Space-Time Block Coding in Wireless communications

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Objectives

(1) To understand principles of MIMO and STBCs
(2) To study detection methods of MIMO
(3) To learn performance evaluation methods
(4) To analyze STBCs performance
(5) To acheive a good performance

Background

(1) MIMO

Channel Model:

$$\begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[N] \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,N} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,N} \end{bmatrix} \times \begin{bmatrix} s[1] \\ s[2] \\ \vdots \\ s[N] \end{bmatrix} + \begin{bmatrix} v[1] \\ v[2] \\ \vdots \\ v[N] \end{bmatrix}$$

Which can be simplified:

$$\vec{y} = \boldsymbol{H}\vec{s} + \vec{v}$$

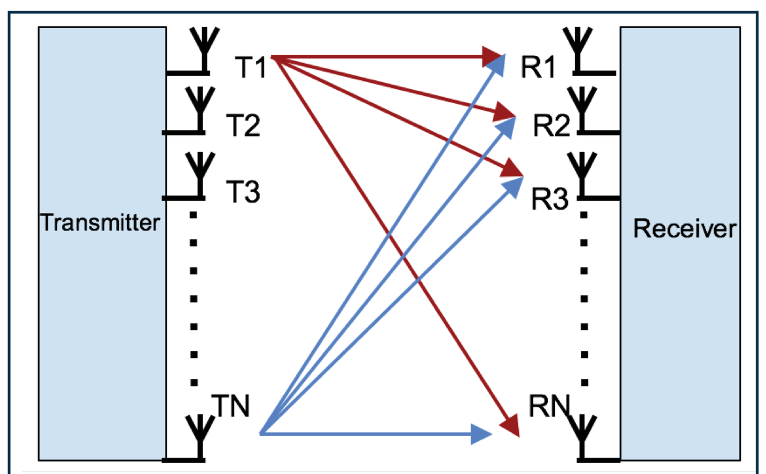

Figure 1: N×N MIMO System Model

At the receiver side, the problem is how to extract the original symbol back. ZF and ML are the ways.

(2) Zero-Forcing Detector

ZF detector objects to reduce channel effect by multiplying the inverse of the channel to $\boldsymbol{y}$:

$$\boldsymbol{H}^{-1}\vec{y} = \boldsymbol{H}^{-1}(\boldsymbol{H}\vec{s} + \vec{v}) = \boldsymbol{I}\vec{s} + \boldsymbol{H}^{-1}\vec{v} \cong \boldsymbol{I}\vec{s}$$

as if noise is small enough.

(3) Maximum Likelihood Detector

ML detector compares the sum-square value of potential noise term $\boldsymbol{v}$, and get the "best guess" of $\boldsymbol{s}$, where in 4×4 case:

$$\vec{s} \in \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

ML detector computes 16 values of sum-squares:

$$\|\vec{y} - \boldsymbol{H}\overrightarrow{s_{det}}\| = \sum_{i=1}^{4} \left|\vec{y} - \boldsymbol{H}\overrightarrow{s_{det}}_i\right|^2 = \sum_{i=1}^{4} |\vec{v}_i|^2$$

as if $\boldsymbol{v}$ is small enough, the above term approaches 0 when $\boldsymbol{s}_{det}$ is the original transmitted symbol.

Space-Time Block Codes

STBCs' idea is to send redundant copies of s by multiple antennas. It does not gain in transmission rate and may cause a slowdown in fact.

(1) Alamouti Code

Alamouti Code encodes symbols s_1 and s_2 in an order-2 form:

$$\boldsymbol{C}(\vec{s}) = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$

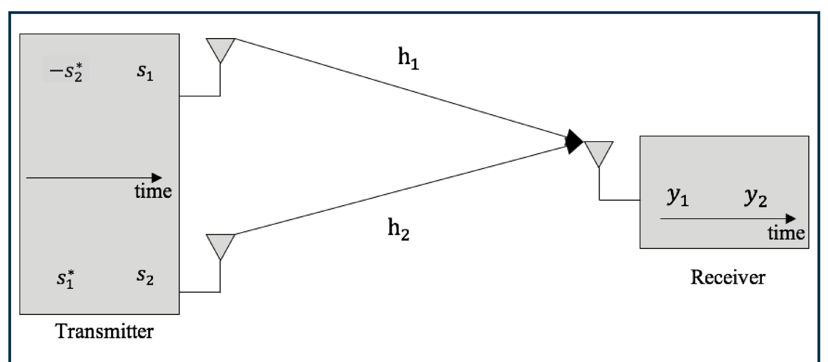

Figure 2: Alamouti Code System Model

and the transmission model is

$$\vec{y} = \boldsymbol{C}(\vec{s}) \times \vec{h} + \vec{v} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Received vector $\boldsymbol{y}$ is obtained over the time for one block transmission.

