CFAR Data Fusion Using Fuzzy Integration

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Abstract

This paper presents a new approach to Constant False Alarm Rate (CFAR) data fusion using Fuzzy Integration. The paper describes how any CFAR scheme may be implemented as part of a fuzzy data fusion scheme by choosing an appropriate membership function to represent the CFAR threshold. Once the threshold membership function of the Fuzzy Integrator has been set up, the false alarm rate of the scheme is independent of fluctuations in interference mean power and depends only on the number of signals integrated by the data fusion unit and the required false alarm rate.

1. Introduction

In the majority of signal detection applications, the primary factor limiting detection accuracy is interference [1]. Interference cannot be represented analytically; rather, it is modelled as a random process. Many signal processing algorithms exist, e.g., maximum-likelihood [2], ordered statistic [3] and cell-averaging [4] detectors, which improve signal-to-interference by taking advantage of various aspects of the orderliness of valid signals. However, a simple concept used for improving signal-to-interference is that of data fusion [5,6]. Data fusion is the process of observing a signal more than once and combining the information gleaned from each observation. It is frequently used in conjunction with CFAR detection schemes. This paper describes CFAR data fusion using fuzzy integration.

The standard method of data fusion is Binary Integration [7]. It was first used as a complete signal detection algorithm, namely the M-out-of-N Detector, and later used in conjunction with other more efficient detection schemes as a method for data fusion. The Binary Integrator can nevertheless be improved by applying fuzzy thresholds [8,9] instead of binary thresholds. A fuzzy threshold is simply a membership function [10] which acts as a measure of possibility that some measurement, in this case some received signal strength, exceeds a conventional threshold. Output is therefore not discretized and so retains more of the input information. This enhances detection performance, with no additional knowledge of the signal or interference required.

In this paper, we develop a rationale for CFAR implementation of the Fuzzy Integrator for the two state problem,

H_0: "No Signal"
H_1: "Signal" ,

in which signals are corrupted by interference of known form, but with some parameter of the distribution unknown.

Data fusion by fuzzy integration is a two-threshold process, as shown in Figure 1 for CFAR detection. First, each input signal is rescaled in terms of an estimate of the unknown interference parameter based on the chosen selection logic of the CFAR scheme. The Fuzzy Integrator differs from the Binary Integrator only in the method by which the first threshold is implemented. In the Binary Integrator, each normalised signal observation is compared to a crisp threshold to produce binary partial decision output, 0 or 1 representing hypotheses H_0 and H_1 respectively. Fuzzy integration replaces the binary threshold with a fuzzy threshold, shown in Figure 2. The logic of this threshold produces partial decision output on the closed interval [0,1], a superset of the discrete output of the binary threshold process; this represents membership to hypothesis H_1, "Signal". The fuzzy threshold, a fuzzy membership function, should be chosen so as to reasonably reflect the possibility of any particular observation representing a valid signal.

After CFAR processing, the single-look partial decision outputs are sent to the Data Fusion Unit where they are integrated. In this paper, we use a simple additive scheme for data fusion; the n partial decision outputs are summed and the integrated output is then compared to a second, conventional threshold to provide an overall detection decision.

2. Data fusion and the fuzzy threshold

The Fuzzy Integrator CFAR Processor is shown in Figure 3. The CFAR Processor accepts n sliding window [7] inputs, each consisting of a Signal Observation bin, y_i, and m Interference Observation bins, e_{ij} ($j = 1, 2, \ldots m$). The signal bin is assumed to consist of either signal-plus-interference, or interference only. The interference bins are assumed to consist of interference only, i.e. there are assumed to be no interfering target signals present in the window in the interference bins. Furthermore, the interference is assumed to be stationary within the window. Standard CFAR procedures call for the interference in the signal bin to be estimated, $\hat{n}_i$, from the interference bin returns. Many such procedures exist with various types of selection logic for interference estimation, e.g. maximum likelihood CFAR [2], ordered-statistic CFAR [3], and cell-averaging CFAR [4]. The signal bin return is normalised, $z_i := y_i / \hat{n}_i$, and then thresholded to generate the i'th signal partial decision.

