

# SIGNAL DETECTION USING FUZZY MEMBERSHIP FUNCTIONS

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## Abstract

*It has recently been investigated that Fuzzy logic provides certain advantages over Binary logic in the formulation of signal detection algorithms. In this paper, the classic M-out-of-N Detector, used in radar detection, is adapted using fuzzy membership functions to replace the binary characteristic functions, first proposed in [1]. Applying membership functions to observations of a signal induces two new spaces, the Fuzzy False Alarm Space and the Fuzzy Detection Space. These spaces allow probability to be visualised in terms of hyper-volume, allowing easy comparison of detection algorithms.*

## 1. Introduction

Despite the existence of optimal and locally optimal signal detection algorithms, it is advantageous to develop less complex detection schemes, more easily and cheaply implemented. The  $M$ -out-of- $N$  Detector, otherwise known as the Binary Integrator, see [2,3], is such a signal detector, relying on very simple, intuitively appealing statistical concepts. It processes  $N$  independent, identically distributed observations, or bins, to determine the presence of a signal. Each bin is thresholded to produce a binary output in which typically  $\{0\}$  represents the hypothesis "No Signal" and  $\{1\}$  the hypothesis "Signal". The output from all  $N$  bins is then integrated and thresholded to give an aggregate decision. For example, a 3-out-of-5 Detector would require that the output from at least 3 of the 5 bins exceed some pre-determined threshold.

In this paper we propose the use of fuzzy membership functions [4] to represent the degree to which a bin is indicative of the conclusions "No Signal" and "Signal". This inevitably leads to the possibility of a Fuzzy  $M$ -out-of- $N$  Detector, or Fuzzy Integrator, for which  $M$  need not be an integer, e.g. 3.6-out-of-5. Such a detector has significantly better performance than the Binary Integrator, and we now discuss its implementation.

## 2. The Fuzzy Integrator

The basic signal detection problem is to test the two hypotheses

$H_0$ : "No Signal"

$H_1$ : "Signal"

based on a set of  $n$  observations,  $S := \{Y_1, \dots, Y_n\}$ . The Binary Integrator defines a crisp characteristic function,  $f: [-\infty, \infty] \rightarrow \{0, 1\}$ , describing the degree to which an observation suggests a "Signal". However, the Fuzzy Integrator defines a fuzzy membership function,  $m: [-\infty, \infty] \rightarrow [0, 1]$ , so retaining more information.

In the case of a fixed signal corrupted by thermal noise, we may write the hypothesis test as

$$\begin{aligned} H_0 : & Y_i \sim N(0, s^2) \\ H_1 : & Y_i \sim N(m, s^2) \end{aligned}$$

where  $Y_i$  is an observation,  $m$  is the signal strength being detected, and  $N(I, s^2)$  represents white Gaussian noise with mean  $I$  and power/variance  $s^2$  [5,6]. We choose the membership function

$$\mathbf{m}(Y_i) := \Phi\left(\frac{Y_i}{s}\right) \equiv \Pr(Z \leq Y_i | Z \sim N(0, s^2)),$$

where  $\Phi$  is the cumulative distribution function of the standard Gaussian distribution. This definition ensures that membership values are distributed uniformly on  $[0,1]$  under  $H_0$ . The Fuzzy Integrator therefore declares a "Signal" only if the integrated output exceeds some threshold,  $K$ ,

$$\text{i.e. only if } \sum_{i=1}^N \mathbf{m}(y_i) \geq K.$$

Comparing the performance of the Fuzzy Integrator with the Binary Integrator, Table 1, we observe a significant gain, increasing as more bins are integrated.