(2) Orthogonal STBC

Higher order STBCs were induced as inspired by Alamouti Code. While orthogonality is maintained, code rate is traded off:

$$\boldsymbol{C}_{4,rate=\frac{3}{4}}(\vec{s}) = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{bmatrix}$$

To compare this class of code with other schemes, special symbols are generated to ensure bits/code is fair:

(3) Quasi-OSTBC

The idea of QSTBC is simply extending Alamouti code into order 2^k:

$$\boldsymbol{C}_{QOSTBC}^{4} = \begin{bmatrix} \boldsymbol{C}_{Alamouti}(s_1, s_2) & \boldsymbol{C}_{Alamouti}(s_3, s_4) \\ -\boldsymbol{C}_{Alamouti}(s_3, s_4)^* & \boldsymbol{C}_{Alamouti}(s_1, s_2)^* \end{bmatrix}$$

is the order-4 codeword. And the order-8 codeword is then:

$$\boldsymbol{C}_{QOSTBC}^{8} = \begin{bmatrix} \boldsymbol{C}_{QOSTBC}^{4}(s_1, s_2, s_3, s_4) & \boldsymbol{C}_{QOSTBC}^{4}(s_5, s_6, s_7, s_8) \\ -\boldsymbol{C}_{QOSTBC}^{4}(s_5, s_6, s_7, s_8)^* & \boldsymbol{C}_{QOSTBC}^{4}(s_1, s_2, s_3, s_4)^* \end{bmatrix}$$

which can always acheive full rate!

(4) DAST

Diagonal-Algebraic Space-Time encodes symbols to each signal to be transmitted, so that channel attenuation cannot ruins the performance. The DAST codeword $\boldsymbol{X}$ is:

$$\vec{\boldsymbol{X}} = \mathrm{diag}(\boldsymbol{U}\vec{s}) = \begin{bmatrix} f_1(s_1, s_2, s_3 \dots s_N) & 0 & \cdots & 0 \\ 0 & f_2(s_1, s_2, s_3 \dots s_N) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f_N(s_1, s_2, s_3 \dots s_N) \end{bmatrix}$$

Where rotation matrix $\boldsymbol{U}$ is defined as:

$$\boldsymbol{U}_N = \boldsymbol{U}_N(\omega_1, \omega_2 \dots \omega_N) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega_1 & \omega_1^2 & \cdots & \omega_1^{N-1} \\ 1 & \omega_2 & \omega_2^2 & \cdots & \omega_2^{N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \omega_N & \omega_N^2 & \cdots & \omega_N^{N-1} \end{bmatrix} \quad \text{with } \omega_i = e^{j\frac{4i-3}{2N}\pi}$$

