Imprecision Analysis of PSK Modulation by Fuzzy Logic[†]

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Abstract

The performance of a classical Phase Shift Keying (PSK) detector may be degraded by parameter imprecision. This paper analyzes the performance variation of PSK schemes due to imprecision, and models the imprecise parameters as fuzzy numbers. Results illustrate the extent of the performance variation of the PSK schemes due to parameter imprecision in analysis.

I. INTRODUCTION

In wireless communication, phase shift keying modulation schemes, such as Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK), are widely used. In general, in the design of a PSK detector, there are several system parameters that are assumed to be known precisely, e.g. the symbol average energy E_b, the carrier frequency f_c, and the symbol period T_b [1]. However, in practice, many detectors cannot reach the devised performance because the values of parameters are not known precisely [2]. For example, system parameters, such as E_b and f_c, can often not be generated stably – there is always a certain amount of imprecision. In this paper, we use a fuzzy method to model such imprecise parameters using fuzzy numbers. The use of fuzzy numbers to model the imprecision of the input system parameters allows us to have a better estimation of system performance in imprecise environments. Similar methods for modeling parameter imprecision using a fuzzy Neyman-Pearson hypothesis test have been derived for radar detection [3, 4]; in this paper, the fuzzy hypothesis test is applied to PSK modulation for imprecision analysis.

II. SUMMARY OF FUZZY NEYMAN-PEARSON HYPOTHESIS TEST

As background for this paper, we briefly summarize the structure of the Fuzzy Neyman-Pearson hypothesis test developed in [4]. In [4], this fuzzy hypothesis test was applied to binary radar detection. When no target is present (hypothesis H_0), the detector returns, z, are assumed to have probability density function (p.d.f.) $f_{Z|H_0}(z\,|\,H_0;P_x)$, which depends on a set of parameters with values $P_x=(p_{x_1},\ldots,p_{x_k})$. However, when a target is present (hypothesis H_1), the returns of the target-plus-interference are assumed to have p.d.f. $f_{Z|H_1}(z\,|\,H_1;P_y)$, where the parameters now have values $P_y=(p_{y_1},\ldots,p_{y_k})$. The likelihood ratio function, Λ_Z, is defined as

$$\Lambda_Z(z;P_x,P_y):=\frac{f_{Z|H_1}(z\,|\,H_1;P_y)}{f_{Z|H_0}(z\,|\,H_0;P_x)}.$$

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Now suppose that the parameter values are imprecise, modeled as fuzzy numbers. The probability density functions $f_{Z|H_0}(z\,|\,H_0;P_x)$ and $f_{Z|H_1}(z\,|\,H_1;P_y)$ are therefore imprecise and consequently so too is the likelihood ratio function, Λ_Z. By applying the Extension Principle for fuzzy numbers [5], we obtain the fuzzy set for Λ_Z, which has membership function

$$\mu_{\Lambda(z)}(t)=\sup_{t=\Lambda(z;P_x,P_y)}\left\{\min\begin{bmatrix}\mu_{\tilde{P}_x}(p_{x_1}),\ldots,\mu_{\tilde{P}_x}(p_{x_k}),\\ \mu_{\tilde{P}_y}(p_{Y_1}),\ldots,\mu_{\tilde{P}_y}(p_{Y_k})\end{bmatrix}\right\}. \quad (1)$$

The decision rule can therefore be written in terms of the fuzzy likelihood ratio as

$$\Lambda_Z \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \quad (2)$$

where γ is a threshold that determines the performance. To implement the above decision rule, the likelihood ratio, Λ_Z, which is fuzzy, and the threshold, γ, which is crisp, must be ordered; any convenient method for ordering fuzzy numbers can be used.

III. IMPRECISION FUZZY MODELING FOR PSK MODULATION

In this paper, we use triangular fuzzy numbers to represent imprecise parameters. Figure 1 shows an example of the membership function of a triangular fuzzy number with 20% imprecision, denoted by <1.0, 0.2>.

We develop a fuzzy maximum likelihood hypothesis test, much like the fuzzy Neyman-Pearson hypothesis test, for BPSK detection in an AGWN channel. Classical coherent PSK detectors multiply the received signal with a reference waveform. The output of the multiplier is then applied to an integrator. Finally, the output symbol energy, x, of the correlation detector

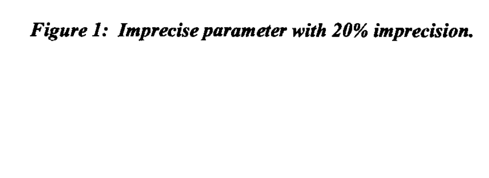


Figure 1: Imprecise parameter with 20% imprecision.

