An Application of Fuzzy Hypothesis Testing to Signal Integration

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***Abstract*- A method of fuzzy likelihood-ratio hypothesis testing is applied to binary signal integration. Imprecise parameters of the distribution under each hypothesis are modeled as fuzzy numbers. The false alarm rate and detection rate are therefore also imprecise. The effects of parameter imprecision on integration performance are examined using an example; the method is applied to envelope detection and integration of fixed amplitude signals in additive white Gaussian noise. For low parameter imprecision, the integration system produces performance which is nearly crisp. However, as parameter imprecision increases significantly, the performance quickly becomes so imprecise that the system may no longer be of practical use.**

I. INTRODUCTION

Signal integration and data fusion are important processes in signal detection systems. They allow data from multiple observations or sensors to be combined to enhance performance [1]. Binary Integration [2], also known as *M*-out-of-*N* Detection, is a simple scheme for performing signal integration when one of two possible signals is transmitted. This scheme can also be used for parallel data fusion [1]. A local decision rule is applied to each observation to identify which signal is received. A target signal is detected globally when at least *M* of the *N* local decisions identify it. When used as a detection scheme only, Binary Integration is a far from optimal; however, it is very simple to implement and requires relatively little memory for digital storage [2]. It is commonly used for data fusion in decentralized detection systems [1].

In practice, the performance of a detection system is not perfect due to ignorance of various types; these include inaccuracy, vagueness and randomness [3]. Interference is usually modeled as random and handled by constructing statistical hypothesis tests, of which Neyman-Pearson and other likelihood ratio tests [4] are probably the most common. Other types of ignorance, such as parameter imprecision, may be modeled more effectively using fuzzy logic. Reference [5] presents a detailed account of statistical inference with imprecise data. In signal detection systems, however, parameter imprecision is not usually modeled. Nevertheless, models for randomness and imprecision have been combined in [6] and, more recently, in [7]. In [6], Neyman-Pearson detection was performed on a signal with imprecise amplitude, modeled as a fuzzy number, in Gaussian noise. This work was extended in [7] to detect an imprecise signal in Gaussian noise with fuzzy variance. The false alarm rate and detection rate were both found to be fuzzy.

In Section II, we adapt the approach taken in [7] to develop a system for signal integration in which imprecise parameters, such as signal strength and noise variance, are modeled as fuzzy numbers. The system allows the effects of parameter imprecision on performance to be analyzed. In Section III, we demonstrate the method and analysis using an example.

II. INTEGRATION OF SIGNALS WITH FUZZY PARAMETERS

In most signal detection applications, a target signal is sought in the presence of interference. When several observations or sensors are available, signal integration can be used to improve detection performance. The Binary Integrator, or *M*-out-of-*N* Detector, is a simple scheme with well-known structure [2] for integrating signals. Observations are quantized to 1 when exceeding a threshold or to 0 otherwise. These local decisions are then summed and again thresholded. The target signal is detected when at least *M* out of *N* available local decisions have value 1. In order to set the thresholds to achieve a prescribed performance, the density function of both the interference and the target-plus-interference should be known precisely. However, most measurable quantities are imprecise by nature. For example, the mean power of interference is not generally known but must be estimated by sampling. Even the power of a transmitted signal can be known only to a certain degree of precision and may fluctuate slightly. We will model such imprecise quantities as fuzzy numbers.

The binary hypotheses tested each observation are

H_0 : Interference,
H_1 : Target + Interference.

Under hypothesis H_0, observations consist of interference only and have probability density function $f_X(x;\boldsymbol{P}_X)$, which depends on a vector of k imprecise parameters

$$\boldsymbol{P}_X = (P_{X,1}, \ldots, P_{X,k}).$$

Under hypothesis H_1, however, observations consist of the target signal plus interference and have density $f_Y(y;\boldsymbol{P}_Y)$, where the imprecise parameters now have values

$$\boldsymbol{P}_Y = (P_{Y,1}, \ldots, P_{Y,k}).$$

Each parameter is modeled as a fuzzy number. Therefore, values of both density functions are fuzzy numbers, and are obtained by applying the extension principle for fuzzy number [8].