In the Fuzzy Integrator, the threshold is specified by a fuzzy membership function, m. It is selected to depend on the type of interference hypothesised. In this paper, we define the threshold membership function to be the interference distribution function of the normalised input, z_i,

$$m(z_i) \ := \ F_I(z_i) . \tag{1}$$

This function is monotonic increasing and so is consistent with the form specified in Figure 2. Significantly, this membership function produces uniform partial decision output on [0,1], i.e.

$$\Pr\big(m(z_i) \geq x\big) = 1 - x , \qquad x \in [0,1] .$$

The practical benefit of this property is great; having matched the membership function to the type of interference, each of the n inputs to the Data Fusion Unit is identically and uniformly distributed. Hence, the output from the Data Fusion Unit is independent of the interference, and is distributed as the sum of n uniform distributions (summation of uniform random variables corresponds to convolution of uniform density functions [11]) in the range $[0,n]$. The overall detection decision is based on a binary comparison of this output to a second, conventional threshold, *TH*. Clearly, there exists a one-to-one function between this threshold and the false alarm rate. Hence, for digital implementation, a single function or look-up table is all that is required to determine the threshold which produces any particular choice of false alarm rate for all possible interference types. The false alarm rate of the whole system depends exclusively on the number of signal observations integrated, n, and the second threshold, *TH*.

It is important to note that the Data Fusion Unit output for Fuzzy Integration is in general not an integer. So although it does not make sense to talk of, say, *7.2-out-of-8* detection for Binary Integration, it makes perfect sense to do so for Fuzzy Integration. Indeed, it is precisely this expansion, from $\{0,1\}^n$ to $[0,1]^n$, of the partial decision space and the definition of the membership function in Eqn. (1) that give rise to the desirable characteristics of the Fuzzy Integrator.

3. Maximum-likelihood CFAR data fusion in Weibull interference

Consider the case of detecting a non-fluctuating signal corrupted by Weibull interference [12] with known shape parameter. That is, hypotheses H_0 and H_1 are given by

H_0: $\quad Y_i \sim \mathrm{W}(p,n_i)$

H_1: $\quad Y_i \sim s + \mathrm{W}(p, n_i)$

and interference is given by

$$e_{ij} \sim W(p,n_i) ,$$

where Y_i is the i'th signal bin random variable with observed value y_i, $\mathrm{W}(p,n_i)$ denotes a Weibull random variable with shape parameter p, scale parameter n_i, and distribution function

$$F_{\mathrm{W}(p,n)}(x; p,n) := 1 - \exp\left(-x^p / n_i{}^p\right) ,$$

and s denotes a non-fluctuating D.C. signal.

The maximum-likelihood estimate of n_i, $\hat{n}_i$, given in [13], is

$$\hat{n}_i = \sqrt[p]{\frac{\sum_j^m e_{ij}{}^p}{m}} .$$

Noting from [13] that

$$\left\{W(p,n_i)\right\}^p \equiv W(1,n_i{}^p) := \mathrm{Exp}(n_i{}^p) ,$$

which is exponential, with density function

$$f_{\mathrm{Exp}(k)}(x;k) := \tfrac{1}{k} \exp(-x/k) \ \text{ on } [0,\infty],$$

and from [11] that

$$\sum_j^m \mathrm{Exp}(n_i{}^p) \sim \mathrm{G}(m,n_i{}^p)$$

where $\mathrm{G}(m, n_i^p)$ denotes a Gamma random variable, with density function

$$f_{\mathrm{G}(m,k)}(x;m,k) := \frac{k^{-m}}{\Gamma(m)} x^{m-1} \exp(-x/k) \ \text{ on } [0,\infty],$$

we find that

$$m\hat{n}_i^p \sim \mathrm{G}(m,n_i{}^p) . \tag{2}$$

Therefore, the distribution function of $\hat{n}_i$ is

$$F_{\hat{n}_i}(x) = F_{\mathrm{G}(m,n_i{}^p)}(mx^p) \tag{3}$$

and hence the density function of $\hat{n}_i$ is

$$f_{\hat{n}_i}(x) = f_{G(m,n_i^p)}(mx^p)\left[mx^p\right]' ,$$
$$= \frac{p}{x\,\Gamma(m)}\left[m\left(\frac{x}{n_i}\right)^p\right]^m \exp\left[-m\left(\frac{x}{n_i}\right)^p\right]. \qquad (4)$$

The normalised signal bin return, Z_i, has observed value $z_i := y_i / \hat{n}_i$. Calculating the joint density function of Z_i and $\hat{n}_i$ gives

$$f_{z_i \hat{n}_i}(z_i, \hat{n}_i) = \hat{n}_i \bullet f_{\hat{n}_i}(\hat{n}_i) \bullet f_{z_i}(\hat{n}_i z_i),$$
$$= \hat{n}_i \frac{p}{\hat{n}_i \Gamma(m)}\left[m\left(\frac{\hat{n}_i}{n_i}\right)^p\right]^m \exp\left[-m\left(\frac{\hat{n}_i}{n_i}\right)^p\right]$$
$$\bullet \frac{p(\hat{n}_i z_i)^{p-1}}{n_i} \exp\left[-\frac{\hat{n}_i}{n_i} z_i\right]^p ,$$
$$= \frac{p^2}{\Gamma(m)} \frac{z_i^{\,p-1}}{n_i^{\,p}} \left(\frac{m}{n_i^{\,p}}\right)^m \hat{n}_i^{\,P(M+1)-1}$$
$$\bullet \exp\left(-\frac{m+z_i^{\,p}}{n_i^{\,n}} \hat{n}_i^{\,p}\right). \qquad (5)$$