**Table 1. Performance Gain of the Fuzzy Integrator**  
False Alarm Rate =  $10^{-6}$

No. Bins ( $N$ )	Detection Rate (%)	Binary Integrator (dB)	Fuzzy Integrator (dB)	GAIN (dB)
3	95	10.0	9.5	0.5
	90	9.3	8.9	0.4
5	95	8.7	7.6	1.1
	90	8.0	6.9	1.1
7	95	8.0	6.1	1.9
	90	7.3	5.5	1.8
9	95	7.4	4.7	2.7
	90	6.7	4.2	2.5

### 3. The Fuzzy CFAR Detector

An alternative description of the performance of the Binary Integrator can be given in terms of false alarm rate. For each bin, output  $\{1\}$  is only produced if the false alarm rate associated with that bin is smaller than some pre-determined threshold,  $P_{fa}$ . By definition, this bin false alarm rate is given by

$$\begin{aligned} \mathbf{m}(Y_i) &:= \Pr(Z \geq Y_i | H_0), \\ &= 1 - \mathbf{m}(Y_i), \end{aligned}$$

and  $\{1\}$  is output when  $\mathbf{m}(Y_i) \leq P_{fa}$ . The Binary Integrator declares a "Signal" only if at least  $M$  of the  $N$  bin false alarm rates are no greater than the threshold,  $P_{fa}$ .

The Fuzzy CFAR detector characterizes each bin by its associated false alarm rate, combining them to calculate an aggregate false alarm rate given by the product of the bin false alarm rates, i.e. the statistic

$$\Theta := \prod_{i=1}^N \mathbf{m}(Y_i).$$

A "Signal" is declared when  $\Theta$  is smaller than some false alarm parameter, which we denote by  $\alpha$ .

The function  $\mathbf{m}$  maps the Observation Space,  $\hat{A}^N$ , to the Fuzzy False Alarm Space,  $\hat{A}^N := [0,1]^N$ . Since  $\mathbf{m}$  is simply a rescaling of  $\mathbf{m}$  membership values,  $\mathbf{m}(y_i)$ , are distributed uniformly on  $[0,1]$  under  $H_0$ . The false alarm rate of the algorithm,  $P^{FA}$ , can thus be found graphically by calculating the hyper-volume of the region  $\Theta \leq \alpha$  |  $H_0$  in  $\hat{A}^N$ , termed the false alarm region. The threshold,  $\Theta = \alpha$ , becomes an  $N-1$  dimensional subspace of  $\hat{A}^N$ . An analytical solution for 2 observations is

$$\begin{aligned} P^{FA} &:= \Pr(\Theta \leq \mathbf{a} \mid H_0), \\ &= \int_0^{\mathbf{a}} f_{\mathbf{m}(y_2) \mid y_2 \sim \mathcal{N}(0, \mathbf{s}^2)}(\mathbf{l}) d\mathbf{l} + \int_{\mathbf{a}}^1 \Pr(\mathbf{m}(y_1) \leq \mathbf{a}/\mathbf{l} \mid y_1 \sim \mathcal{N}(0, \mathbf{s}^2)) f_{\mathbf{m}(y_2) \mid y_2 \sim \mathcal{N}(0, \mathbf{s}^2)}(\mathbf{l}) d\mathbf{l}, \\ &= \int_0^{\mathbf{a}} d\mathbf{l} + \int_{\mathbf{a}}^1 \left(\frac{\mathbf{a}}{\mathbf{l}}\right) d\mathbf{l}, \\ &= \mathbf{a} [1 - \ln \mathbf{a}]. \end{aligned}$$

where  $f$  represents the conditional density function. For the general  $N$ -bin algorithm, the false alarm rate is found by equating coefficients of powers of  $\ln \alpha$ , and is given by

$$P^{FA} = \mathbf{a} \sum_{i=0}^{N-1} (-1)^i \frac{(\ln \mathbf{a})^i}{i!}.$$

A second membership function,  $\mathbf{m}^D$ , describing the degree to which an observation suggests a "Signal", is defined by

$$\mathbf{m}^D(Y_i) := \Phi\left(\frac{Y_i - m}{s}\right) \equiv \Pr(Z \leq Y_i \mid Z \sim \mathcal{N}(m, \mathbf{s}^2)).$$

This membership function,  $\mathbf{m}^D$ , is a map from the Observation Space,  $\hat{A}^N$ , to a Fuzzy Detection Space,  $D^N := [0,1]^N$ . Membership values are distributed uniformly on  $[0,1]$  under  $H_1$ . Hence the detection rate of the algorithm,  $P^D$ , can be found graphically by calculating the hyper-volume of the region  $T \leq \alpha$  |  $H_1$  in  $D^N$ , termed the detection region. However, there is no analytical solution for the detection rate, which must therefore be calculated numerically. Table 2 shows a comparison of the performance of the Binary Integrator and the Fuzzy Detector.