Given a set of N observations $\boldsymbol{z} = (z_1, \ldots, z_N)$, a local decision can be obtained using the procedure for fuzzy likelihood ratio hypothesis testing in [7]. The likelihood ratio function is defined as the ratio of the probability density functions under each hypothesis [4],

$$\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y) := \frac{f_Y(z_i;\boldsymbol{P}_Y)}{f_X(z_i;\boldsymbol{P}_X)}. \tag{1}$$

By the extension principle [8], the likelihood ratio function can be written in terms of its membership function,

$$m_{\Lambda(z_i)}(t) = \sup_{t=\Lambda(z_i)}\left\{\min\begin{bmatrix} m_{P_{X,1}}(p_{X,1}), \ldots, m_{P_{X,k}}(p_{X,k}), \\ m_{P_{Y,1}}(p_{Y,1}), \ldots, m_{P_{Y,k}}(p_{Y,k}) \end{bmatrix}\right\}, \tag{2}$$

where $m_{P_{X,1}}(p_{X,1})$ is the membership of the fuzzy parameter $P_{X,1}$ at value $p_{X,1}$. Conventionally, each observation would be quantized to either 0, representing hypothesis H_0, or 1, representing hypothesis H_1, according to a local decision rule,

$$\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y) \underset{H_0}{\overset{H_1}{\gtrless}} g\ . \tag{3}$$

However, imprecision in the values of the parameters renders the likelihood ratio fuzzy. Some form of fuzzy ordering is therefore required to compare the likelihood ratio to the crisp threshold. One method is to choose one of three possible decisions; H_0, H_1, or an uncertain decision, much like the approach taken in [6]. An alternative method explored in [7], which we use here, is to determine a membership value for the degree of preference for selecting hypothesis H_1. A fuzzy ordering membership function Ω assigns a degree of preference for ordering the likelihood ratio greater than the crisp threshold, with output

$$\Omega(\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y),g), \tag{4}$$

which we term the fuzzy threshold. Fig. 1 shows the general appearance of a conventional crisp threshold compared with a fuzzy threshold.

We integrate the signals and obtain a global decision by thresholding the sum of the local decisions, in much the same way as the Binary Integrator,

$$\textstyle\sum_i^N \Omega(\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y),g) \underset{H_0}{\overset{H_1}{\gtrless}} M\ ; \tag{5}$$

here N is the number of observations, and z_i is the value of i^{th} observation, and M is a real-valued threshold defined on the interval $[0, N]$ rather than an integer, as required by the Binary Integrator. The use of global decision rule (5) means that the global decision is crisp, even though the local decisions are not.

The false alarm rate can be defined in terms of the threshold parameters g and M as

$$P^F(g,M) := \Pr\left(\textstyle\sum_i^N \Omega(\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y),g) \geq M \,\middle|\, H_0\right) \tag{6}$$

while the detection rate is

$$P^D(g,M) := \Pr\left(\textstyle\sum_i^N \Omega(\Lambda(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y),g) \geq M \,\middle|\, H_1\right). \tag{7}$$

The false alarm and detection rates are both fuzzy quantities as they depend on the imprecise parameter vectors, $\boldsymbol{P}_X$ and $\boldsymbol{P}_Y$. As in [7], the false alarm rate is calculated by applying the extension principle to (6),

$$m_{P^F(g,M)}(t) = \sup_{t=P^F(g,M)}\left\{\min\begin{bmatrix} m_{P_{X,1}}(p_{X,1}), \ldots, m_{P_{X,k}}(p_{X,k}), \\ m_{P_{Y,1}}(p_{Y,1}), \ldots, m_{P_{Y,k}}(p_{Y,k}) \end{bmatrix}\right\}. \tag{8}$$

The detection rate is obtained from (7) similarly.

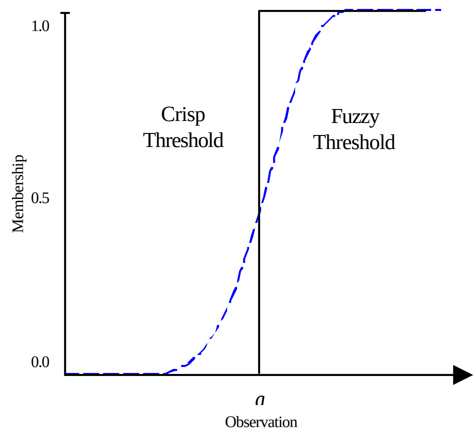

Fig. 1. Comparison of crisp and fuzzy thresholds.