We now obtain the membership function for Weibull interference:

$$m(z_i; m, p) := F_{Z_i}(z_i),$$
$$= \int_0^{z_i} dz \int_0^{\infty} d\hat{n}_i \, f_{Z_i \hat{n}_i}(z, \hat{n}_i),$$
$$= \int_0^{z_i} \frac{p^2}{\Gamma(m)} \left(\frac{m}{n_i^{\,p}}\right)^m \frac{z^{\,p-1}}{n_i^{\,p}} \bullet$$
$$\int_0^{\infty} \hat{n}_i^{\,p(m+1)-1} \exp\left[-\frac{m+z_i^{\,p}}{n_i^{\,p}} \hat{n}_i^{\,p}\right] d\hat{n}_i \, dz ,$$
$$= \int_0^{z_i} \frac{p^2}{\Gamma(m)} \left(\frac{m}{n_i^{\,p}}\right)^m \frac{z^{\,p-1}}{n_i^{\,p}} \bullet$$
$$\frac{\Gamma(m+1)}{p} \left(\frac{m+z^{\,p}}{n_i^{\,p}}\right)^{-(m+1)} dz ,$$
$$= \int_0^{z_i} p \left(\frac{m}{m+z^{\,p}}\right)^{m+1} z^{\,p-1} \, dz ,$$
$$= -\left[\frac{m}{m+z^{\,p}}\right]^m \Bigg|_0^{z_i} ,$$
$$= 1 - \left(\frac{m}{m+z_i^{\,p}}\right)^m . \qquad (6)$$

Figure 4 shows a graph of the membership function for $p = 1, 2, 3, 4$ and $m = 8$. The case $p = 1$ corresponds to Exponential interference

$$m(z_i; m, 1) = 1 - \left(\frac{m}{m+z_i}\right)^m , \qquad (7)$$

while $p = 2$ corresponds to Rayleigh interference

$$m(z_i; m, 2) = 1 - \left(\frac{m}{m+z_i^{\,2}}\right)^m . \qquad (8)$$

Table 1 shows the procedure for fuzzy data fusion of 8 hypothetical interference-only inputs with $p = 2$, $n_i = 0.5$ ($i = 1, 2, \ldots 8$).

Input	y_i	$\hat{n}_i$	z_i	$m(z_i)$
1	0.8073	0.5706	1.4148	0.832
2	0.4159	0.3629	1.1460	0.704
3	0.4891	0.7483	0.6536	0.340
4	0.5664	0.4722	1.1995	0.734
5	0.1501	0.4373	0.3432	0.110
6	0.3985	0.4794	0.8312	0.485
7	0.4379	0.5267	0.8314	0.485
8	0.1233	0.5408	0.2280	0.050
OUTPUT	–	–	–	3.740

Table 1. Fuzzy data fusion for 8 inputs (interference-only), $s = 0$, $p = 2$, $n_i = 0.5$.

The output from the data fusion unit is 3.740, so setting the second threshold to 3.740 or less will produce a false alarm, while setting the threshold to more than 3.740 will produce a correct “No Signal” decision.

Table 2 shows the procedure for fuzzy data fusion of 8 hypothetical signal-plus-interference inputs with D.C. signal $s = 1$, $p = 2$, $n_i = 0.5$ ($i = 1, 2, \ldots 8$).

Input	y_i	$\hat{n}_i$	z_i	$m(z_i)$
1	1.1609	0.4166	2.7866	0.996
2	1.4616	0.7724	1.8923	0.948
3	1.6985	0.4034	4.2105	1.000
4	1.1161	0.5964	1.8714	0.945
5	1.4719	0.4024	3.6578	1.000
6	1.7126	0.6013	2.8482	0.996
7	1.1742	0.4809	2.4417	0.988
8	1.7720	0.5008	3.5383	0.999
OUTPUT	–	–	–	7.872

Table 2. Fuzzy data fusion for 8 inputs (signal-plus-interference), $s = 1$, $p = 2$, $n_i = 0.5$.

The output from the data fusion unit is 7.872, so setting the second threshold to 7.872 or less will produce a correct “Signal” decision, while setting the threshold to more than 7.872 will produce a missed detection.

4. Conclusion

This paper has described a method for applying fuzzy signal integration techniques to CFAR data fusion.

Derivation of the fuzzy threshold has been demonstrated by example.

CFAR data fusion with Fuzzy Integration may be implemented under the following conditions:

1. the type of interference is known, or adequately modelled, *a priori*;
2. the normalised interference distribution function can be found;
3. the interference is stationary within the window interference bins (interference parameter may vary from input to input).