**Table 2. Comparison of the Binary Integrator and Fuzzy CFAR Detector**  
Signal-to-Noise Ratio Required for Detection (dB)  
Two-Bin Scheme

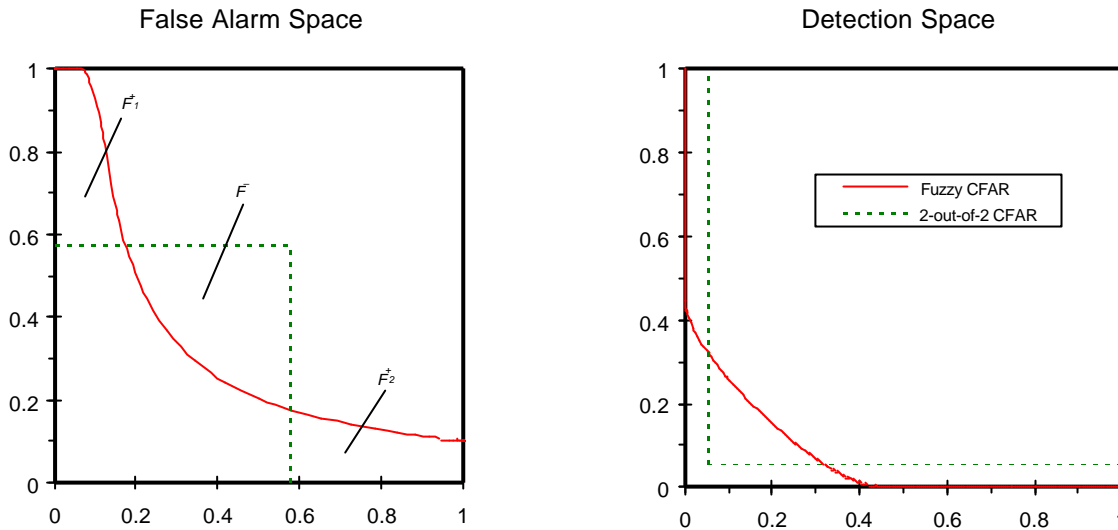
False Alarm Rate	Detection Rate (%)	Binary Integrator (dB)	Fuzzy CFAR Detector (dB)	GAIN (dB)
$10^{-4}$	99	10.8	9.8	1.0
	90	8.9	8.1	0.8
$10^{-6}$	99	12.1	11.1	1.0
	90	10.5	9.7	0.8
$10^{-8}$	99	13.0	12.1	0.9
	90	11.6	10.9	0.7

#### 4. Using Fuzzy Spaces

A sample plot of the False Alarm Space and Detection Space for the Binary Integrator and Fuzzy CFAR Detector is shown in Figure 1. Here, a two-bin scheme is considered at a signal-to-noise ratio of 0 dB with 33% false alarm rate.

Although the false alarm rates of the two detectors shown are equal, the false alarm regions are quite different. It is precisely this difference that accounts for the improved detection rate of the Fuzzy CFAR Detector; the detection rate associated with regions  $F_1^+$  &  $F_2^+$  exceeds that associated with region  $F^-$ .

**Figure 1. Fuzzy Spaces for the Binary Integrator and Fuzzy CFAR Detector**  
(0 dB, 33% false alarm rate)



## 5. Conclusion

This paper has discussed two new signal detectors, applying the principle of fuzzy membership functions. Both detectors have significant gain over the Binary Integrator on which they are based, with the performance of the Fuzzy CFAR Detector comparable to that of the optimal Neyman-Pearson detector [5,6]. The fuzzy spaces induced by the choice of membership function, chosen so as to map the hypothesised noise distribution to a uniform distribution, allow simple graphical comparison of detectors, demonstrating the advantageous application of fuzzy logic to signal detection.

## References

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