III. THE EFFECTS OF PARAMETER IMPRECISION ON SIGNAL INTEGRATION

As shown in [7] and discussed earlier, parameter imprecision in detection systems causes performance measures such as the false alarm rate and detection rate to be imprecise also. We believe that as parameter imprecision rises, performance may become so imprecise that the system is of no practical use. We now test this belief by considering an example.

A common interference model used in signal detection systems is to treat the interference as additive white Gaussian noise. At the output of a band-pass filter, the envelope of such noise has Rayleigh distribution [9]. When the noise is combined with a constant amplitude target signal, the received envelope has Rice distribution [9]. The density functions of these distributions are

$$f_X\left(x;s^2\right)=\frac{x}{s^2}\exp\left(-\frac{x^2}{2s^2}\right) \tag{9}$$

for the Rayleigh noise, and

$$f_Y\left(y;m,s^2\right)=\frac{y}{s^2}\exp\left(-\frac{y^2+m^2}{2s^2}\right)I_0\left(y\frac{m}{s^2}\right) \tag{10}$$

for the Rice target-plus-noise; m is the strength of the target signal, s^2 is the Gaussian noise variance, and $I_0(\cdot)$ represents the modified Bessel function of the first kind of order zero. The parameter vectors are $\boldsymbol{P}_X=\left(0,s^2\right)$ and $\boldsymbol{P}_Y=\left(m,s^2\right)$. Inserting (9) and (10) into (1), we obtain the likelihood ratio function,

$$\Lambda\left(z_i;m,s^2\right)=\exp\left(-\frac{1}{2}\frac{m^2}{s^2}\right)I_0\left(z_i\frac{m}{s^2}\right). \tag{11}$$

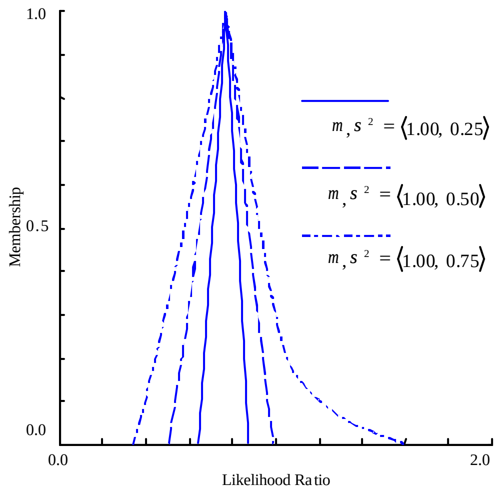

Fig. 2. Fuzzy likelihood ratio for various values of m and s^2.

Now suppose that the target signal strength, m, and the noise variance, s^2, are triangular fuzzy numbers rather than crisp numbers. The fuzzy likelihood ratio can be obtained from (2) and (11) using the numerical approaches described in [10]. Fig. 2 shows a plot of the fuzzy likelihood ratio for several combinations of m and s^2; the notation $\langle C,w\rangle$ denotes a symmetric triangular fuzzy number with support $[C-w,C+w]$. Both the signal strength and noise variance are set approximately to 1. Clearly, as the parameters become less precise, the value of the likelihood ratio also becomes less precise, particularly in the upper bound.

The fuzzy threshold $\Omega\left(\Lambda\left(z_i;\boldsymbol{P}_X,\boldsymbol{P}_Y\right),g\right)$ can also be obtained numerically after applying the extension principle to (4) and (11). Fig. 3 shows several plots of the fuzzy threshold, together with a plot of the crisp threshold of the Binary Integrator, for $g = 1.0$. The plots demonstrate how the fuzzy threshold diverges from the ideal crisp threshold. When the parameters become less precise, the fuzzy threshold generates non-crisp output over a wider interval. Observations from systems with a high degree of parameter imprecision tend to produce a relatively large number of non-crisp local decisions.