In short, Fuzzy Integration is valid whenever the threshold membership function, defined in Eqn. (1), can be found. In addition to offering efficient detection performance, the Fuzzy Integrator allows simple False Alarm control independent of interference.

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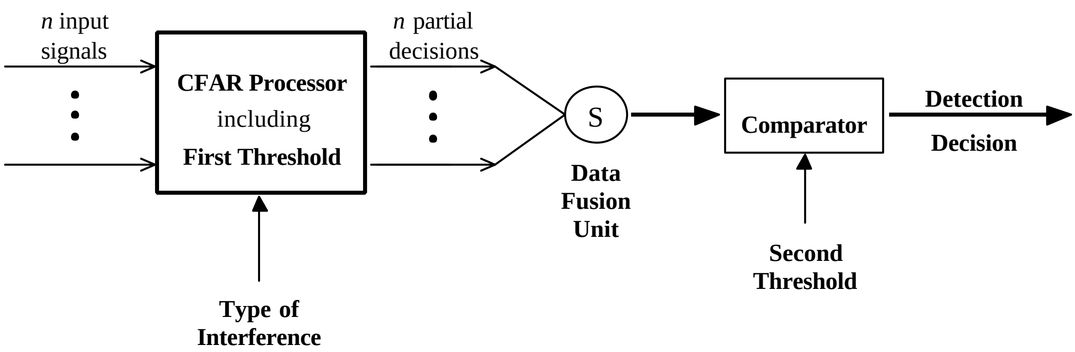

Figure 1 Standard CFAR processor with data fusion unit

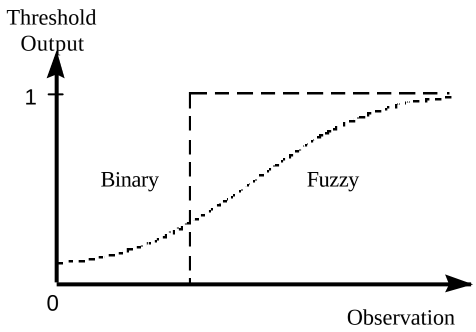

Figure 2 Comparison of binary & fuzzy thresholds

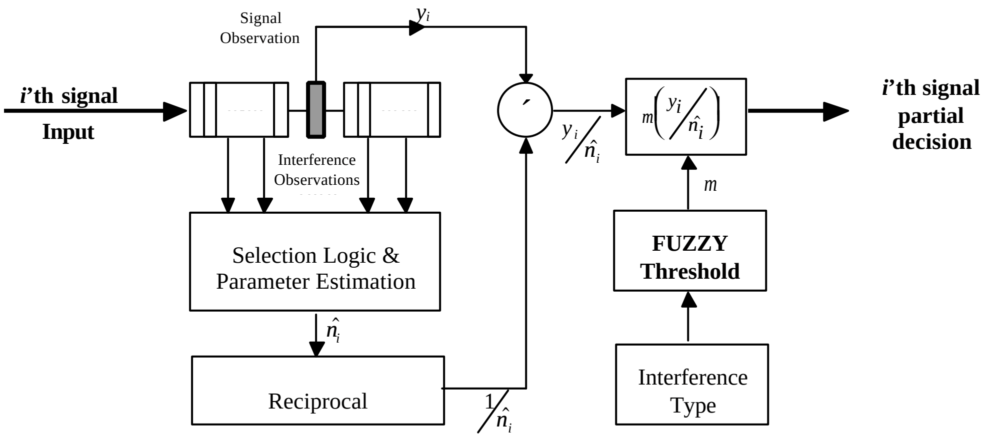

Figure 3 Block diagram of fuzzy integrator CFAR processor

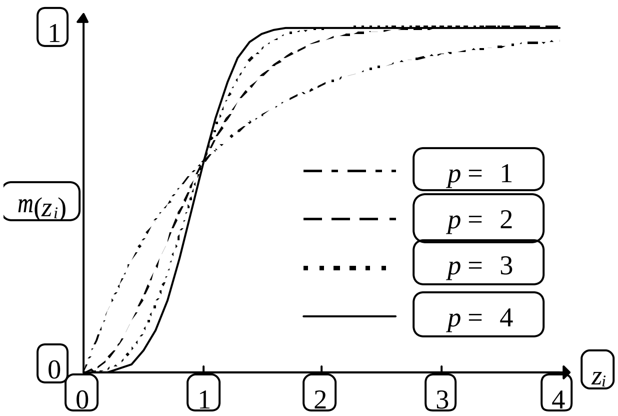

Figure 4 Plot of fuzzy threshold for Weibull interference, (interference-only), *s* = 0, *p* = 1, 2, 3, 4.