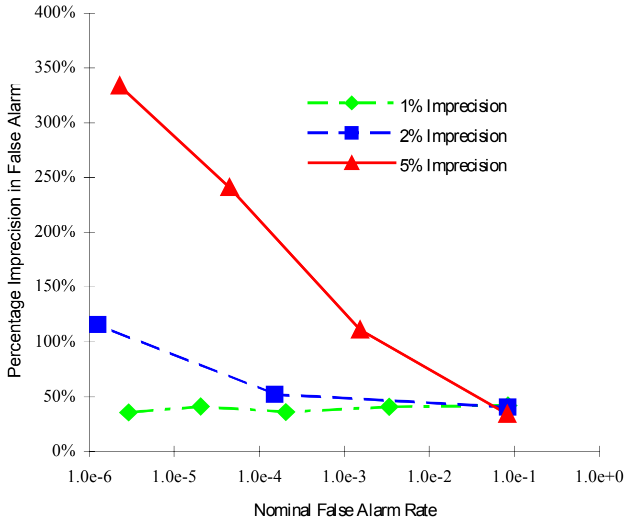

Fig. 3. Plot of false alarm rate imprecision of the Fuzzy Integrator as a function of nominal false alarm rate, SNR = 0dB.

nominal false alarm rate is reduced, the imprecision increases. For example, at nominal false alarm rate about 10^{-5}, the false alarm rate imprecision rises to more than 300%. Note also that for 1% parameter imprecision, the false alarm rate imprecision is roughly constant, indicating that parameter imprecision of up to 1% will not adversely effect the detector's performance.

IV. CONCLUSION

The effects of parameter imprecision on the detection performance of the Fuzzy Integrator for a Rician target in Rayleigh background is presented. We observe that the imprecision of the performance (both the false alarm rate and detection rate) can be many times greater than imprecision of the parameters, highlighting the importance of evaluating or estimating parameters as precisely as practicable. The methods described here can also be used for other interference and target models, and detectors to assess the relationship between parameter imprecision and performance imprecision.

REFERENCES

[1] M. H. DeGroot, *Probability and Statistics*, 2nd ed., Reading : Addison-Wesley, 1984.

[2] J.J. Saade and H. Schwarzlander, "Fuzzy hypothesis testing with hybrid data," *Fuzzy Sets and Systems*, 35, 1990, pp. 197-212.

[3] J.J. Saade and H. Schwarzlander, "Application of fuzzy hypothesis testing to signal detection under uncertainty," *Fuzzy Sets and Systems*, 62, 1994, pp. 9-19.

[4] J.J. Saade and H. Schwarzlander, "Extension of fuzzy hypothesis testing with hybrid data," *Fuzzy Sets and Systems*, 63, 1994, pp. 57-71.

[5] A. Dziech and M. B. Gorzałczany, "Decision making in signal transmission problems with interval-valued fuzzy sets," *Fuzzy Sets and Systems*, Vol. 23, 1987, pp. 191-203.

[6] J. C. Son, I. Song, S. Y. Kim, and S. I. Park, "An application of the generalized Neyman–Pearson fuzzy test to stochastic-signal detection," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 23, No. 5, 1993, pp. 1474-1481.

[7] J. R. Boston, "Effects of membership function parameters on the performance of a fuzzy signal detector," *IEEE Transaction on Fuzzy Systems*, Vol. 5, No. 2, 1997, pp. 249-255.

[8] J. W. Minett, S. W. Leung, and P. W. Wong, "Detecting fixed amplitude signals in a fuzzy Gaussian Environment," *Proceedings of the 1999 IEEE International Conference on Systems, Man, and Cybernetics*, Tokyo, Japan, October 12-15, 1999.

[9] T. Terano, K. Asai, and M. Sugeno, *Fuzzy Systems Theory and Its Applications*, San Diego, CA : Academic Press, 1992.

[10] S. W. Leung, J. W. Minett, and M. K. Lee, "An application of fuzzy hypothesis testing to signal integration," *Proceedings of the 2000 IEEE International Conference on Fuzzy Systems*, San Antonio, Texas, U.S.A., May 7-10, 2000, Paper No. 107.

[11] J. Minkoff, *Signals, Noise, and Active Sensors*, New York: John Wiley & Sons, Inc., 1992.

[12] A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications*, New York: Van Nostrand Reinhold, 1985.