We now examine the effects of parameter imprecision on performance. We estimate both the false alarm rate and detection rate by Monte-Carlo simulation. Fig. 4 shows the false alarm rate of the Fuzzy Integrator for the same values of m and s^2 with crisp threshold $g = 1.0$. The false alarm rate of the Binary Integrator with the same crisp threshold is shown for comparison. We now examine the alpha cuts of the false alarm rate. For $m,s^2=\langle 1.00,0.25\rangle$, the false alarm rate has support roughly $[10^{-2}, 10^{-1}]$, while at membership grade 0.5 the alpha cut is about $[2 \times 10^{-2}, 10^{-1}]$. This demonstrates that at this level of imprecision the false alarm rate is accurate only to about one order of magnitude. As the parameters become less precise, the precision in the false alarm rate

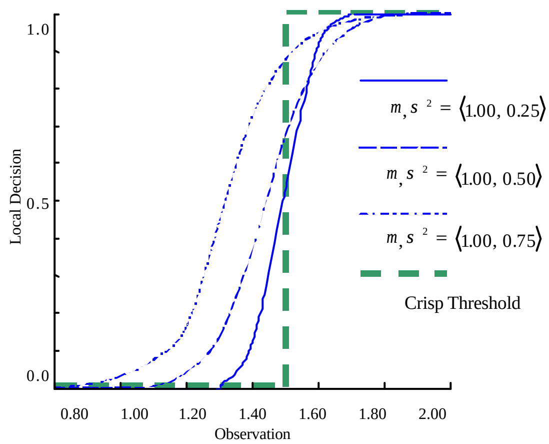

Fig. 3. Fuzzy threshold for various values of m and s^2.

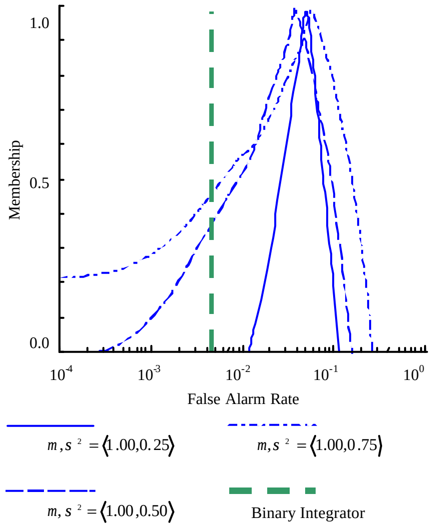

Fig. 4. False alarm rate of fuzzy integrator for various values of m and s^2.

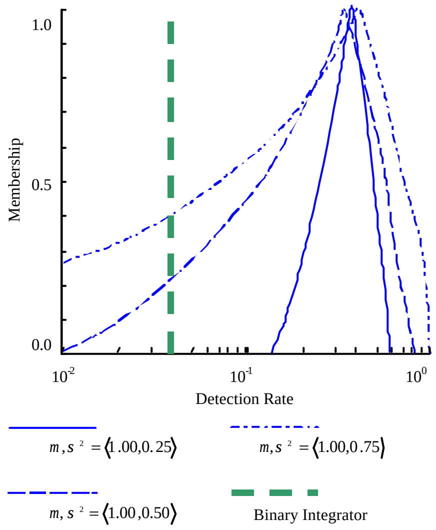

Fig. 5. Detection rate of fuzzy integrator for various values of m and s^2.

drops even further. For $m, s^2 = \langle 1.00, 0.75 \rangle$, the false alarm rate is so imprecise that both 10^{-1} nor 10^{-4} are permissible values.

Fig. 5 shows that the detection rate is effected similarly. Indeed, for $m, s^2 = \langle 1.00, 0.75 \rangle$, the support of the detection rate is practically the entire unit interval. We cannot say with any certainty that the detection rate lies within any sub-interval of [0,1]. Even for $m, s^2 = \langle 1.00, 0.50 \rangle$, the support of the detection rate is about [0.01, 0.80], which may be too vague to be useful. Without nearly precise performance measures, signal integration is of little practical use.

IV. Discussion

We have demonstrated a method for signal integration based on fuzzy hypothesis testing to analyze the effects of parameter imprecision. Having applied the method to envelope detection of a fixed amplitude signal in additive white Gaussian noise, we observe that performance imprecision parameter can be many times greater than parameter imprecision. This amplification of imprecision may lead to performance which is so vague that the integration system is of little use. This behavior highlights the importance of both maintaining high precision when measuring physical quantities and accounting for imprecision in the signal integration process. We are continuing to investigate ways to limit the impact of parameter imprecision on integration performance.

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