(multiplier and integrator) is applied to a decision device to produce a decision [1]. Figure 2 summarizes the correlation process for a BPSK detector.

In the ideal situation (no noise or imprecision), the detector either receives $+\sqrt{E_b}$, in which case the received symbol should be classified as bit '1' (hypothesis H_1), or receives $-\sqrt{E_b}$, in which case the received symbol should be classified as bit '0' (hypothesis H_0). In an AWGN channel model with zero mean, the p.d.f. of the random noise is a normal distribution, $N(0,\sigma^2)$. The two hypotheses can be stated as

$$H_1: \; X \sim N\left(+\sqrt{E_b},\sigma^2\right),$$
$$H_0: \; X \sim N\left(-\sqrt{E_b},\sigma^2\right).$$

The p.d.f. of the symbol energy under hypothesis H_1 is

$$f(x|H_1) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)\exp\left[-\frac{\left(x-\sqrt{E_b}\right)^2}{2\sigma^2}\right], \quad (3)$$

while p.d.f. of the symbol energy under hypothesis H_0 is

$$f(x|H_0) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)\exp\left[-\frac{\left(x+\sqrt{E_b}\right)^2}{2\sigma^2}\right], \quad (4)$$

both of which are shown in Figure 3.

However, when the symbol energy, now written $\tilde{E}_b$, and the noise variance, written $\tilde{\sigma}^2$, are imprecise, the two hypotheses to be tested become

$$H_1: \; X \sim N\left(\tilde{A},\tilde{\sigma}^2\right) \qquad \tilde{A} = \sqrt{\tilde{E}_b},$$
$$H_0: \; X \sim N\left(\tilde{B},\tilde{\sigma}^2\right) \qquad \tilde{B} = -\sqrt{\tilde{E}_b}.$$

where $\tilde{E}_b$ and $\tilde{\sigma}^2$ are the triangular fuzzy numbers. The p.d.f. of the symbol energy under hypothesis H_1 therefore becomes

$$f(x|H_1) = \left(\frac{1}{\sqrt{2\pi\tilde{\sigma}^2}}\right)\exp\left[-\frac{\left(x-\tilde{A}\right)^2}{2\tilde{\sigma}^2}\right], \quad (5)$$

while under hypothesis H_0 it becomes

$$f(x|H_0) = \left(\frac{1}{\sqrt{2\pi\tilde{\sigma}^2}}\right)\exp\left[-\frac{\left(x-\tilde{B}\right)^2}{2\tilde{\sigma}^2}\right]. \quad (6)$$

The likelihood ratio function is therefore given by

$$\Lambda(x) = \frac{f(x|H_1)}{f(x|H_0)} = \exp\left(\frac{\tilde{B}^2-\tilde{A}^2+2x(\tilde{A}-\tilde{B})}{2\tilde{\sigma}^2}\right). \quad (7)$$

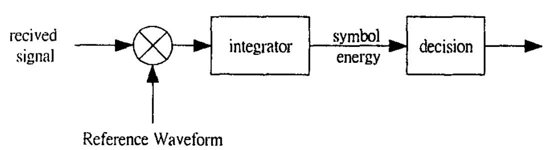


Figure 2: BPSK correlation receiver.

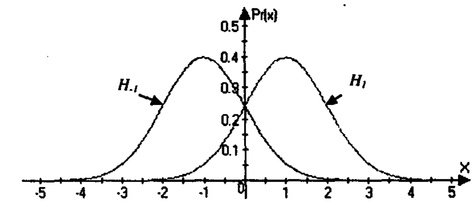


Figure 3: The p.d.f. of received symbol energy in an AWGN channel with $E_b = 1$ and $\sigma = 1$.

The received symbol is classified as bit "1" (hypothesis H_1) when $\Lambda > 1$, and as bit "0" (hypothesis H_0) when $\Lambda < 1$. When Λ intersects the threshold, 1, a fuzzy ordering system, such as that used in [3], can be used to obtain a binary decision.

