ERG 2012B
Advanced Engineering Mathematics II

Part IV
Introduction to Probability & Statistics

Lectures #21
Probability & Statistics Basics

Probability & Statistics

An experiment is a process by which a measurement or observation is obtained. A single performance of an experiment is called a **trial.**

Examples are:

- Inspecting a light bulb to see if it defective or not
- Rolling a dice and observing what number appears
- Making a measurement of daily rainfall
- Measuring the tensile strength of steel wire
- Randomly selecting a person and asking whether he or she likes a new car model

Thus *experiment*s used in a very broad sense. We are interested here in experiments that involve **randomness** – chance effects – so that we cannot predict the results exactly.

The result is then called an **outcome** or **sample point** and the set of all outcomes is the **sample space** *S* of the experiment

Probability & Statistics

In our examples:

- Inspecting a light bulb to see if it defective or not
  - $S = \{D,N\}$ where D = defective N = nondefective
- Rolling a dice and observing what number appears
  - $S = \{1,2,3,4,5,6\}$
- Making a measurement of daily rainfall
  - S the nonnegative numbers in some interval $0 \leq x \leq K$
- Measuring the tensile strength of steel wire
  - S the numbers in some interval $a \leq x \leq b$
- Randomly selecting a person and asking whether he or she likes a new car model
  - $S = \{L,D,U\}$, L = like, D = dislike, U = undecided

The subsets of S are called **events** and the outcomes **simple events**

In 2nd example events are {1,3,5} {2,4,6) {5,6} simple events are {1}, {2}, {3}, {4}, {5}, {6}.

Subsets, Unions & Intersections

Subsets

If A is a subset of B – written $A \subseteq B$ – means that all points of A are also points of B. If A happens then B also happens.

Unions, Intersections, Complements

From events A, B, C,.... of a given sample space S we can derive further events for practical or theoretical reasons as follows

The **union** $A \cup B$ of A and B consists of all points in A or B or both

The **intersection** $A \cap B$ of A and B consists of all points that are both in A and B

If A and B have no common points [as $A=\{1,3,5\}$ & $B=\{2,4,6\}$] we call them **mutually exclusive** because if one event happens the other will not and $A \cap B = \emptyset$ where Ø is the *empty set*

Subsets, Unions & Intersections

The **complement** A^c of an event A consists of all the points of S **not** in A. If $S = \{1,2,3,4,5,6\}$, then the complement of $A = \{1,3,5\}$ is $A^c = \{2,4,6\}$

Note: an event and its complement are *always* mutually exclusive and their union is the whole space S

$$A \cap A^c = \emptyset, \quad A \cup A^c = S$$

The union and intersection of multiple events are defined similarly so that:

$$\bigcup_{j=1}^{m} A_j = A_1 \cup A_2 \cup \cdots \cup A_m$$

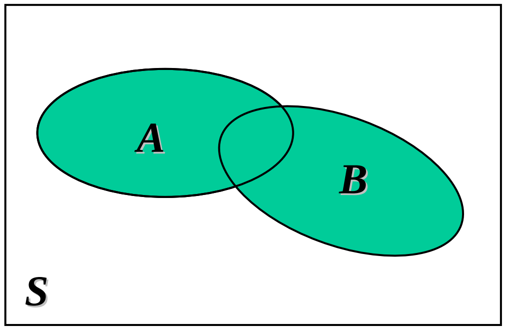
consists of all points in at least one A_j and the intersection

$$\bigcap_{j=1}^{m} A_j = A_1 \cap A_2 \cap \cdots \cap A_m$$

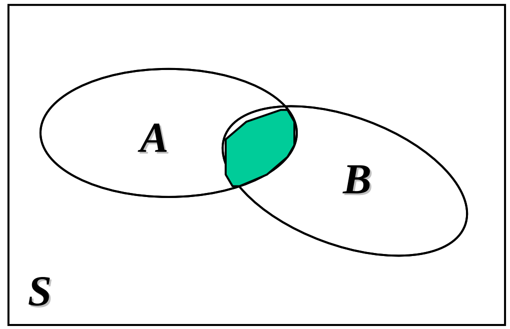
consists of points of S that are common to all all events A_j

Venn Diagrams

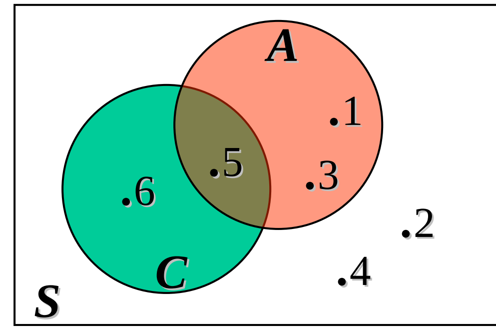
Venn Diagrams are graphical representations of events in a sample space and are useful in working with events



Union $A \cup B$



Intersection $A \cap B$



$A=\{1,3,5\}$, $C=\{5,6\}$, $S=\{1,2,3,4,5,6\}$ shows $A \cup C$ and $A \cap C$

Unions and Intersections of 3 events

Example: In rolling a dice, consider the events:

A: Number greater than 3. B: Number less than 6. C: Even number

Then $A \cap B = \{4,5\}$, $B \cap C = \{2,4\}$, $C \cap A = \{4,6\}$,

$A \cap B \cap C = \{4\}$

$A \cup B = S$ hence $A \cup B \cup C = S$ - why?

Probability

Definition: If the sample space S of an experiment consists of finitely many outcomes (points) that are equally likely, then the probability $P(A)$ of and event A is:

$$P(A) = \frac{\text{Number of points in } A}{\text{Number of points in } S}$$

Thus, in particular, $P(S) = 1$

Example: In rolling a fair dice, what is the probability $P(A)$ of A of obtaining at least a 5? The probability of B: *Even number*?

Solution: The six outcomes are equally likely, so that each has probability 1/6.

Thus $P(A) = 2/6 = 1/3$ because $A = \{5,6\}$ has 2 points.

Thus $P(B) = 3/6 = 1/2$

Probability

Definition: Given a sample space S with each event A of S (subset of S) there is associated a number P(A), called the **probability of A** such that the following axioms hold:

1. For every A in S

$$0 \leq P(A) \leq 1$$

2. The entire sample space S has the probability

$$P(S) = 1$$

3. For mutually exclusive events A and B

$$P(A \cup B) = P(A) + P(B) \qquad (A \cap B = \emptyset)$$

or more generally for mutually exclusive events $A_1, A_2, \ldots$

$$P(A_1 \cup A_2 \cup \ldots..) = P(A_1) + P(A_2) + \ldots\ldots.$$

Conditional Probability

Often it is required to find the probability of an event B under the condition that an event A occurs. This probability is called the **conditional probability of *B* given *A*** and is denoted $P(B|A)$. In this case A serves as a new (reduced) sample space, and that probability is the fraction of $P(A)$ which corresponds to $A \cap B$:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad [P(A) \neq 0]$$

Similarly, the conditional probability of A given B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad [P(B) \neq 0]$$

Solving these two equations for $P(A \cap B)$ gives:

Theorem: Multiplication rule: If A and B are events in a sample space S and $P(A) \neq 0$, $P(B) \neq 0$ then:

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$