Simulation Results

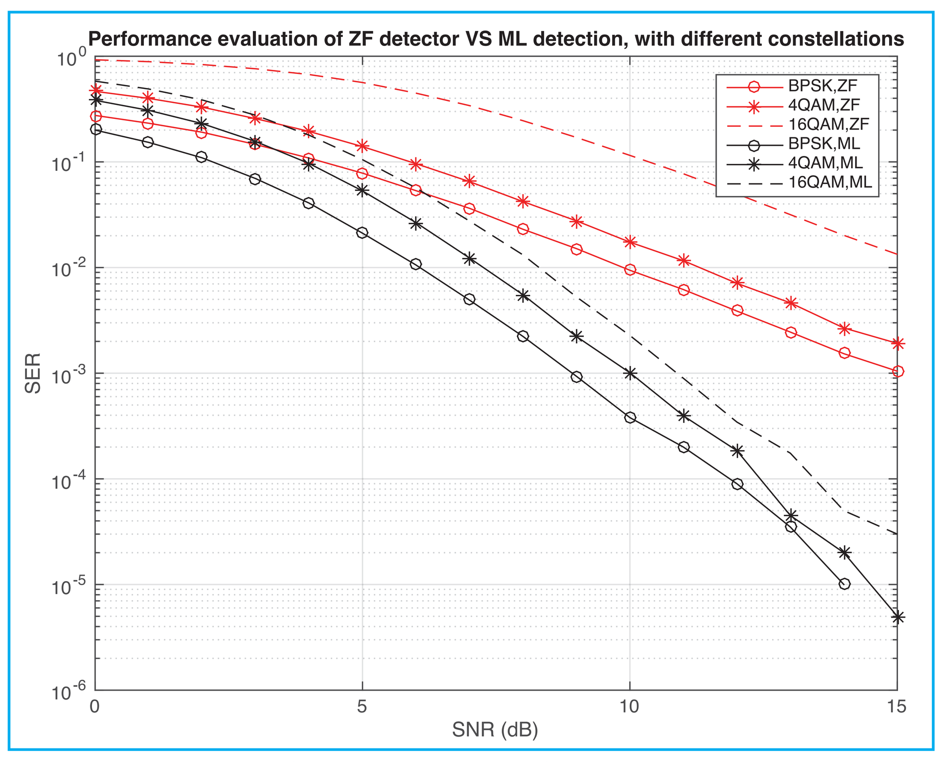

Figure 3: Performance evaluation of ZF and ML detector

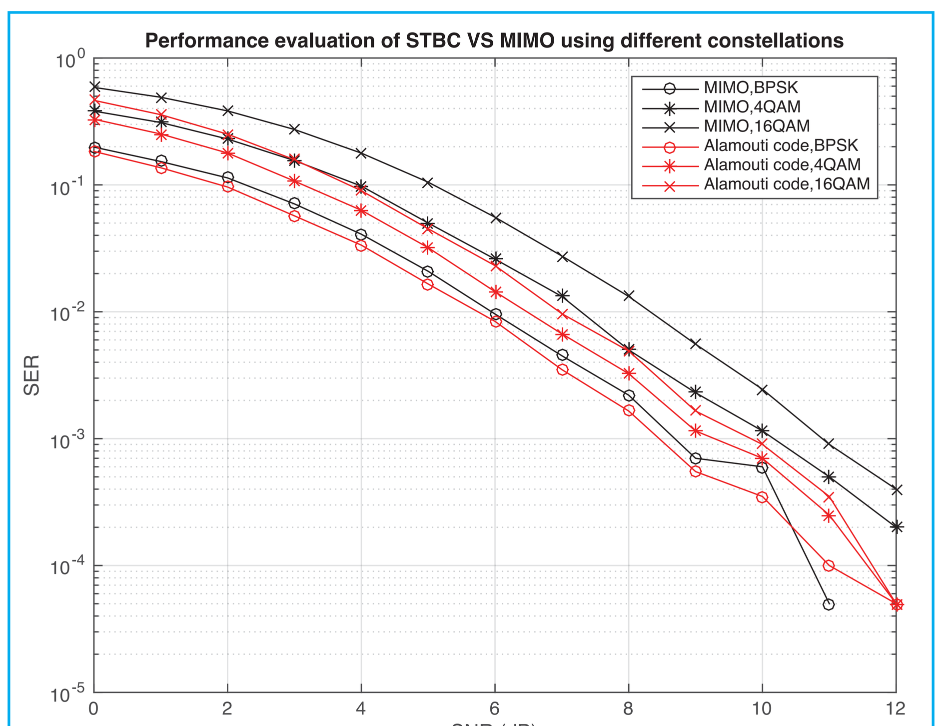

Figure 4: Performance evaluation of Alamouti Code and 2x2 MIMO

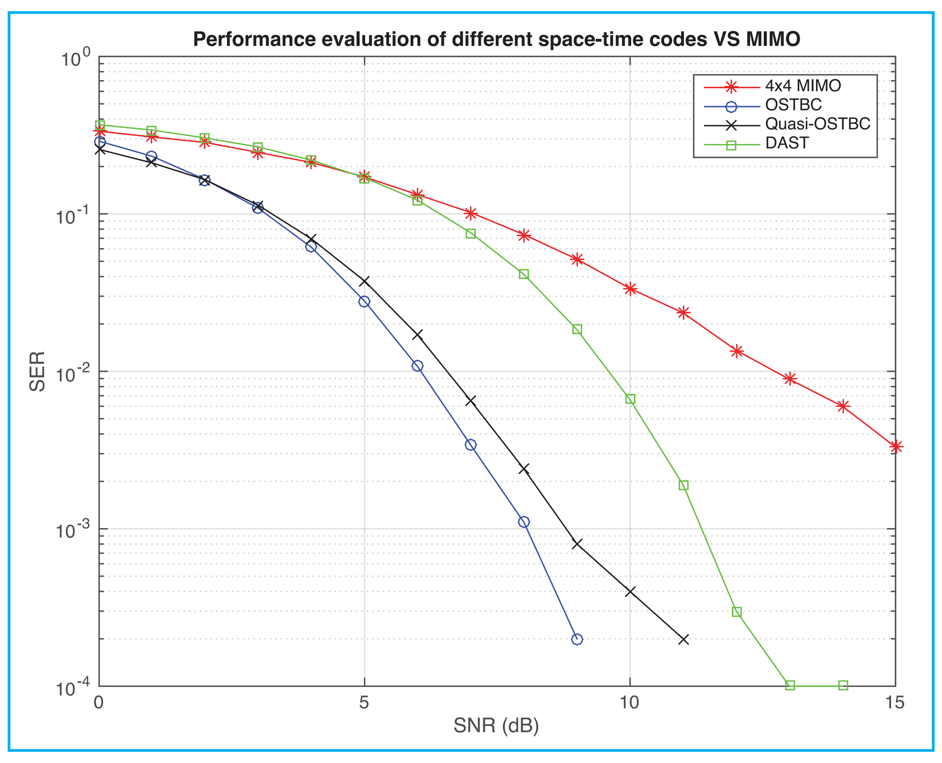

Figure 5: Performance evaluation of different STBCs under order-4 case

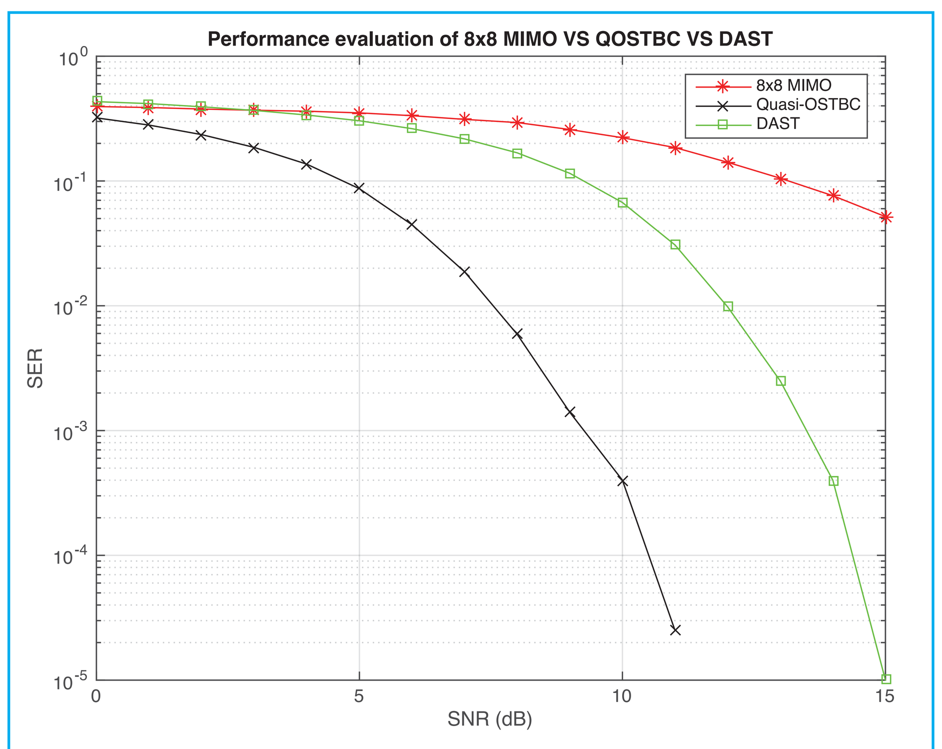

Figure 6: Performance evaluation of different STBCs under order-8 case

Conclusion

(1) MIMO and 4 types of STBCs examined
(2) ML performs better than ZF detector
(3) Monte-Carlo method is used in simulations
(4) Alamouti's code is of the best performance
(5) STBCs are always better than MIMO
(6) OSTBC peforms the best in error rate, but cannot acheive full rate
(7) QSTBC acheives full rate, but not full diversity even at high SNR range
(8) DAST demonstrates a "sharp-cut" at high SNR range, more obvious with higher order
(9) Code rate, diversity, performance and complexity are the 4 most essential parameters to evaluate STBCs
(10) Not about the best, but the most suitable

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