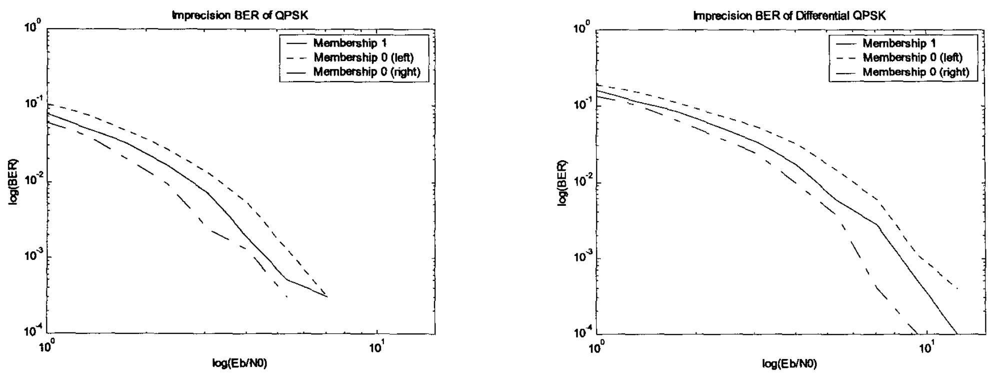
IV. Results

In QPSK detection, the modulation is divided into two independent channels — the in-phase channel and the quadra-phase channel. The resultant detector therefore uses two independent binary hypothesis tests, so the imprecision model is identical to that described for BPSK detection.

We proceed by supposing that either the received symbol energy E_b or the noise variance σ^2 are imprecise, modeled as triangular fuzzy numbers, and perform a simulation for QPSK and differential QPSK (DQPSK) modulated detection to calculate their respective response to parameter imprecision. The results of these simulations are shown in Figures 4 and 5. Figures 4(a) and 4(b) display the bit-error-rate (BER) of QPSK and DQPSK, respectively, against the signal-to-noise ratio (E_b / N_0) with 20% imprecision in the symbol energy, E_b. Figures 5(a) and 5(b) display the BER of QPSK and DQPSK, respectively, against the signal-to-noise ratio (E_b / N_0) with 20% imprecision in the noise variance, σ^2.

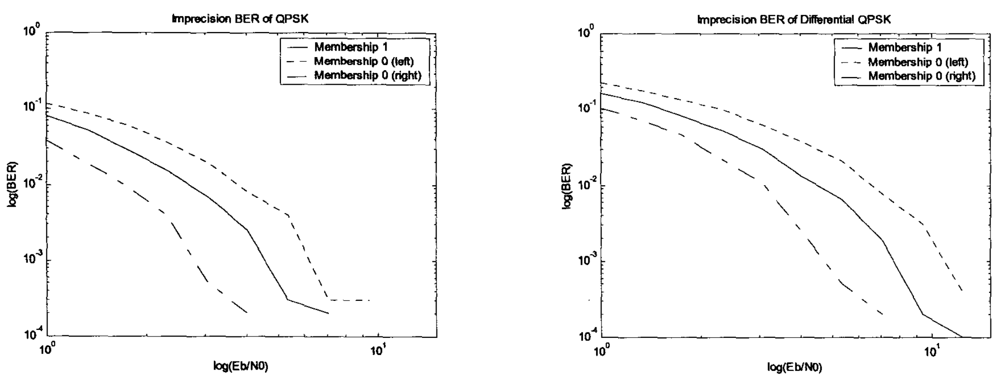
The results show the degree to which the performance of each PSK detector becomes imprecise due to the parameter imprecision. Performance imprecision becomes particularly large when the signal-to-noise ratio is high. However, the effect of parameter imprecision on QPSK is similar to that on DQPSK, implying that fluctuation of the noise channel will affect the performance of QPSK rather more than that of DQPSK.

For 20% imprecision in symbol energy, the performance imprecision, shown in Figure 4(b), approaches 300% at E_b/N_0 about 10 dB. When the noise variance imprecision reaches 20%, the performance imprecision at the same signal-to-noise ratio rises to more than 1400%, as shown in Figure 5(b). This shows that the effect of imprecision in the noise variance, σ^2, can be far greater than that of the symbol energy, E_b.



(a) (b)

Figure 4: Imprecision BER curves with 20% imprecision of E_b, (a) QPSK (b) DQPSK.



(a) (b)

Figure 5: Imprecision BER curves with 20% imprecision of σ^2, (a) QPSK (b) DQPSK.

V. CONCLUSION

A fuzzy method for modeling the effects of parameter imprecision on the performance of Phase Shift Keying modulation detection schemes has been developed and demonstrated, in particular, for QPSK and DQPSK detectors. This fuzzy method allows the effect of imprecise parameter on the performance of PSK detectors to be analyzed. Results of the analysis show that the performance imprecision of the QPSK and DQPSK detectors can be many times greater than the imprecision of the input parameters. Moreover, the results also imply that the performance of the DQPSK system is more stable than that of the QPSK system in imprecise situations, and that noise variance imprecision is potentially far more damaging to performance than symbol energy imprecision.

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